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MILMAN'S QUESTION CONCERNING QUASIREPRESENTATIONS OF THE GROUPS $SO(2n+1,\ n \geq 1$

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Abstract. We present a short elementary proof of the answer in the affirmative of "a half" of the Milman question concerning quasirepresentations of the groups $SO(2n+1)$, $n \geq 1$.

§ 1. Introduction

I his 1982 paper [1], Kazhdan formulated a question posed to him by Milman as follows. "V. Milman asked me the following question: Let $\rho\colon \mathrm{O}(n) \to \mathrm{O}(N)$ be a map which is "almost" a representation, that is,

$$|\rho(gg') - \rho(g)\rho(g')|$$

is small for all $g, g' \in \mathrm{O}(n)$. Is it true that ρ is near to an actual representation of $\mathrm{O}(n)$?"

An affirmative answer to this question was obtained for semisimple connected compact Lie groups [2]. However, the group $\mathrm{O}(n)$ is only a finite extension of the semisimple connected compact Lie group $\mathrm{SO}(n)$.

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The main objective of the present note is to give a short elementary proof of the fact that the answer to the question for the group $SO(2n+1)$, $n \geq 1$, is also affirmative.

§ 2. Preliminaries

For the fundamentals concerning quasirepresentations of topological groups, see [2]–[4].

Since an ordinary continuous representation of a compact topological group G is bounded, it follows that the necessary condition for the existence of a uniformly close ordinary representation to a given quasirepresentation of G is that this quasirepresentation be bounded.

§ 3. Main Theorem

Theorem. *Every bounded quasirepresentation of $O(2n+1)$, $n \geq 1$, in a finite-dimensional Hilbert space H is uniformly close to an ordinary continuous representation of $O(2n+1)$ in H, where the closeness depends on the defect of the quasirepresentation and the supremum of the norms of the quasirepresentation operators and tends to zero as the supremum of the norms of the quasirepresentation operators is bounded and the defect tends to zero.*

proof. By Theorem 3.2.1 of [2], the restriction to $SO(2n+1)$ of a bounded quasirepresentation π of $O(2n+1)$ is uniformly close to some ordinary continuous representation of $SO(2n+1)$. Let L be the center of the group $O(2n+1)$. It consists of two elements, the identity element e of $SO(2n+1)$ and the diagonal element s with -1 on the main diagonal. Then, obviously, the formula

$$\sigma(g) = \begin{cases} \rho(g) & \text{if } g \in SO(2n+1), \\ \pi(s)\rho(sg) & \text{if } g \in sSO(2n+1), \end{cases}$$

is a continuous quasirepresentation of G which is obviously uniformly close to π and whose defect is small if the defect of π is. By the well-known property of continuous quasirepresentations, of compact topological groups, with small defect, there is a continuous ordinary representation of G close to σ [3], [4], as was to be proved.

§ 4. Comments

For the arbitrary finite extension of a compact connected semisimple Lie group, the assertion is proved in a similar way in [5].

§ 5. Concluding Remarks

It is obvious from the proof of Theorem 3.2.1 in [2] that the nature of the target group is inessential; in the theorem of this paper, this is simply a Lie group.

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