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Introduction index as a new social topological graph index

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Abstract

Almost four thousands topological graph indices have been defined in the last 80 years some of which are used for several practical purposes and some have only mathematical beauty. The applications of such indices include chemistry, pharmacology, city planning, neuroscience, networks etc. In this paper, we introduce and investigate such a new topological graph index which has applications in social sciences and network studies. If two members in a network are acquainted and a new friend of one of these two members is joined to this group, then the common friend introduces the new member to the existing member so that all three members get to know each other. This idea is used to define the new graph called the introduction graph. Intuitionally, the introduction graph is obtained by adding a new edge between each vertex pair which have distance 2. More technically, it is obtained by replacing every path of length three by a triangle by adding the two endpoints of each such path. That way, two members knowing a common member become new friends. We study the fundamental properties of this graph including common graph parameters, metric and topological properties and some relations with topological graph indices.

Keywords: introduction graph, topological graph index

2000 Mathematics Subject Classification: 05C07, 05C10, 05C30, 68R10

1 Introduction

In this section, we recall some fundamental notions and related results about graphs from the literature. For a more detailed treatment of such notions, one may refer to [6].

Throughout this paper, we let $G = (V, E)$ be a finite, simple, connected and undirected graph with the vertex set $V(G) = \{v_1, v_2, \cdots, v_n\}$ and edge set $E(G) = \{v_i v_j : v_i, v_j \in V(G)\}$. Let $|V(G)| = n$ and $|E(G)| = m$ be the order and the size of G, respectively. The vertex degree of a vertex v is the number of edges meeting at v, and it is denoted by $d_G v$ or briefly by dv. If, in particular, v has only one adjacent vertex, then it is called a pendant vertex. Hence, a pendant vertex is a vertex of degree 1. Pendant vertices mostly appear in trees as they are connected acyclic graphs. Two vertices u and v are called adjacent if there is an edge e between them. In such a case, we denote this edge by $e = uv$. We also say that e is incident to u (and v). Adjacency and incidency are useful notions which help to transfer a graph into a computer program to be able to work on the given graph by much faster methods.

There are many different graph classes in literature and everyday, new ones are added to the list. Researchers choose the convenient class or classes for different purposes and studying their mathematical properties, they reach to the applications of graphs. We denote by P_n, C_n, K_n, S_n, $K_{r,s}$, $T_{r,s}$, W_n, L_n, W_a^b, P and CP_{2n} the path, cycle, complete, star, complete bipartite, tadpole, wheel, ladder, windmill, Petersen and cocktail graphs, respectively, which are usually the most frequently used graph classes.

Another important graph theoretical notion that will be needed here is the distance. Many notions in graph theory are related to either vertex degrees or distances. The distance between two vertices u and v is denoted by $d(u, v)$ and is defined to be the length of a shortest path between u and v.

The set of all vertices which are adjacent to a vertex v is called the (first) neighbourhood of v and usually denoted by $N_G(u)$ or $N(u)$. Actually, $N(u)$ can also be thought as the set of all vertices in G which are at distance 1 from u. In another words, the number of vertices in the first neighbourhood of v is equal to the degree of v. That is,

$N(u) = \{v \in V(G)|d(u,v) = 1\}$. Neighbourhoods are needed when the information obtained from the vertex itself are not sufficient enough and all the studies related to the vertex degrees can easily be extended to neighbourhoods.

Similary to the first neighbourhood of a vertex, other neighbourhoods can also be defined and used. For example, the second neighbourhood of v denoted by $N^2(v)$ is the set of vertices which are at distance 2 from v. That is,

$$N^2(v) = \{u \in V(G)|d(u,v) = 2\}.$$

The notion of second neighbourhood can also be used in defining the introduction graph of a graph. Actually, the square G^2 of any graph G is the graph with the same vertex set as G, in which two vertices are adjacent if their distance is at most 2 in G. For a given vertex $v \in V(G)$, the number of vertices u such that $d(u,v) = 2$ is the second neighbourhood degree of v. As a natural generalization, similarly to G^2, we can define G^k for an arbitrary positive integer k. The first and second neighbourhoods are important in the study of introduction graphs which are the essential notions in this paper.

