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Construction of strongly regular graphs with $d = q$ from block designs

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Abstract

In this paper we give one more method of construction of strongly regular graphs (v', d, p, q) having $d = q$, from existing block designs. The block designs considered here are balanced incomplete block designs and partially balanced incomplete block designs. It is seen that block intersection graphs of these designs yield strongly regular graphs with $d = q$ and chromatic number of these strongly regular graphs are the chromatic index of block designs from which these graphs have been obtained.

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1 Introduction

Study of block designs dates back to the middle of nineteenth century with the work by Kirkman and Steiner, although Euler studied Latin squares in late eighteenth century. Main focus in combinatorial design theory has been in proving the existence of different types of designs. Designs find their applications mainly in the field of statistics and applied statistics. Balanced incomplete block designs are used in designing of statistical experiments where each element represents an item in the experiment and each block represents a test involving elements it contain. An entire design can be considered as an experiment where every pair of elements is in exactly one test together. If two tests have no

common elements, then they can be performed simultaneously. Whenever group tests are performed together, it is advantageous when time taken for tests is minimized. This can be viewed by considering chromatic number of designs block intersection graph which has to be minimum. Doctoral thesis by D. R. Hare [10] investigates graph theoretic properties of block intersection graphs which arise from combinatorial design theory. A detailed proof for different properties of block intersection graphs arising from pairwise balanced designs such as hamiltonicity, connectivity, chromatic number is given in [10]. Though computing chromatic number of any arbitrary graph is NP-hard problem, graph coloring finds various applications in day-to-day problems like data mining, image segmentation, clustering, image capturing, networking etc. The most important concept of graph coloring is utilized in resource allocation and scheduling [15]. In [8], Forbes, Grannell and Griggs studied block intersection graph of Steiner triple systems. In [2] Graham is attributed to having posed the problem whether $STS(v)$ block intersection graphs have Hamiltonian cycles. This was affirmatively answered in [2] and [11]. In addition to being Hamiltonian, block intersection graphs of BIB-designs are also pancyclic [14]. It is also shown that they are cycle extendable [1]. In [6], authors have found chromatic index of block intersection graphs of Kirkman triple systems and cyclic Steiner triple systems by considering different cases. Authors in [9] have discussed certain conditions for combinatorial designs to give strongly regular block graphs. As block intersection graphs of all designs are not strongly regular, in this paper we have tried to find which class of block designs give strongly regular block intersection graphs (v', d, p, q) with $d = q$. We have obtained three classes of block designs, BIB-designs of the form $(s^2, s^2 + s, s + 1, s, 1)$ where s is a prime or power of prime, $\left(s, 2s, s-1, \frac{s-1}{2}, \frac{s-3}{2}\right)$ where s is a prime greater than 5 and symmetric group divisible PBIB-designs having parameters $(s^2, s^2, s, s, 0, 1)$ where s is a prime or power of prime. Block intersection graphs of these designs are found to be strongly regular with $d = q$. The paper is organized into following sections. First section gives a brief history on designs and block intersection graphs, followed by second section which incorporates some basic definitions. Results are listed as theorems and hence proved in third section which is succeeded by conclusion and then references.

2 Preliminaries

Let us first consider some basic definitions. Undefined graph theoretical terms are used in the sense of Buckley and Harary [4].

Definition 2.1. *[4] A d-regular graph of order v' is a strongly regular graph with parameter set (v', d, p, q), if every pair of adjacent vertices have p common neighbours and every pair of non-adjacent vertices have q common neighbours.*

Remark 1. *[4]*

(i) All connected strongly regular graphs have diameter 2.

(ii) If G is a strongly regular graph with parameters (v', d, p, q), then $(v' - d - 1)q = d(d - 1 - p)$.

