Advanced Studies in Contemporary Mathematics
36 (2026), No. 4, pp. 299 – 301
www.jangjeon.or.kr
http://dx.doi.org/10.17777/ascm2026.36.4.002

NONIDENTITY CONNECTED LIE GROUPS CANNOT HAVE COMPACT (NOT NECESSARILY HAUSDORFF) DUAL SPACES

A. I. Shtern

Abstract. We prove that a connected Lie group cannot have nonidentity compact (not necessarily Hausdorff) dual space.

§ 1. Introduction

We prove here that the dual spaces of nonidentity connected Lie groups [1] cannot have compact (not necessarily Hausdorff) dual spaces. This agrees with the fact that the known examples of nondiscrete locally compact groups [2] are nonconnected Lie groups and totally disconnected locally compact groups.

§ 2. Preliminaries

As is well known, a locally compact group with compact (not necessarily Hausdorff) dual space is discrete if all continuous irreducible unitary representations of the group are finite-dimensional.

For examples of nonconnected nondiscrete locally compact groups with compact dual spaces, see [3].

2020 *Mathematics Subject Classification.* Primary 22E45, Secondary 22E15.
Submitted July 23, 2026.
Key words and phrases. Dual space of a locally compact group, compact (not necessarily Hausdorff) dual space, connected Lie group..

Typeset by $\mathcal{AMS}$-TEX

§ 3. Main Results

Theorem 1. *A nonidentity connected Lie group cannot have a compact dual space.*

Proof. Let G be a nonidentity connected Lie group with compact dual space. Then the semisimple quotient Lie group also has a compact dual space. This is impossible if this quotient is not the identity group. Therefore, G is solvable and connected. Hence the connected commutative group of G has compact dual space. However, if this quotient is nonidentity, then it is a product of a finite-dimensional torus and a finite-dimensional vector space. The dual group of this product is not compact, since this product is not discrete (by the Pontryagin duality). Thus, every commutative quotient group of the solvable group G is the identity group, which means that G is the identity group. A contradiction.

Theorem 2. *A nonidentity connected locally compact group cannot have a compact dual space.*

Proof. Let G be a nonidentity connected locally compact group with compact dual space. Then G is the projective limit of the quotient groups of G by the compact normal subgroups for which the corresponding quotient group is a Lie group [4]. However, as shown above, these quotient groups are connected lie groups with compact dual space, and thus are the identity groups. Therefore, G is also the identity group. A contradiction.

§ 4. Discussion

A short proof shows that the assertion remains valid for almost connected locally compact groups.

This shows that connectivity of almost connectivity of a locally compact group is an obstacle for having a compact dual space.

Acknowledgments

I thank Professor Taekyun Kim for the invitation to publish this paper in the Advanced Studies of Contemporary Mathematics.

Funding

The research was supported by the Scientific Research Institute for System Analysis of the National Research Centre “Kurchatov Institute” according to the project FNEF-2024-0001.

References

1. J. Dixier, *Les C^*-algèbres et leurs représentations*, Éditions Jacques Gabay, Paris, 1996.
2. A. I. Shtern, *Finite-dimensional quasirepresentations of connected Lie groups and Mishchenko's conjecture*, J. Math. Sci. (N. Y.) **159** (2009), no. 5, 653–751.
3. A. I. Shtern, *Groups with bicompact dual space*, Uspekhi Mat. Nauk **26** (1971), no. 3, 217–218. (Russian)
4. D. Montgomery, L. Zippin, *Topological Transformation Groups*, Interscience Publishers, New York–London, 1955.

Moscow Center for Fundamental and Applied Mathematics, Moscow, 119991 Russia
Department of Mechanics and Mathematics,
Moscow State University,
Moscow, 119991 Russia
NRC "Kurchatov Institute" – SRISA,
Moscow, 117312 Russia
E-mail: aishtern@mtu-net.ru, rroww@mail.ru