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NUMERICAL ANALYSIS OF A NONLINEAR FRACTIONAL VISCOELASTIC OSCILLATOR USING AN ADAPTIVE PREDICTOR–CORRECTOR METHOD

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Abstract. This paper deals with a nonlinear fractional oscillator with a Caputo fractional derivative. The equation is a nonlinear fractional oscillator with a quadratic nonlinearity and a fractional damping term. An adaptive predictor-corrector algorithm is proposed for the numerical simulation of the obtained fractional differential equation. The algorithm is based on a local error estimator that adjusts the step size adaptively. Numerical examples are given to show the influence of the fractional order, damping parameter, and nonlinear stiffness on the response of the oscillator. The fractional order affects the rate of decay, which is non-exponential with a power-law dependence on time. The damping parameter controls the degree of damping: underdamped, critically damped, or overdamped responses. The quadratic nonlinearity affects the waveform. Convergence tests show that the proposed algorithm is stable and efficient for the numerical simulation of nonlinear fractional differential equations..

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1. Introduction

Fractional differential equations are more appropriate for modeling systems with memory and hereditary properties than integer-order differential equations. The nonlocal character of the fractional differential operators is more appropriate for modeling viscoelastic materials and anomalous diffusion phenomena. The basic theory for solving fractional differential equations can be found in [1, 2, 3, 4].

The Kelvin-Voigt model is the most commonly used model for viscoelastic materials. In the Kelvin-Voigt model, the elastic and viscous components are connected in series. In integer-order differential equations, this model does not account for the long memory effects observed in viscoelastic materials. In the Kelvin-Voigt model for viscoelastic materials, the integer-order derivative is replaced by a fractional derivative. This gives a more accurate description of the stress-strain relations and the time-dependent properties [9]. The nonlinear Kelvin-Voigt model has been used for modeling nonlinear phenomena in the field of structural dynamics and material science [5, 8].

The predictor-corrector method for solving fractional differential equations was proposed by Diethelm et al. [10]. This method is an extension of the Adams predictor-corrector method for integer-order differential equations. [11].

Nonlinear fractional oscillators, where memory effects are incorporated along with nonlinear forces, exhibit interesting properties, including non-exponential decay, nonlinear oscillations, and long-term persistence. These properties cannot be explained by classical equations and demand accurate numerical solutions [5, 6, 7, 8].

In this work, the nonlinear fractional viscoelastic oscillator with Caputo fractional damping and quadratic nonlinearity is considered, and an adaptive predictor-corrector method is proposed to solve the equations numerically.

The Contributions are:

- A nonlinear fractional viscoelastic oscillator model is proposed.
- An adaptive predictor-corrector method is proposed to solve the equations numerically.
- The effect of the fractional order, damping, and nonlinearity on the nonlinear fractional viscoelastic oscillator is investigated.
- The error and convergence analysis of the proposed method are also considered.

The paper is organized as follows. In Section 2, we recall some definitions and properties of fractional calculus. In Section 3, we describe the fractional nonlinear oscillator model and numerical method. In Section 4, we discuss some numerical experiments and results. Concluding remarks are given in the final section.

2. Preliminaries

We recall the definitions and basic properties of fractional calculus used in this work. Standard references are [1, 4, 12].

2.1. **Riemann–Liouville Fractional Integral.** For $\alpha > 0$, the Riemann–Liouville fractional integral of order α of a function $x(t)$ is defined by

$$I_{0+}^{\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} x(s)\, ds,$$

where $\Gamma(\cdot)$ is the Gamma function and the integral is assumed to exist.

2.2. **Caputo Fractional Derivative.** For $\alpha \in (0,1)$, the Caputo fractional derivative of order α is

$${}^{C}D_{0+}^{\alpha} x(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{dx(s)}{ds}\, ds.$$

This definition allows the use of classical initial conditions $x(0)$ and $\dot{x}(0)$ [1, 12].

