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HAMMING INDEX OF A GRAPH WITH SELF-LOOPS

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ABSTRACT. Let G_S be a graph with self-loops, obtained by attaching a self-loop to each vertex of simple graph G. Let $A(G_S)$ denote the adjacency matrix of G_S. Each row of the matrix is a string of finite length, denoted by $s(v_i)$. The Hamming distance between two strings $s(v_i)$ and $s(v_j)$, where $i < j$ is defined as the number of positions at which the strings differ. The sum of Hamming distances between all the pairs of vertices is the Hamming index. In this paper we obtained the Hamming distance and the Hamming index of a class of graphs with self-loops, and explore various operations on graphs with self-loops.

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1. INTRODUCTION

Recently, graphs with self-loops have gained significant attention from researchers due to their applications in chemistry [1, 2, 3, 4]. A self-loop graph is constructed by attaching a self-loop to each vertex of the subset of the vertex set of a simple graph. In chemical graph theory, a self-loop represents a hetero-molecule. In control systems, neural networks, and dynamical systems, self-loops represent self-feedback. A node influencing its own future state is often modeled as a self-loop, essential in modeling recurrence or memory. In Graph Neural Networks, self-loops are often added artificially to ensure that a node's own features are included during message passing or aggregation.

Let G be a simple graph on n vertices and m edges with vertex set V. The self-loop graph G_S with σ self-loops is obtained by attaching a self-loop to each vertex of the set $S \subseteq V$. The number of vertices, edges and self-loops of G_S are n, m, σ, respectively. Let $\deg_G(v)$ be the degree of the vertex v in the graph G. Then, $\deg_{G_S}(v) = \deg_G(v) + 2$, if v has a self-loop and $\deg_{G_S}(v) = \deg_G(v)$ otherwise.

For a simple graph on n vertices, the associated adjacency matrix is the square matrix of order n, whose entries are 1, if the corresponding vertices are adjacent and it is 0 otherwise. The Hamming distance between two vertices u and v is defined as the number of positions where the corresponding entries in the rows of the adjacency matrix of the graph differ. It is denoted by $H_d(u, v : G)$. The Hamming index is the sum of Hamming distances between all the pair of vertices of a graph. It is denoted by $H_A(G)$ [6].

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In [8], the authors obtained expression foe Hamming distance between two strings in terms of number common and non-common neighbors. Hamming index of various product graphs can be found in [9, 10]. Hamming index of derived graphs are studied in [11]. The authors of [12], obtained Hamming index of few class of graphs and operation of two graphs.

Hamming weight analysis of bits has a significant role in information theory, coding theory and cryptography. The minimum Hamming distance is used as error detecting and error correcting codes. The Hamming distance is also used in biological systematic as a measure of genetic distance. For more information on Hamming graphs, one can refer [13, 14, 15].

The Hamming distance between two vertices in terms of degree of the vertices and number of common neighbors between the vertices is given in Theorem 1.1.

Theorem 1.1. [7] *The Hamming distance between the vertices u and v is*

$$H_d(u, v : G) = \deg_G(u) + \deg_G(v) - 2|N_G(u, v)|.$$

Where, $|N_G(u, v)|$ is the number of common neighbors between u and v in G.

For a graph G, the first Zagreb index is,

$$M_1(G) = \sum_{i=1}^{n} (\deg_G(v_i))^2.$$

For a graph G_S with σ self-loops, the adjacency matrix is $A(G_S) = (a_{ij})_S$, where

$$(a_{ij})_S = \begin{cases} 1, & \text{if } v_i \sim v_j \\ 1, & \text{if } v_i \text{ has a self-loop} \\ 0, & \text{otherwise.} \end{cases}$$

Hamming distance between two vertices of G_S is the number of positions where the corresponding entries in the rows of $A(G_S)$ differ and the sum of all Hamming distances gives the Hamming index of G_S based on adjacency matrix.

This article extends the concept of Hamming distance and Hamming index, originally defined using the adjacency matrix for simple graphs, to graphs that include self-loops. It investigates how the inclusion of self-loops alters the Hamming distance by comparing it with the Hamming distance of the corresponding original graph without self-loops. The study explores basic properties of this extended definition, focusing on how self-loops contribute to the differences in adjacency matrices and thus impact the overall Hamming distance.

2. Basic properties of Hamming index of a graph with self-loops

The following theorem gives the Hamming distance between two vertices of the graph G_S in terms of Hamming distance between two vertices of the graph G.

Theorem 2.1. *The Hamming distance between two vertices u and v in self-loops graph G_S is*

$$H_d(u,v:G_S) = H_d(u,v:G) + x.$$

Where,

$$x = \begin{cases} 0, & \text{if } u,v \notin S \\ -1, & \text{if } uv \in E(G),\ u \in S \text{ and } v \notin S \\ 1, & \text{if } uv \notin E(G),\ u \in S \text{ and } v \notin S \\ -2, & \text{if } uv \in E(G)\ u,v \in S \\ 2, & \text{if } uv \notin E(G)\ u,v \in S. \end{cases}$$

Proof. Consider the adjacency matrix of G_S. The sum of each row of $A(G_S)$ is $\deg_G(u)$ if $u \notin S$ and is $\deg_G(u)+1$ if $u \in S$. The number of common neighbors between two vertices $u,v \in V(G_S)$ depends on the presence of self-loops. The following are the different cases based on the presence of self-loops.