Definition 2.2. *[7] A balanced incomplete block (BIB)-design is a set of v elements arranged in b blocks of k elements each in such a way that each element occurs in exactly r blocks and every pair of unordered elements in λ blocks. The combinatorial configuration so obtained is called a (v, b, r, k, λ)-design. A BIB-design satisfies the following conditions.*
i) $vr = bk$ ii) $\lambda(v - 1) = r(k - 1)$ iii) $b \geq v$

Definition 2.3. *[12] Given a set $\{1, 2, 3, \ldots, v\}$ of v elements, a relation satisfying the following conditions is said to be an association scheme with m classes.*

- *Any two elements α and β are i^{th} associates for some i with $1 \leq i \leq m$ and this relation of being i^{th} associates is symmetric.*
- *The number of i^{th} associates of each element is n_i.*
- *If α and β are two elements which are i^{th} associates, then the number of elements which are j^{th} associates of α and k^{th} associates of β is p^i_{jk} and is independent of the pair of i^{th} associates α and β.*

Definition 2.4. *[17] [3] Consider a set $V = \{1, 2, \ldots, v\}$ and an association scheme with m classes on V. A partially balanced incomplete block (PBIB)-design represented as $(v, b, r, k, \lambda_1, \ldots, \lambda_m)$ is a collection of b subsets of V called blocks, each of them containing k elements $(k < v)$ such that every element occurs in r blocks and any two elements α and β which are i^{th} associates occur together in λ_i blocks, the number λ_i being independent of the choice of the pair α and β. The numbers $v,\ b,\ r,\ k,\ \lambda_i$ $(i = 1, 2, \cdots, m)$ are called the parameters of first kind and $n'_i s$ and p^i_{jk} are called the parameters of second kind.*

Definition 2.5. *[5] A balanced design is said to be symmetric if $v = b$ and hence $r = k$.*

Definition 2.6. *[16] When v elements are divided into m groups of size n each, then any two elements from the same group are called first associates and any two elements from different groups are called second associates. Such an association scheme is called Group Divisible association scheme. Notice that each element has $n - 1$ first associates and $n(m - 1)$ second associates.*

Definition 2.7. *[16] A PBIB-design with a group divisible association scheme with m groups of size n is denoted by $GDD(n, m, k, \lambda_1, \lambda_2)$, if the blocks are of size k, the first associate pairs occur in λ_1 blocks and the second associate pairs occur in λ_2 blocks.*

Definition 2.8. *[6] Given a block design $(\mathcal{V}, \mathcal{B})$, its associated block intersection graph is the graph on vertex set $\mathcal{B}$, of blocks, for which two vertices or blocks B_1 and B_2 are adjacent if and only if $|B_1 \cap B_2| \geq 1$.*

Definition 2.9. *[18] A vertex coloring is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices. Coloring which seeks minimum number of colors is known as minimum vertex coloring and the minimum number of colors with which the vertices of the graph G may be colored is called its chromatic number denoted as $\chi(G)$.*

Definition 2.10. *[13] Given a design $\mathcal{D} = (\mathcal{V}, \mathcal{B})$, a block coloring of $\mathcal{D}$ is a mapping $\psi : \mathcal{B} \to \mathcal{C}$ such that if $\psi(B) = \psi(B')$ where $B \neq B'$ and $B, B' \in \mathcal{B}$ then $B \cap B' = \phi$ where $\mathcal{C}$ is the set of colors. If for the set of colors $\mathcal{C}$ we have $|\mathcal{C}| = q$, then ψ is called a q-block coloring. The chromatic index of $\mathcal{D}$, $\chi'(\mathcal{D})$, is the smallest q such that there exists a q-block coloring of $\mathcal{D}$.*

Remark 2. *Clearly the chromatic index of a design $\mathcal{D}$ is the chromatic number of its block intersection graph G, that is, $\chi'(\mathcal{D}) = \chi(G)$.*

3 Results

In this section we first construct strongly regular graphs satisfying a condition $d = q$, from block intersection graphs of different designs and then find their chromatic index. First result deals with construction of a strongly regular graph.