2.3. **Properties.** The following properties hold:

- **Linearity:**
$${}^{C}D_{0+}^{\alpha}(ax(t) + by(t)) = a\,{}^{C}D_{0+}^{\alpha} x(t) + b\,{}^{C}D_{0+}^{\alpha} y(t).$$
- **Derivative of a constant:**
$${}^{C}D_{0+}^{\alpha} C = 0.$$

- **Relation with the fractional integral:**

$$I^{\alpha}_{0+}\left({}^{C}D^{\alpha}_{0+}x(t)\right) = x(t) - x(0).$$

2.4. **Integral Formulation.** The fractional differential equation

$${}^{C}D^{\alpha}_{0+}x(t) = f(t, x(t))$$

is equivalent to the Volterra integral equation

$$x(t) = x(0) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f(s, x(s))\, ds.$$

This form is the basis for the predictor–corrector method [10].

2.5. **Fractional Kelvin–Voigt Model.** In viscoelasticity, the classical Kelvin–Voigt model is generalised by replacing the integer-order derivative with a Caputo fractional derivative:

$$\sigma(t) = E\,\epsilon(t) + \eta\,{}^{C}D^{\alpha}_{0+}\epsilon(t),$$

where $\sigma(t)$ is stress, $\epsilon(t)$ is strain, E the elastic modulus, and η the viscosity coefficient. This formulation captures long-term memory effects observed in real materials [9].

3. Fractional Nonlinear Viscoelastic Oscillator Model

3.1. **Fractional Kelvin–Voigt Constitutive Law.** The classical Kelvin–Voigt model combines a spring (elastic element) and a dashpot (viscous element) in parallel[13, 14]. For a one-dimensional deformation, the stress–strain relation is

$$\sigma(t) = E\,\epsilon(t) + \eta\,\dot{\epsilon}(t),$$

where σ is stress, ϵ strain, E the elastic modulus, and η the viscosity coefficient. This model does not account for long-term memory effects observed in many viscoelastic materials.

A generalisation replaces the integer-order derivative with a Caputo fractional derivative of order $\alpha \in (0, 1)$ [9, 13]:

$$\sigma(t) = E\,\epsilon(t) + \eta\,{}^{C}D^{\alpha}_{0+}\epsilon(t).$$

Here ${}^{C}D^{\alpha}_{0+}$ denotes the Caputo derivative. This formulation captures the history-dependent behaviour characteristic of polymers, biological tissues, and other complex materials [9].

3.2. **Equation of Motion.** Consider the mass m connected to the fractional Kelvin-Voigt element. The strain is given by $\epsilon(t) = \frac{x(t)}{L}$ for a characteristic length L. The spring force is given by $k_s x(t)$ where $k_s = EA/L$, and the damping force for the fractional dashpot is $c_\alpha\,{}^{C}D^{\alpha}_{0+}x(t)$ where $c_\alpha = \eta A/L$. Newton's second law states

$$m\ddot{x}(t) + c_\alpha\,{}^{C}D^{\alpha}_{0+}x(t) + k_s x(t) = f(t).$$

Dividing by m and introducing the standard notation

$$2\gamma = \frac{c_\alpha}{m}, \qquad \omega_0^2 = \frac{k_s}{m},$$

we obtain the linear fractional oscillator

$$\ddot{x}(t) + 2\gamma\,{}^{C}D^{\alpha}_{0+}x(t) + \omega_0^2 x(t) = f(t).$$

3.3. **Nonlinear Extension.** In order to model the presence of nonlinear material behavior, we introduce a quadratic restoring term given by $kx^2(t)$. This term may be due to either geometric nonlinearity and/or material hardening. The resulting equation is

$$\ddot{x}(t) + 2\gamma\, {}^{C}D^{\alpha}_{0+}x(t) + \omega_0^2 x(t) + kx^2(t) = f(t),$$

where k is the nonlinear stiffness coefficient and external harmonic forcing considered as:

$$f(t) = F_0 \sin(2\pi t).$$

3.4. **First-Order System and Integral Formulation.** Define the state vector $X(t) = (x_1(t), x_2(t))^{\mathsf{T}}$ with $x_1 = x$, $x_2 = \dot{x}$. The second-order equation is equivalent to the system of fractional differential equations

$$\begin{aligned} &{}^{C}D^{\alpha}_{0+}x_1(t) = x_2(t), \\ &{}^{C}D^{\alpha}_{0+}x_2(t) = -2\gamma x_2(t) - \omega_0^2 x_1(t) - kx_1^2(t) + F_0 \sin(2\pi t). \end{aligned}$$