In this paper, we shall introduce the introduction graph of a given graph. The idea of introducing members in a community who have not met before is a useful idea in network theory. In any network, the members which could be anything from neurons to planets, from people to countries, from companies to cities, could have been acquainted earlier or may not have any idea about each other. Introducing a new member to an existing network who knows some existing members in the network increases the effectiveness of the network and the interrelations between the members. But if the new member does not know any member in the network, then the connections between this new member and the network will be weaker. So the introduction graph structure will be an important notion in determining the strength of a given network.

2 Introduction graph as a new derived graph

Sometimes the graph we use to model a real life problem may not give us enough clues. In such cases, we go for some other graph which is deduced from the first graph according to some rule and is more informative. Such a graph is called a derived graph. Line, total, middle, subdivision, core, jump, image graphs are well-known examples of derived graphs. Next, using the ideas of first and second neighbourhoods of a vertex, we define the introduction graph as a new type of derived graph as follows:

Definition 2.1. *Given a graph G, the graph with the same vertex set with G obtained by adding a new edge between each pair (u, v) of vertices in G having distance $d(u, v) = 2$ is called the introduction graph of G and denoted by $IN(G)$.*

Let G be a graph. Note that by the definition, $IN(G)$ is the graph with the same vertex set with G where two non-adjacent vertices in G are joined with a new edge if the distance between them is 2. That is, to obtain $IN(G)$, one must add the edges between all pairs of vertices having distance 2 to the graph itself. Note that, by the definition of the introduction graph, we must have G as a connected graph at the beginning. Some examples of the introduction graphs for path, cycle and tadpole graphs can be found in Figures 1-3.