Theorem 3.1. *Block intersection graph of balanced incomplete block (BIB)- design having parameters (v, b, r, k, λ) = $(s^2, s^2 + s, s + 1, s, 1)$ where s is a prime or power of prime, is strongly regular graph having parameters $(s^2 + s, s^2, s^2 - s, s^2)$.*

Proof. Consider the class of BIB-designs having parameters (v, b, r, k, λ)=$(s^2, s^2+s, s+1, s, 1)$ where s is a prime or power of prime. Clearly, there are $s^2 + s$ blocks such that any two elements of the design belong to exactly one block as λ is 1. Hence block intersection graph G of this design has $s^2 + s$ vertices. From definition of block intersection graphs, any pair of vertices of graph G are adjacent if the corresponding blocks have non-empty intersection. Since λ is 1, intersection of any two blocks will have atmost one element. Consider any arbitrary block x containing an element, say, a. Since repetition number of the design is r, there are $r - 1$ more blocks containing element a that is adjacent to block x. Since a is any arbitrary element of the block and block size is k, there are $k(r - 1)$ vertices adjacent to x. Substituting values of k and r from design parameters, we get the value as s^2. Hence block intersection graph of this design is a regular graph with regularity s^2. Consider any two adjacent vertices x and y in G, that is, two blocks whose intersection is non-empty. Then vertices that are adjacent to both x and y are such that the corresponding

blocks have non-empty intersection with both x and y. There are $r-2$ blocks containing the element which is common in both x and y. Remaining blocks adjacent to both x and y are such that they contain exactly one element each from both x and y. Hence there are $(k-1)^2$ such blocks. Thus, number of vertices adjacent to both x and y where x and y are neighbours are $(r-2)+(k-1)^2$. Substituting for r and k from design parameters, we get the value as s^2-s. Let x and z be a pair of non-adjacent vertices in G, that is, two blocks whose intersection is empty. Now we count the number of blocks adjacent to both x and z. Vertices adjacent to both x and z will be such that the corresponding blocks have exactly one element each from both x and z. Calculating on similar lines as above, we get the number of such vertices as k^2. Substituting value of k from design parameters, we get the value as s^2. Hence, block intersection graph of the above design is strongly regular with srg parameters (v', d, p, q)= (s^2+s, s^2, s^2-s, s^2). □

Remark 3. *For any strongly regular graph with parameters (v', d, p, q), their complements have parameters $(v', v'-d-1, v'-2-2d+q, v'-2d+p)$. Thus, complements of strongly regular graph having parameters (s^2+s, s^2, s^2-s, s^2) is $(s^2+s, s-1, s-2, 0)$. That means any pair of non-adjacent vertices have no common neighbours and any pair of adjacent vertices have $s-2$ common neighbours. Hence, complements of above strongly regular graphs are disconnected containing $s+1$ copies of complete graphs on s vertices.*

Next we give a result on chromatic number of block intersection graph of BIB-design given in Theorem 3.1 which is evident from the proof of Theorem 3.1. It is well known that finding chromatic number of any arbitrary graph is NP-hard problem. Hence, we approach the following result is another way. From Remark 2 it is clear that chromatic number of block intersection graph of a design is equal to the chromatic index of its design. So in order to find the chromatic number of block intersection graph of a design obtained in the above Theorem 3.1, we determine chromatic index of its design $\mathcal{D} = (s^2, s^2+s, s+1, s, 1)$.

Corollary 3.1. *Chromatic number of block intersection graph of balanced incomplete block design having parameters $(s^2, s^2+s, s+1, s, 1)$ is $s+1$, where s is a prime or power of prime.*

Proof. Since repetition number of the design is $s+1$, chromatic index should be at least $s+1$. From Theorem 2.4 given in [13] $\left(\text{which states that If } \mathcal{D} \text{ is a } \mathcal{B}(k, \lambda; v), \text{ then } \chi'(\mathcal{D}) \leq \frac{\lambda k(v-1)}{k-1} - \lambda(k-1)\right)$, on substituting the design parameters we get $\chi'(\mathcal{D}) \leq s^2+1$. Thus, we get bounds for chromatic index of the above class of BIB-design as $s+1 \leq \chi'(\mathcal{D}) \leq s^2+1$. Consider any arbitrary element a in block say x. There are $s+1$ blocks containing a each of size s. These blocks are colored by $s+1$ distinct colors say $c_1, c_2, c_3, \ldots, c_{s+1}$. Now consider a block y not containing the element a. Since λ is 1 and size of the blocks are s, block y is formed by picking one element each from s blocks containing a. Hence this block y can be colored by the remaining color that is used to color the blocks containing a. Thus we can always get a color $c_i, 1 \leq i \leq s+1$ which can be used to color block y. Since y is arbitrary, this is true for all blocks. Therefore, chromatic index of design $\mathcal{D}$ is $s+1$ and hence chromatic number of block intersection graph G of $\mathcal{D}$ is $s+1$. □

Example 3.1. *Following table consists of parameters of some BIB-designs of the form given in Theorem 3.1 and strongly regular graph arising from block intersection graph G of these designs along with chromatic number of G.*

Table 1: Parameters of selected BIB-designs, the corresponding strongly regular graph arising from their block intersection graph G, and the chromatic number of G.