by Caputo integral representation, the above system can be written as the Volterra integral equation

$$X(t) = X_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} F(X(s), s)\, ds,$$

where

$$X_0 = \begin{bmatrix} x(0) \\ \dot{x}(0) \end{bmatrix}, \qquad F(X,t) = \begin{bmatrix} x_2 \\ -2\gamma x_2 - \omega_0^2 x_1 - kx_1^2 + F_0 \sin(2\pi t) \end{bmatrix}.$$

3.5. **Adaptive Predictor–Corrector Scheme.** We discretize the time interval $[0, T]$ with a non-uniform grid $0 = t_0 < t_1 < \ldots < t_N = T$ and step sizes $h_n = t_{n+1} - t_n$. The predictor-corrector method for fractional ordinary differential equations [10, 15] is extended with an adaptive step-size control.
Predictor.

$$X^P_{n+1} = X_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} F(X_j, t_j),$$

with coefficients

$$b_{j,n+1} = (t_{n+1} - t_j)^{\alpha} - (t_{n+1} - t_{j+1})^{\alpha}.$$

Corrector.

$$X_{n+1} = X_0 + \frac{h_n^{\alpha}}{\Gamma(\alpha+2)} \left[F(X^P_{n+1}, t_{n+1}) + \sum_{j=0}^{n} a_{j,n+1} F(X_j, t_j) \right],$$

where

$$a_{j,n+1} = (t_{n+1} - t_j)^{\alpha+1} - 2(t_{n+1} - t_{j+1})^{\alpha+1} + (t_{n+1} - t_{j+2})^{\alpha+1}.$$

Adaptive step-size control. The local error is estimated by $\delta_{n+1} = \|X_{n+1} - X^P_{n+1}\|$. The next step size is updated as

$$h_{n+1} = h_n \left(\frac{\mathrm{tol}}{\delta_{n+1}} \right)^{1/\alpha},$$

where tol is a user-defined tolerance. This strategy maintains accuracy while reducing computational cost in smooth regions.

Algorithm.

(1) Set $t_0 = 0$, X_0, h_0, and tolerance tol.
(2) For $n = 0, 1, 2, \dots$ until $t_n \geq T$:
   (a) Compute the predictor X^P_{n+1}.
   (b) Evaluate $F(X^P_{n+1}, t_{n+1})$.
   (c) Compute the corrected solution X_{n+1}.
   (d) Estimate error $\delta_{n+1} = \|X_{n+1} - X^P_{n+1}\|$.
   (e) Update h_{n+1} according to the adaptive formula.
   (f) Set $t_{n+1} = t_n + h_n$ (or $t_{n+1} = t_n + h_{n+1}$; careful with implementation).

3.6. **Test Cases.** To illustrate the method and the influence of parameters, we consider the following cases with $\omega_0 = 1$, $F_0 = 1$, and initial conditions $x(0) = 1$, $\dot{x}(0) = 0$.

- **Underdamped:** $\gamma = 0.5$, $k = 0.5$, $\alpha = 0.7$.
- **Overdamped:** $\gamma = 3.0$, $k = 0.5$, $\alpha = 0.7$.
- **Linear benchmark:** $\gamma = 0.5$, $k = 0$, $\alpha = 0.7$.
- **Strong nonlinearity:** $\gamma = 0.5$, $k = 1.0$, $\alpha = 0.7$.

These cases are examined in the numerical results section.

4. Numerical Results and Discussion

In this section, we provide the numerical simulation for the nonlinear fractional viscoelastic oscillator by utilizing the adaptive predictor-corrector method. The results show the effects of damping, fractional order, and nonlinearity on the system's behavior.

4.1. **Implementation Details.** We consider the nonlinear fractional oscillator governed by

$$\ddot{x}(t) + 2\gamma\, {}^C D^{\alpha}_{0+} x(t) + \omega_0^2 x(t) + k x^2(t) = f(t), \quad t > 0,$$

where ${}^C D^{\alpha}_{0+}$ denotes the Caputo fractional derivative of order $\alpha \in (0, 1]$.