Case 1: Let $u,v \notin S$. Then, $\deg_{G_S}(u) = \deg_G(u)$ and $\deg_{G_S}(v) = \deg_G(v)$ and common neighbors between u and v remains as in G. Therefore,

$$H_d(u,v:G_S) = H_d(u,v:G).$$

Case 2: Let $uv \in E(G)$, $u \in S$ and $v \notin S$. Then, row sum corresponding to u is $\deg_G(u)+1$ and row sum corresponding to v is $\deg_G(v)$. Since u has a self-loop, $u \sim u$ and $v \sim u$. Therefore, u is a common neighbor to u and v. $|N_{G_S}(u,v)| = |N_G(u,v)|+1$. Hence,

$$\begin{aligned} H_d(u,v:G_S) &= \deg_G(u)+1+\deg_G(v) - 2(|N_G(u,v)|+1) \\ &= \deg_G(u)\deg_G(v) - 2|N_G(u,v)| - 1 \\ &= H_d(u,v:G) - 1. \end{aligned}$$

Case 3: Let $uv \notin E(G)$, $u \in S$ and $v \notin S$. Then, row sum corresponding to u is $\deg_G(u)+1$ and row sum corresponding to v is $\deg_G(v)$. The common neighbors between u and v remains as in G. Therefore,

$$\begin{aligned} H_d(u,v:G_S) &= \deg_G(u)+1+\deg_G(v) - 2|N_G(u,v)| \\ &= H_d(u,v:G) + 1. \end{aligned}$$

Case 4: Let $uv \in E(G)$ $u,v \in S$. Then, row sum corresponding to u is $\deg_G(u)+1$ and row sum corresponding to v is $\deg_G(v)+1$. Since u has a self-loop, $u \sim u$ and $v \sim u$. Since v has a self-loop, $v \sim v$ and $u \sim v$. Therefore, u and v are common neighbors to u and v. Hence,

$$\begin{aligned} H_d(u,v:G_S) &= \deg_G(u)+1+\deg_G(v)+1 - 2(|N_G(u,v)|+2) \\ &= \deg_G(u)\deg_G(v) - 2|N_G(u,v)| - 2 \\ &= H_d(u,v:G) - 2. \end{aligned}$$

Case 4: Let $uv \in E(G)$ $u,v \in S$. Then, row sum corresponding to u is $\deg_G(u)+1$ and row sum corresponding to v is $\deg_G(v)+1$. The common neighbors between u and v remains as in G. Therefore,

$$\begin{aligned} H_d(u,v:G_S) &= \deg_G(u)+1+\deg_G(v)+1 - 2|N_G(u,v)| \\ &= H_d(u,v:G) + 2. \end{aligned}$$

The theorem follows by combining all the cases. □

Remark 2.2. *The Hamming index of G_S based on adjacency matrix is*

$$\begin{aligned} H_A(G_S) &= \sum_{1\le i<j\le n} H_d(v_i, v_j : G_S) \\ &= \sum_{1\le i<j\le n} (H_d(v_i, v_j : G) + x) \\ &= H_A(G) + \sum_{1\le i<j\le n} x. \end{aligned}$$

Note that, $H_A(G_S) = H_A(G)$ if and only if $\sum_{1\le i<j\le n} x = 0$.

Theorem 2.3. *Let G_S be a graph with σ self-loops. Then*

$$H_A(G_S) = 2mn + (n-1)\sigma - M_1(G) - 2\sum_{v_i\in S} \deg_G(v_i),$$

where $M_1(G)$ is the first Zagreb index of G.

Proof. Let v_i and v_j be any two vertices of G_S. Then

$$H_d(v_i, v_j : G_S) = \deg_G(v_i) + \deg_G(v_j) + a - 2(|N_G(v_i, v_j)| + b).$$

Where,

$$a = \begin{cases} 0, & \text{if } v_i, v_j \notin S \\ 1, & \text{if either } v_i \in S \text{ or } v_j \in S \\ 2, & \text{if } v_i, v_j \notin S \end{cases}$$

and

$$b = \begin{cases} 0 & \text{if } u, v \notin S \text{ or } uv \notin E(G) \\ 1 & \text{if } uv \in E(G),\ u \in S \text{ and } v \notin S \\ 2 & \text{if } uv \in E(G)\ u, v \in S. \end{cases}$$

Therefore

$$\begin{aligned} H_A(G_S) &= \sum_{1\le i<j\le n} H_d(v_i, v_j : G_S) \\ &= \sum_{1\le i<j\le n} (\deg_G(v_i) + \deg_G(v_j) + a) - 2\sum_{1\le i<j\le n} (|N_{G_S}(v_i, v_j)| + b). \end{aligned} \tag{1}$$