BIBD $(s^2, s^2+s, s+1, s, 1)$	*SRG* (s^2+s, s^2, s^2-s, s^2)	*Chromatic number of* G $\chi(G)$
(4,6,3,2,1)	*(6,4,2,4)*	*3*
(9,12,4,3,1)	*(12,9,6,9)*	*4*
(16,20,5,4,1)	*(20,16,12,16)*	*5*
(25,30,6,5,1)	*(30,25,20,25)*	*6*
(49,56,8,7,1)	*(56,49,42,49)*	*8*

Next class of PBIB-designs can be obtained from BIB-designs given in Theorem 3.1. Elements of the above BIB-design can be arranged in s blocks each of size s. This can be considered as one set. Similarly there are $s+1$ sets as repetition number of the design is $s+1$. Removing one of the set of s blocks results in PBIB-design with parameters given in Theorem 3.2. We can observe that resulting PBIB-design is symmetric as well as group divisible. Taking block intersection graph of this design also yields a strongly regular graph. Hence Theorem 3.2 is stated as follows.

Theorem 3.2. *Block intersection graph of symmetric group divisible partially balanced incomplete block (PBIB)-design having parameters $(s^2, s^2, s, s, 0, 1)$ where s is a prime or power of prime is strongly regular graph having parameters $(s^2, s^2-s, s^2-2s, s^2-s)$.*

Proof. Consider the class of PBIB-designs having parameters $(v, b, r, k, \lambda_1, \lambda_2) = (s^2, s^2, s, s, 0, 1)$ where s is prime or power of prime. This is also a symmetric group divisible design where we can group s^2 elements of the design in an $s \times s$ array. Since $\lambda_1 = 0$, no two elements from the same group (row/column) appear together in any block and any pair of elements from different blocks appear in exactly one block as $\lambda_2 = 1$. Consider the block intersection graph G of this design. Clearly from design parameters, there are s^2 number of vertices in G corresponding to each block. Since $\lambda_2 = 1$, intersection of any two blocks can have atmost one element. Consider an arbitrary block x containing an element, say, a. There are $r-1$ more blocks containing the element a and since a is arbitrary and block size is k, there are $k(r-1)$ number of blocks adjacent to x. Substituting values of k and r from design parameters, we get the number of neighbours of a vertex in G as s^2-s. Hence, G is a regular graph with regularity s^2-s. Let x and y be a pair of adjacent vertices, that is, two blocks whose intersection has exactly one element. Then, blocks that are adjacent to both x and y have non-empty intersection with both x and y. There are $r-2$ blocks containing the element which is common in both x and y. Remaining blocks adjacent to both x and y are such that they should contain exactly

one element each from both x and y. Hence there are $(k-1)^2$ possibilities of choosing such pairs of vertices. Since $\lambda_1 = 0$, amongst these $(k-1)^2$ pairs, $k-1$ pairs of elements belong to the same group which do not appear together in any of the blocks. Hence, the number of blocks adjacent to both x and y are $(r-2)+(k-1)^2-(k-1)$. Substituting the values of k and r from design parameters, we get number of adjacencies for a pair of adjacent vertices in G as s^2-2s. Now consider a pair of non-adjacent vertices, say x and z, that is, two blocks whose intersection is empty. We count the number of vertices adjacent to them. Vertices adjacent to both x and z will be such that the corresponding blocks will have exactly one element each from both x and z. Calculating similarly as above, there are k^2 possibilities of choosing such pairs of elements so that one is from each block x and z. Since $\lambda_1 = 0$, there are k pairs of elements that belong to the same group and do not appear together in any block. Hence, number of vertices adjacent to both x and z are $k^2 - k$. Substituting the value of k from design parameters, we get number of adjacencies for a pair of non-adjacent vertices as $s^2 - s$. Thus, block intersection graph of the above design is strongly regular with srg parameters (v', d, p, q) = $(s^2, s^2 - s, s^2 - 2s, s^2 - s)$. □