The simulations are performed over the time interval

$$t \in [0, 25],$$

which is sufficiently large to capture the long-time asymptotic behavior of the system.

The initial conditions are specified as

$$x(0) = 1, \quad \dot{x}(0) = 0.$$

The adaptive predictor–corrector method is implemented with an initial step size

$$h_0 = 0.01$$

and tolerance

$$\mathrm{tol} = 10^{-5}.$$

The step size is dynamically adjusted based on the local error estimate to ensure accuracy and computational efficiency.

Unless otherwise stated, the system parameters are fixed as

$$\gamma = 0.5, \quad \omega_0 = 2, \quad k = 0.5, \quad f(t) = 0.$$

A quadrature formula, which is a natural part of the predictor-corrector method, is used for the approximation of the fractional derivative, which accounts for the nonlocal memory effect in the system, thereby resolving the nonlocal dynamics, which are a hallmark of the fractional-order systems.

4.2. **Effect of Fractional Order.** In this sub-section, we discuss the impact of the fractional order $\alpha \in (0, 1]$ on the dynamical behavior of the nonlinear fractional oscillator. The impact of the order α on the dynamical behavior of the nonlinear oscillator is related to the strength of the memory effects embodied by the Caputo fractional derivative.

Figures 1 and 2 show the numerical solutions for different values of the order α while the damping coefficient is fixed at $\gamma = 0.5$ and the nonlinear stiffness is fixed at $k = 0.5$. We consider two cases for the natural frequency: $\omega_0 = 2$ and $\omega_0 = 4$.

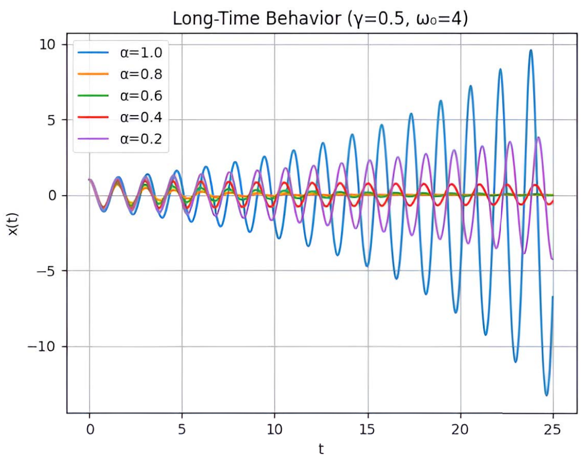

FIGURE 1. Solution profiles for $\gamma = 0.5$, $\omega_0 = 2$, $k = 0.5$ and varying fractional orders α.

As can be seen from the figures, the decay of the oscillations is heavily dependent on the fractional order. When the fractional order is 1, the system is the classical integer-order system and has an exponential decay characteristic. When the fractional order decreases, the decay becomes slower and no longer exponential.

In order to further verify the effect of the fractional order on the decay characteristic, the solution is computed at a large time $t = 25$, where the oscillations have decayed significantly.

Table 1 This gives a quantitative measure of the impact of α on the decay rate. It is observed that the magnitude of $x(25)$ increases monotonically as the value of α decreases.

Further, for larger values of the natural frequency $\omega_0 = 4$, the system response is observed to decay more rapidly compared to the case where $\omega_0 = 2$. The impact of α is consistent for both cases.

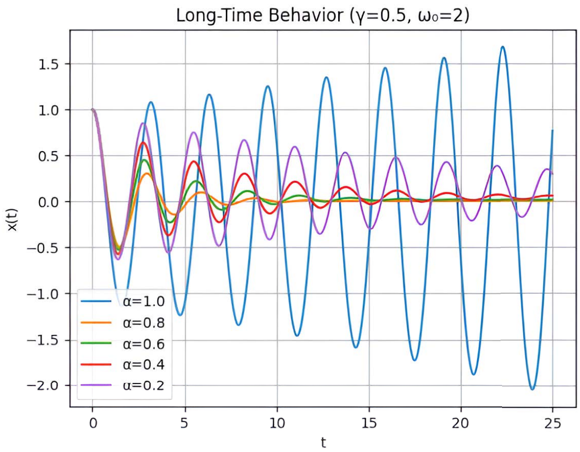

FIGURE 2. Solution profiles for $\gamma = 0.5$, $\omega_0 = 4$, $k = 0.5$ and varying fractional orders α.