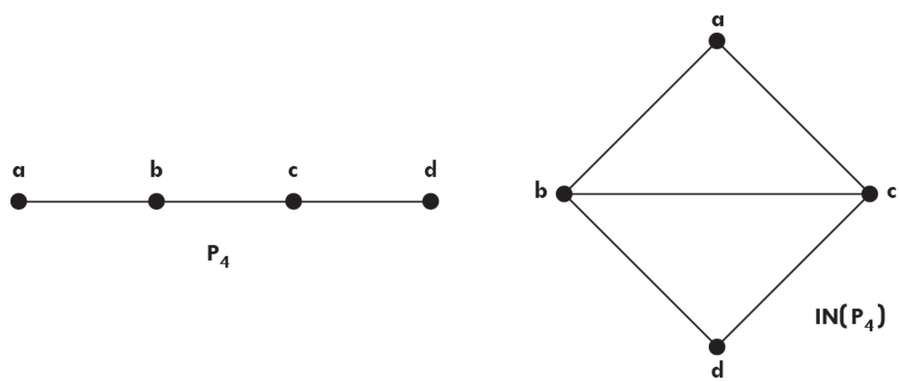

Figure 1 Path graph P_4 and its introduction graph

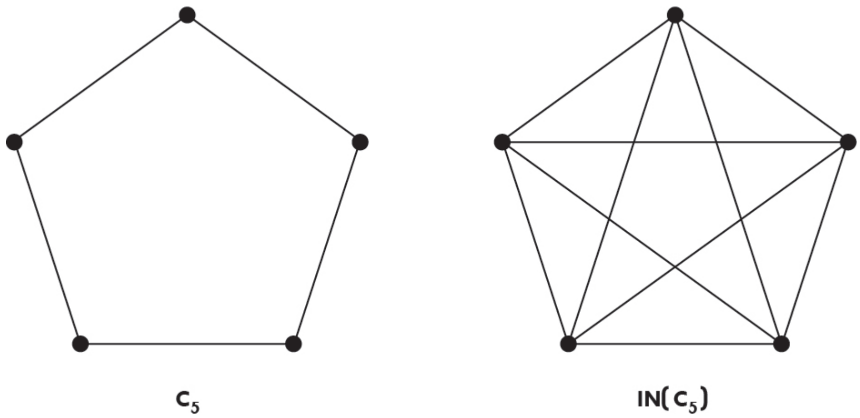

Figure 2

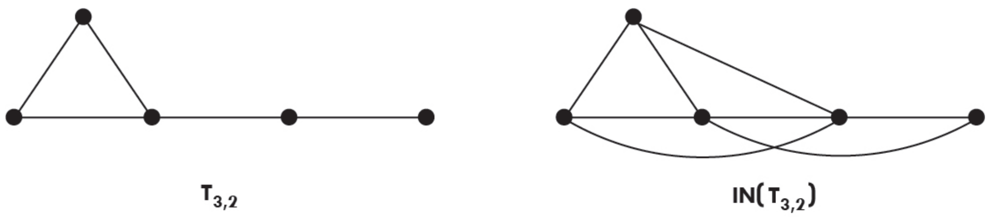

Figure 3

Clearly, the introduction graph $IN(G)$ of a given graph G can be stated as follows in terms of neighbourhoods:

$$IN(G) = \bigcup_{v \in V(G)} [\{v\} \cup N(G) \cup N^2(G)].$$

Note that it is obtained by replacing every path of length three by a triangle by adding two endpoints of each such path. Doing this for all possible paths P_3 of length 2, each pair of members knowing a common member become new friends. This is why this new derived graph is named as the introduction graph.

We now recall some graph parameters in terms of distances within the graph which are going to be utilized in the remaining sections. The eccentricity denoted by $ecc(v)$ of a vertex v in G is the greatest distance from v to any other vertex. The radius $rad(G)$ of G

is the smallest eccentricity amongst the vertices of G. The diameter denoted by $diam(G)$ of a graph G is the greatest eccentricity within the graph. It corresponds to the highest distance between all vertices of the graph. The center of the graph G is the set consisting of the vertices v having smallest possible eccentricity. That is, each vertex in the center of the graph has eccentricity equal to the radius of the graph which is $ecc(v) = rad(G)$.

Now we present a general result about the introduction graph of graphs with a maximum value for the diameter.

Theorem 2.2. *Let G be a graph of order n with diameter less than or equal to 2. Then $IN(G) = K_n$.*

Proof. As the diameter is less than or equal to 2, it is either 1 or 2. If diameter is 1, by definition, all vertices are adjacent to each other giving a complete graph. If diameter is 2, the maximum distance between all vertex pairs is 2 and this time, by the definition of the introduction graph, all vertex pairs will become joined again giving a complete graph K_n. □

The following are obvious results of this theorem:

Lemma 2.3. *If G is a complete graph K_n, then $IN(K_n) = K_n$. Also*

$$diam(IN(K_n)) = diam(K_n) = 1$$

and

$$rad(IN(K_n)) = rad(K_n) = 1.$$

Proof. Let us consider a complete graph K_n. As each vertex is adjacent to all other vertices in K_n and these adjacencies will also appear in $IN(K_n)$ by the definition, the result follows. □

Lemma 2.4. *If G is a star graph S_n, then $IN(S_n) = K_n$. Also $diam(S_n) = 2$ and*

$$diam(IN(S_n)) = diam(K_n) = 1$$

and $rad(S_n) = 1$ *and*

$$rad(IN(S_n)) = rad(K_n) = 1.$$

Proof. There are two types of vertices in the star graph, the central one and $n-1$ outer vertices. Let us label the central vertex by v_n and the outer vertices by v_i for $i = 1, 2, \cdots, n-1$. v_n is connected to all other vertices. Next consider an outer vertex v_i with $i = 1, 2, \cdots, n-1$. Each such outer vertex is adjacent only to v_n in S_n, but it will be adjacent to all other $n-2$ outer vertices in $IN(S_n)$. The result then follows for star graphs. □

Note that the introduction graphs of the star graph S_n and complete graph K_n are equal to the complete graph K_n. This property holds for many other graph classes and therefore, needs special attention:

Definition 2.5. *Let G be a graph. If $IN(G) = K_n$, then G is called an almost complete graph.*

The fact that the introduction graph $IN(G)$ of a graph G is a complete graph means that G is very near to be complete. So this is a new kind of test for completeness. So far, we have seen that star and complete graphs are almost complete. Now we can state the following result to generalize the final results:

Corollary 2.6. *The Petersen graph P, wheel graphs W_n, complete bipartite graphs $K_{r,s}$, windmill graphs W_a^b and cocktail party graphs CP_n are almost complete. That is,*

$$IN(P) = IN(W_n) = IN(K_{r,s}) = IN(W_a^b) = IN(CP_n) = K_n.$$

Proof. It all follows by Theorem 2.2. □

Cycle graphs are one of the most popular few graph types and they deserve a special result for their introduction graphs. To give this result, we must first recall an important class of graphs having maximal symmetry as this result depends completely on the circulant graphs. A circulant graph $C(n, k)$ is the graph obtained by joining every vertex v of an n-gon with the vertex at distance k to v. That is, every vertex pair having distance k on the cycle graph C_n are joined by an edge. Then we have

Theorem 2.7. *Let C_n be a cycle graph of order n. Then the introduction graph of C_n is the circulant graph $C(n,2)$. Also for $n \leq 5$, $IN(C_n)$ is almost complete.*

3 Some fundamental properties of the introduction graphs

In this section, we study some fundamental graph parameters related to introduction graphs. As the definition is related to the notion of distance, we first study distance related parameters.

Corollary 3.1. *If G is a graph, then*

$$rad(IN(G)) = \lceil \frac{rad(G)}{2} \rceil.$$

Corollary 3.2. *If P_n is path graph, then*

$$diam(IN(P_n)) = \lceil \frac{diam(P_n)}{2} \rceil = \lceil \frac{n-1}{2} \rceil.$$

In fact, the above result which is given to be valid for path graphs is also valid for many other graphs as given below:

Theorem 3.3. *If G is a graph with $diam(G) = k$, then*

$$diam(IN(G)) = \lceil \frac{k}{2} \rceil.$$

Proof. There are two vertices $u, v \in V(G)$ such that $d(u,v) = k$ and the distance between all other vertex pairs is $\leq k$. Let the shortest path between u and v be $uv_1v_2 \cdots v_{k-1}v$. As $uv_1v_2, \cdots, v_{k-1}v$ is a path of length k, by Corollary 3.1, we know that $diam(IN(G)) = \lceil \frac{k}{2} \rceil$, which concludes the proof. $\square$

Using the fact that $diam(G) = k$, we can restate the above result as below:

$$diam(IN(G)) = \begin{cases} \frac{diam(G)}{2}, & if\ diam(G)\ is\ even \\ \frac{diam(G)+1}{2}, & if\ diam(G)\ is\ odd \end{cases}.$$

Let t be the number of edges which are added to the edge set of G to obtain the introduction graph $IN(G)$. t is also the number of unordered vertex pairs (u, v) having distance between u and v equal to 2. Therefore,

$$m(IN(G)) = m(G) + t.$$

Now we present another important property of the introduction graphs. As the introduction graph is defined by distance idea, it is natural to ask the question that such graphs form a metric space or not. The answer is affirmative as given in the following result:

Theorem 3.4. *$IN(G)$ is a metric space.*

Proof. **(M1)** $d(u, v) \geq 0$ is obvious from the notion of the distance in graphs.

(M2) The statement $d(x, y) = 0$ in a simple graph means that the vertices x and y are the same vertex as otherwise, the distance between them would be a positive integer equal to the length of a shortest path between these vertices. On the contrary, if $x = y$, then by the definition of distance in simple graphs, $d(x, y)$ must be zero.

(M3) $d(x, y) = d(y, x)$ is again clear by the definition of distance. Recall that the adjacency matrix is a symmetrical matrix because of this property.

(M4) The last property to be proven is the triangle inequality $d(v_1, v_2) \leq d(v_1, v_3) + d(v_3, v_2)$. As these three quantities in this inequality are also equal to the diameter of the corresponding paths, that is, e.g. $d(v_1, v_2)$ is also the diameter of the path graph between v_1 and v_2, we can use Corollary 3.2 as below. Let $d(v_1, v_2)$ be the length of a shortest path $P_f = v_1 a_2 a_3 \cdots a_{f-1} v_2$ between v_1 and v_2, let $d(v_1, v_3)$ be the length of a shortest path $P_g = v_1 b_2 b_3 \cdots b_{g-1} v_3$ between v_1 and v_3 and let $d(v_3, v_2)$ be the length of a shortest path $P_h = v_3 c_2 c_3 \cdots c_{h-1} v_2$ between v_3 and v_2 which are $f-1$, $g-1$ and $h-1$, respectively, see Fig. 4.