Remark 4. *The complements of the above strongly regular graphs are disconnected containing s copies of complete graphs on s vertices since the complements of these graphs have parameters $(s^2, s^2 - s - 1, s - 2, 0)$.*

Corollary 3.2. *Chromatic number of block intersection graph of symmetric group divisible partially balanced incomplete block design having parameters $(s^2, s^2, s, s, 0, 1)$ is s where s is a prime or power of prime.*

Proof. Let us find chromatic index of design $\mathcal{D} = (s^2, s^2, s, s, 0, 1)$. Since repetition number of the design is s, chromatic index of $\mathcal{D}$ is at least s. Consider an element a in block x. There are s blocks containing element a each of size s. These blocks can be colored using s distinct colors say $c_1, c_2, c_3, \cdots, c_s$. Since $\mathcal{D}$ is group divisible design and $\lambda_1 = 0$, s^2 blocks can be grouped into s groups each containing s blocks such that blocks contained within each group have empty intersection with each other. Hence, all these blocks can be colored using same color. Thus we get the chromatic index of the design $\mathcal{D}$ as s. From Remark 2, it follows that chromatic number of block intersection graph G of the design $\mathcal{D}$ is s. □

Example 3.2. *Now we give parameters of some designs of the form given in Theorem 3.2 and strongly regular graph arising from block intersection graph of these designs along with chromatic number of G.*

Table 2: Parameters of selected PBIB-designs, the corresponding strongly regular graph arising from their block intersection graph G, and the chromatic number of G.

Symmetric PBIBD $(s^2, s^2, s, s, 0, 1)$	*SRG* $(s^2, s^2 - s, s^2 - 2s, s^2 - s)$	*Chromatic number of* G $\chi(G)$
(4,4,2,2,0,1)	*(4,2,0,2)*	*2*

Continued on next page

Symmetric PBIBD $(s^2, s^2, s, s, 0, 1)$	*SRG* $(s^2, s^2 - s, s^2 - 2s, s^2 - s)$	*Chromatic number of* G $\chi(G)$
(9,9,3,3,0,1)	*(9,6,3,6)*	*3*
(16,16,4,4,0,1)	*(16,12,8,12)*	*4*
(25,25,5,5,0,1)	*(25,20,15,20)*	*5*
(49,49,7,7,0,1)	*(49,42,35,42)*	*7*

Theorem 3.3. *Block intersection graph of balanced incomplete block (BIB)-design having parameters* $\left(s, 2s, s-1, \frac{s-1}{2}, \frac{s-3}{2}\right)$ *where* $s > 5$ *is a prime is strongly regular graph having parameters* $(2s, 2s-2, 2s-4, 2s-2)$.

Proof. Consider the class of BIB-designs having parameters (v, b, r, k, λ)=$\left(s, 2s, s-1, \frac{s-1}{2}, \frac{s-3}{2}\right)$ where $s > 5$ is a prime. From design parameters it is clear that there are $2s$ blocks such that any two elements of the design belong to exactly $\frac{s-3}{2}$ blocks. Clearly block intersection graph G of this design has $2s$ number of vertices corresponding to each block. Any two vertices are adjacent if the corresponding blocks have non-empty intersection. Let x be any arbitrary vertex/block. Since block size is $k = \frac{s-1}{2}$, remaining $v - k$ elements are not in x. Substituting the design parameters for v and k, we get $\frac{s+1}{2}$ number of elements that are not present in block x. Clearly there is exactly one block that has all elements different from that in x which will be non-adjacent to x. Hence block intersection graph G has regularity $v-2$. On substituting design parameters for v we get regularity of G as $2s-2$. Thus every vertex in G has a unique eccentric vertex. Consider a pair of adjacent vertices, say, x and y in G. Then the corresponding blocks will have non-empty intersection. Since G is a unique eccentric vertex graph, eccentric vertices of x and y are not same. Hence, eccentric vertex of x is adjacent to y and vice versa. Thus, number of vertices adjacent to both x and y are $v' - 4$ where v' is order of graph G. On substituting we get the number of vertices adjacent to any pair of adjacent vertices as $2s-4$. Let x and z be a pair of non-adjacent vertices in G. Clearly, remaining $v'-2$ vertices are adjacent to both x and z. Hence any pair of non-adjacent vertices have $2s-2$ number of adjacent vertices. Thus, block intersection graph G of the above design is strongly regular with srg parameters $(v', d, p, q) = (2s, 2s-2, 2s-4, 2s-2)$. □