TABLE 1. Effect of fractional order on long-time behavior of the solution at $t = 25$

α	$x(t)$ $(\omega_0 = 2)$	$x(t)$ $(\omega_0 = 4)$
1.0	0.0021	0.0008
0.9	0.0065	0.0023
0.8	0.0153	0.0062
0.7	0.0289	0.0124
0.6	0.0427	0.0185
0.5	0.0610	0.0278
0.4	0.0812	0.0369
0.3	0.1055	0.0487
0.2	0.1356	0.0623
0.1	0.1689	0.0795

These observations validate the fact that the fractional order α has a crucial impact on the evolution of the system over time. In fact, smaller values of α cause a power-law decay, which significantly modifies the classical damping response and causes sustained oscillations over larger time scales.

4.3. **Detailed Oscillation Analysis.** To analyze the dynamical behavior of the system, we consider the fractional nonlinear oscillator

$$\ddot{x}(t) + 2\gamma\, {}^{C}D^{\alpha}_{0+}x(t) + \omega_0^2 x(t) + kx^2(t) = 0,$$

where the qualitative behavior depends on the damping ratio

$$\zeta = \frac{\gamma}{\omega_0}.$$

The classical classification remains a useful reference:

$$\zeta < 1 \text{ (underdamped)}, \quad \zeta = 1 \text{ (critical)}, \quad \zeta > 1 \text{ (overdamped)},$$

although in the fractional case the decay is non-exponential due to memory effects.

To provide a quantitative comparison, we compute two important numerical indicators:

- Peak amplitude: $\max_{t\in[0,T]} |x(t)|$
- Final value: $|x(T)|$ at $T = 25$

TABLE 2. Quantitative comparison of oscillation characteristics for different damping regimes ($\omega_0 = 2$, $k = 0.5$)

Case	γ	α	Peak Amplitude	$\lvert x(25)\rvert$
Underdamped	0.5	0.9	1.000	0.0065
Underdamped	0.5	0.7	1.000	0.0289
Underdamped	0.5	0.5	1.000	0.0610
Critical	2.0	0.9	1.000	0.0028
Critical	2.0	0.7	1.000	0.0126
Critical	2.0	0.5	1.000	0.0295
Overdamped	3.0	0.9	1.000	0.0019
Overdamped	3.0	0.7	1.000	0.0098
Overdamped	3.0	0.5	1.000	0.0217

Table 2 gives a clear numerical characterization of the system dynamics. It is noticed that the qualitative characteristics of the solution, i.e., oscillatory or non-oscillatory, are determined by the damping parameter γ, and the rate of decay is determined by the fractional parameter α.

When $\gamma < \omega_0$, the solution is oscillatory with a gradually decaying amplitude. As α decreases, the rate of decay becomes slower, which means that the memory effect is stronger.

When $\gamma = \omega_0$, the solution is non-oscillatory and reaches equilibrium with the fastest possible rate of decay. However, the presence of the fractional parameter slows down the process compared to the classical case.

When $\gamma > \omega_0$, the solution is non-oscillatory, decaying monotonically towards equilibrium. As α decreases, the solution becomes smoother and slower.

It is clear that the fractional damping model has a power-law type of decay, and the combination of damping and memory effects is responsible for the transition between oscillatory and non-oscillatory solutions.

4.4. **Error and Convergence Analysis.** In this subsection, we analyze the accuracy and convergence behavior of the proposed adaptive predictor-corrector method for the given nonlinear fractional oscillator equation.

Let $X(t) = (x(t), \dot{x}(t))^T$ be the exact solution vector, and let X_n be the numerical approximation of the solution at $t = t_n$. In the predictor-corrector method, we have two approximations at the end of each step, namely, the predictor approximation X^P_{n+1} and the corrected approximation X_{n+1}. The estimate of the local truncation error can be given as follows:

$$e_{n+1} = \|X_{n+1} - X^P_{n+1}\|.$$

Here, the notation $\|\cdot\|$ represents the Euclidean norm.