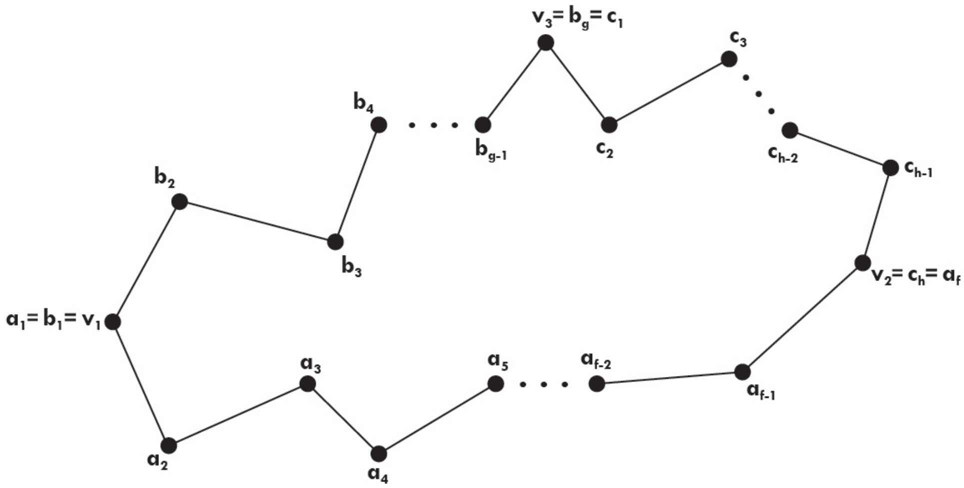

Figure 4

This gives us $f-1 \leq g-1 \;+\; h-1$ which is equivalent to

$$f \leq g+h-1. \tag{1}$$

By Corollary 3.2, it is sufficient to show that

$$diam(IN(P_f)) \leq diam(IN(P_g)) + diam(IN(P_h)) \tag{2}$$

which is equivalent to

$$\lceil \frac{f-1}{2} \rceil \leq \lceil \frac{g-1}{2} \rceil \;+\; \lceil \frac{h-1}{2} \rceil. \tag{3}$$

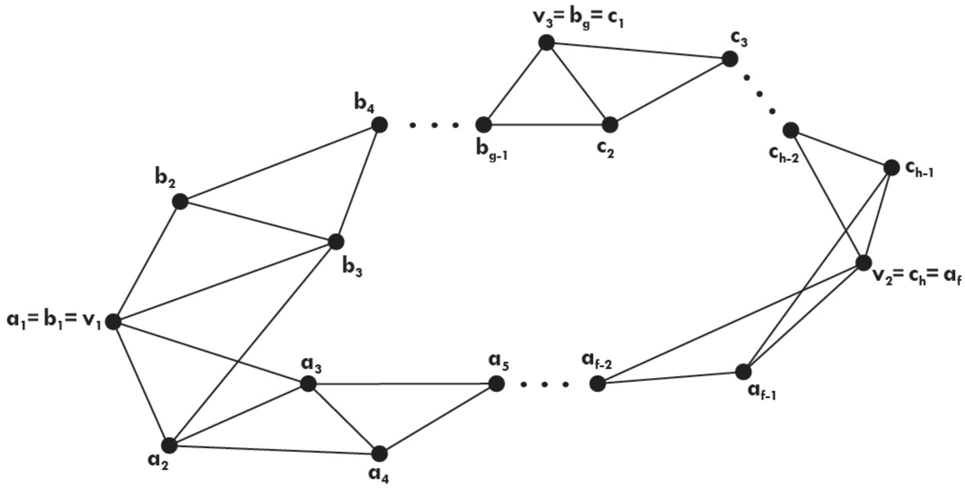

Figure 5

Now, the shortest path between v_1 and v_2 in $IN(P_f)$ has length $\lceil \frac{f-1}{2} \rceil$. There are three cases:

Case 1. g and h have different parities: In this case, w.l.o.g. we assume that g is even and h is odd. Then $\lceil \frac{g-1}{2} \rceil = \frac{g}{2}$ and $\lceil \frac{h-1}{2} \rceil = \frac{h-1}{2}$. If f is odd, then Eqn. 2 becomes $\frac{f-1}{2} \leq \frac{g+h-1}{2}$ which is correct by Eqn. 1 and if f is even, then Eqn. 2 becomes $\frac{f}{2} \leq \frac{g+h-1}{2}$ which is again correct by Eqn. 1 showing that the inequality in Eqn. 3 is true.

Case 2. g and h are both odd: In this case, the right hand side of Eqn. 3 becomes $\frac{g-1}{2} + \frac{h-1}{2} = \frac{g+h-2}{2}$. If f is odd, then Eqn. 2 becomes $\frac{f-1}{2} \leq \frac{g+h-2}{2}$ which is correct by Eqn. 1 and if f is even, then Eqn. 2 becomes $\frac{f}{2} \leq \frac{g+h-2}{2}$. In this case, the proof is complete if $f < g+h-1$. In the single case where $f = g+h-2$, the path P_f and the path obtained by joining two paths P_g and P_h both have the same length proving the inequality in Eqn. 3.

Case 3. g and h are both even: In this case, the right hand side of Eqn. 3 becomes $\frac{g}{2} + \frac{h}{2} = \frac{g+h}{2}$. If f is odd, then Eqn. 2 becomes $\frac{f-1}{2} \leq \frac{g+h}{2}$ which is correct by Eqn. 1 and if f is even, then Eqn. 2 becomes $\frac{f}{2} \leq \frac{g+h}{2}$ proving the inequality in Eqn. 3. $\square$

4 Some topological graph indices of introduction graphs

Topological indices, also called molecular descriptors by chemists, are the mathematical formulae giving us an invariant number as a result for all isomorphic graphs. The first such index is known as the Wiener index defined by H. Wiener in 1947 to compare the boiling degrees of some alkane isomers, see [14]. Since then, over 3000 such indices have taken their place in chemical databases for several applications. For some fundamental chemical applications of such indices, one can refer to [12, 13].

The Wiener index $W(G)$ is defined as follows:

$$W(G) = \sum_{u \in V(G)} d_u.$$

In this paper, we shall calculate the introduction graphs for some well-known graph classes such as first and second Zagreb indices, Narumi-Katayama index, Wiener index and forgotten index. We shall also calculate the degree sequences of these graphs.

The first and second Zagreb indices of a graph G were defined by Gutman and Trinajstic in 1972, [5], respectively by

$$M_1(G) = \sum_{e=uv \in E(G)} (d_u + d_v) \qquad \text{and} \qquad M_2(G) = \sum_{e=uv \in E(G)} d_u d_v.$$

The Narumi-Katayama index $NK(G)$ is defined as follows:

$$NK(G) = \prod_{u \in V(G)} d_u.$$

More information on this index can be found in [1, 7, 8, 9].

The forgotten index abbreviated by F-index of a graph G denoted by $F(G)$ is defined in [4] as the sum of the cubes of the degrees of the vertices of the graph as

$$F(G) = \sum_{u \in V(G)} d_u^3.$$

More information on this index can be found in [2].

To calculate the above indices, we need to determine the vertex degrees and degrees of pairs of vertices forming an edge. First we list the degree sequences of some graph classes and their introduction graphs in Table 1 which are going to be needed in the calculation of the selected topological graph indices:

G	$\mathbf{D(G)}$	$\mathbf{D(IN(G))}$
P_n	$\{1^{(2)},\ 2^{(n-2)}\}$	$\{2^{(2)},\ 3^{(2)},\ 4^{(n-4)}\}$
C_n	$\{2^{(n)}\}$	$\{4^{(n)}\}$
K_n	$\{(n-1)^{(n)}\}$	$\{(n-1)^{(n)}\}$
S_n	$\{1^{(n-1)},\ (n-1)^{(1)}\}$	$\{(n-1)^{(n)}\}$
$K_{r,s}$	$\{r^{(s)},\ s^{(r)}\}$	$\{(r+s-1)^{(r+s)}\}$
$T_{r,s}$	$\{1^{(1)},\ 2^{(r+s-2)},\ 3^{(1)}\}$	$\{2^{(1)},\ 3^{(1)},\ 4^{(r+s-6)},\ 5^{(3)},\ 6^{(1)}\}$
W_n	$\{3^{(n-1)},\ (n-1)^{(1)}\}$	$\{(n-1)^{(n)}\}$
L_n	$\{3^{(2n)}\}$	$\{7^{(2n)}\}$
W_a^b	$\{(a-1)^{(n-1)},\ (n-1)^{(1)}\}$	$\{(n-1)^{(n)}\}$
P	$\{3^{(10)}\}$	$\{9^{(10)}\}$
CP_{2n}	$\{(2n-2)^{(2n)}\}$	$\{(2n-1)^{(2n)}\}$

Table 1 Degree sequences of some graphs and their introduction graphs

G	$\mathbf{NK(IN(G))}$	$\mathbf{M_1(IN(G))}$
P_n	$2^{2n-6}.3^2$	$16n-38$
C_n	2^{2n}	$16n$
K_n	$(n-1)^n$	n^3-2n^2+n
S_n	$(n-1)^n$	n^3-2n^2+n
$K_{r,s}$	$(r+s-1)^{r+s}$	$(r+s)(r+s-1)^2$
$T_{r,s}$	$2^{2r+2s-11}.3.5^3.6$	$16(r+s)+28$
W_n	$n.5^n$	$n.(n-1)^2$
L_n	7^{2n}	$98n$
W_a^b	$(n-1)^n$	$n.[(a-1)+(a-1).(b-1)]^2$
P	9^{10}	810
CP_{2n}	$(2n-1)^{2n}$	$2n(2n-1)^2$

Table 2 NK and First Zagreb indices of the introduction graphs of some graphs

G	$\mathbf{W(IN(G))}$	$\mathbf{M_2(IN(G))}$	$\mathbf{F(IN(G))}$
P_n	$4n-6$	$32n+60$	$64n-186$
C_n	$4n$	$32n$	$64n$
K_n	n^2-n	$\frac{n(n-1)^3}{2}$	$n(n-1)^3$
S_n	n^2-n	$\frac{n(n-1)^3}{2}$	$n(n-1)^3$
$K_{r,s}$	$(r+s)^2-(r+s)$	$\frac{(r+s)(r+s-1)^3}{2}$	$(r+s)(r+s-1)^3$
$T_{r,s}$	$4r+4s+2$	$16(2r+s)-159$	$64(r+s)+242$
W_n	$6n-6$	$\frac{n(n-1)^3}{2}$	$(n-1)[125+(n-1)^2]$
L_n	$14n$	$98n$	$686n$
W_a^b	$(n-1)(ab-b+1)$	$\frac{n(n-1)^3}{2}$	$(n-1)[b^3(a-1)^3+(n-1)^2]$
P	90	810	7290
CP_{2n}	$2n(2n-1)$	$2n(2n-1)^2$	$2n(2n-1)^3$

Table 3 Wiener and second Zagreb indices and forgotten indices of the introduction graphs of some graphs

5 Conclusions

21st century will have been remembered by the incredibly fast advancements in technology and in particular, in AI and social networks. As each network is an example of a graph, it is very natural to study such topics by the parameters and methods of graph theory. Social media platforms are extremely large examples of social networks where high number of followers or likers is deadly important for many people. Therefore, suggestions are being made to each of us by such network programs so that our connection circle gets larger everyday and the network relatedly becomes larger and larger. In this paper, we introduced a new derived graph for any given graph which helps us in calculating the speed of growth of the considered social network when new relations are established between some pairs of members of the network. One can use this novel notion to extend the calculations made here to more general examples. For some recent examples of derived graphs in relation with topological graph indices and other graph parameters, one can refer to [3, 10, 11].

Conflict of interest

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