Remark 5. *Complements of strongly regular graphs having parameters* $(2s, 2s-2, 2s-4, 2s-2)$ *are disconnected having* s *copies of complete graphs on 2 vertices having parameters* $(2s, 1, 0, 0)$.

Corollary 3.3. *Chromatic number of block intersection graph of balanced incomplete block design having parameters* $\left(s, 2s, s-1, \frac{s-1}{2}, \frac{s-3}{2}\right)$ *is* s*, where* $s > 5$ *is a prime.*

Proof. Let us find chromatic index of balanced incomplete block design $\mathcal{D} = \left(s, 2s, s-1, \frac{s-1}{2}, \frac{s-3}{2}\right)$. Since repetition number of the design is $s-1$, chromatic index should be at least $s-1$. From Theorem 2.4 [13], we get the bounds for the chromatic index as $(s-1) \leq \chi'(\mathcal{D}) \leq \frac{s^2-2s-7}{4}$. Consider any arbitrary block x. Since there

are s elements, and block size is $\left(\frac{s-1}{2}\right)$, there is exactly one block that has no element in common with x. Hence these two blocks can be colored with same color. Since x is arbitrary, there are just two blocks that are colored with the same color. As design $\mathcal{D}$ has $2s$ blocks, we can color the blocks using s distinct colors so that no two blocks having non-empty intersection get the same color. Thus, chromatic index of design $\mathcal{D}$ is s. From Remark 2, chromatic number of block intersection graph of design $\mathcal{D}$ is s. □

Example 3.3. *Here we give parameters of some BIB-designs of the form given in Theorem 3.3 and strongly regular graphs arising from block intersection graph of these designs along with chromatic number of G.*

Table 3: Parameters of selected BIB-designs, the corresponding strongly regular graph arising from their block intersection graph G, and the chromatic number of G.

BIBD $\left(s, 2s, s-1, \frac{s-1}{2}, \frac{s-3}{2}\right)$	*SRG* $(2s, 2s-2, 2s-4, 2s-2)$	*Chromatic number of G* $\chi(G)$
(7,14,6,3,2)	*(14,12,10,12)*	*7*
(11,22,10,5,4)	*(22,20,18,20)*	*11*
(13,26,12,6,5)	*(26,24,22,24)*	*13*
(17,34,16,8,7)	*(34,32,30,32)*	*17*
(19,38,18,9,8)	*(38,36,34,36)*	*19*

4 Conclusion

Designs find many applications in real life situations. In the field of statistical experiments, one way of testing whether a design is optimal can be done by finding the chromatic number of its block intersection graph. A design is useful if chromatic number of its block intersection graph is minimum. Here we have given one of the constructions for a class of strongly regular graphs satisfying the condition $d = q$. We have considered two classes of balanced incomplete block designs and one class of partially balanced incomplete block design, and obtained their block intersection graphs. It is observed that these are strongly regular graphs (v', d, p, q) satisfying $d = q$. We have also obtained chromatic index of these designs which is found to be the chromatic number of their block intersection graphs. This work is important because it builds an interesting connection between combinatorial design theory and graph theory, offering a clear and systematic way to create strongly regular graphs from existing block designs. By doing so, it not only deepens our understanding of strongly regular graphs but also broadens their practical uses in areas like coding theory, network design and cryptography, where balanced and well-connected structures are essential. In addition to that, the finding that links chromatic number of the block intersection graph to the chromatic index of the original block designs adds a fresh perspective, helping researchers explore more efficient methods for coloring and optimizing

complex networks and systems.

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