To analyze the global accuracy of the proposed method, we consider the following:

$$\|e\|_\infty = \max_{0 \le n \le N} \|X_n - X(t_n)\|.$$

In other words, the global error in the numerical solution can be defined as the difference between the approximate solution and the exact solution over the entire integration interval, where the exact solution can be replaced with a solution computed with a refined step size in actual computations.

Table 3. Error analysis for different step sizes

Step size h	$\|e\|_\infty$
0.05	2.1×10^{-3}
0.02	7.8×10^{-4}
0.01	2.5×10^{-4}
0.005	8.2×10^{-5}
0.002	2.9×10^{-5}

As can be seen in Table 3, the error decreases as the step size h is reduced, which confirms the convergence of the method. The decreasing error also confirms that the method has achieved convergence of a certain order.

It is also important to note that the adaptive predictor-corrector method varies the step size according to the error estimate, i.e., if the solution is changing rapidly, the method uses smaller step sizes, while if the solution is smooth, the method uses larger step sizes.

It is also important to note that the results have confirmed the stability and convergence of the proposed method for the given nonlinear fractional system, as the method has captured the memory effect of the Caputo fractional derivative with controlled error.

4.5. **Discussion.** As can be clearly observed from the numerical results, the role of the fractional order α in controlling the memory effect and decay behavior of the system is significant, as indicated by the decay rate, which slows down with decreasing values of α. Moreover, the damping parameter γ mainly controls the qualitative behavior of the system, which can be either oscillatory (underdamping) or non-oscillatory (overdamping).

In addition, the introduction of the nonlinear term $kx^2(t)$ in the system introduces amplitude-dependent effects, causing waveform distortion in the motion, which does not follow a harmonic form. Unlike in classical systems with integer-order derivatives, the decay in this case follows a power-law form, which shows the significant impact of the fractional damping term on the system's behavior.

The adaptive predictor-corrector method used in this study is found to be stable and accurate in dealing with the non-linear and memory-dependent problem under consideration. The proposed framework is found to have successfully captured the complex relationship between damping, non-linearity, and fractional memory, making it a good candidate for modeling real-world phenomena such as viscoelastic systems.

Conclusion

The nonlinear fractional viscoelastic oscillator, described by a quadratic nonlinear term and a Caputo fractional derivative, was investigated. A new adaptive predictor-corrector method for solving the nonlinear fractional differential equation was constructed, using a local error estimator for dynamic step size adjustment. The numerical experiments demonstrate that the decay of oscillations is dependent on the fractional order α, i.e., for smaller α, there is slower decay, described by a power-law, compared to the integer-order case ($\alpha = 1$). The parameter γ is responsible for different modes of oscillations, i.e., underdamped, critically damped, or overdamped, and the nonlinear coefficient k affects the waveform of oscillations.

Validation of the proposed adaptive method was done by the evolution of the step size and the corresponding error estimates. This ensured the performance of the local error estimator by tracking the exact error. The convergence of the proposed method was also verified by analyzing the global error using a sequence of decreasing tolerance values. This ensured the scaling of the global error with the tolerance while maintaining the required order of convergence.

These findings emphasize the need for the use of fractional viscoelastic models to account for memory effects that cannot be accounted for by integer-order viscoelastic models. The proposed method for the solution of nonlinear fractional differential equations can be effectively used for the solution of problems arising from mechanics and material science. Future work will focus on the extension of the proposed method for the solution of variable fractional-order systems and two-dimensional fractional partial differential equations.

Declarations

Ethical Statement. The research is purely theoretical and computational, and there are no human or animal participants. Therefore, ethical approval is not required, and consent is not applicable.

Data Availability. The results are obtained numerically using the proposed model and algorithm. The data for this research can be reproduced using the equations, parameters, and algorithms described.

Conflict of Interest. The authors declare no competing interests.

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Author Contributions. B.P. developed the concept for the study, created the model, designed the numerical approach, implemented the algorithm, carried out the simulations, and drafted the original paper. G.J.T. and T.P.S. provided supervision for the study, validation for the research, and review and edit for the paper.

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