But

$$\begin{aligned} \sum_{1\le i<j\le n} (\deg_G(v_i) + \deg_G(v_j)) &= (n-1)\sum_{i=1}^{n} \deg_G(v_i) \\ &= 2m(n-1), \end{aligned} \tag{2}$$

$$\sum_{1\le i<j\le n} a = (n-1)\sigma, \tag{3}$$

$$2\sum_{1\le i<j\le n}|N_G(v_i,v_j)| = 2\sum_{i=1}^{n}\binom{\deg_G(v_i)}{2}$$
$$= \sum_{i=1}^{n}(\deg_G(v_i))^2 - \sum_{i=1}^{n}(\deg_G(v_i))$$
$$= M_1(G) - 2m \tag{4}$$

and

$$\sum_{1\le i<j\le n} b = \sum_{v_i\in S}\deg_G(v_i). \tag{5}$$

By substituting Equations (2), (3), (4), and (5) in Equation (1),

$$H_A(G_S) = 2m(n-1) + (n-1)\sigma - M_1(G) + 2m - 2\sum_{v_i\ inS}\deg_G(v_i)$$
$$= 2mn + (n-1)\sigma - M_1(G) - 2\sum_{v_i\in S}\deg_G(v_i).$$

□

3. Hamming index of graphs with self-loops

Theorem 3.1. *Let G be an r-regular graph on n vertices and G_S be a graph with σ self-loops. Then*

$$H_A(G_S) = n^2r - nr^2 - (n-1-2r)\sigma,$$

where $\sigma = |S|$.

Proof. From Theorem 2.3,

$$H_A(G_S) = 2mn + (n-1)\sigma - M_1(G) - 2\sum_{v_i\in S}\deg_G(v_i).$$

If G_S is a self-loop graph obtained by an $r-$regular graph G, then, $2m = nr$, $M_1(G) = nr^2$, and $\sum_{v_i\in S}\deg_G(v_i) = \sigma r$. Therefore,

$$H_A(G_S) = n^2r - (n-1)\sigma - nr^2 - 2\sigma r$$
$$= nr(n-r) + (n-1-2r)\sigma.$$

□

Corollary 3.2.
(1) *$H_A(K_n)_S = (n-1)(n+\sigma)$. Where, K_n is a complete graph on n vertices.*
(2) *$H_A(K_{n,n})_S = 2n^2 + \sigma$. Where $K_{n,n}$ is a complete bipartite graph on $2n$ vertices.*
(3) *$H_A(S_n^0)_S = 2n(n^2-1) - \sigma$. Where, S_n^0 is a crown graph which is obtained by removing a perfect matching from $K_{n,n}$.*
(4) *$H_A(C_n)_S = 2n(n-2) - (n-5)\sigma$. Where, C_n is a cycle on n vertices, $n \ge 5$.*

Proof. From Theorem 3.1, for a graph G_S obtained by attaching a self-loop to each vertex of the set S of an $r-$regular graph,

$$H_A(G_S) = n^2r - nr^2 - (n-1-2r)\sigma.$$

(1) Consider K_n which is an $(n-1)-$regular graph. Then
$$\begin{aligned} H_A(K_n)_S &= n^2(n-1) - n(n-1)^2 - (n-1-2(n-1))\sigma \\ &= (n-1)(n+\sigma). \end{aligned}$$
(2) Consider $K_{n,n}$ which is an $n-$regular graph on $2n$ vertices. Then
$$\begin{aligned} H_A(K_{n,n})_S &= (2n)^2(n) - 2n(n)^2 - (2n-1-2n)\sigma \\ &= 2n^2 + \sigma. \end{aligned}$$
(3) Consider S_n^0 which is an $(n-1)-$regular graph on $2n$ vertices. Then
$$\begin{aligned} H_A(S_n^0)_S &= (2n)^2(n-1) - 2n(n-1)^2 - (2n-1-2(n-1))\sigma \\ &= 2n(n^2-1) - \sigma. \end{aligned}$$
(4) Consider C_n, $n \geq 5$, which is a $2-$regular graph. Then
$$\begin{aligned} H_A(C_n)_S &= n^2(2) - n(2)^2 - (n-1-2(2))\sigma \\ &= 2n(n-2) - (n-5)\sigma. \end{aligned}$$

□

Theorem 3.3. *Let $(K_{p,q})_S$ be the complete bipartite graph with self-loops and U and W be the partition of $V(G)$ such that $|U| = p$ and $|W| = q$ and σ be the total number of self-loops. Then,*
$$H_A(K_{p,q})_S = pq(p+q) + (p-q-1)\sigma_1 + (q-p-1)\sigma_2.$$
Where σ_1 and σ_2 are the number of self-loops in U and W.

Proof. We know that
$$H_A(G_S) = 2mn + (n-1)\sigma - M_1(G) - 2\sum_{v_i \in S} \deg_G(v_i). \tag{6}$$
But in $(K_{p,q})_S$, $m = pq$, $n = p+q$, $\sigma = \sigma_1 + \sigma_2$, $M_1(G) = p^2q + pq^2$, and $\sum_{v_i \in S} \deg_G(v_i) = \sigma_1 q + \sigma_2 p$.

By substituting the values in Equation (6), we get
$$\begin{aligned} H_A(K_{p,q})_S &= 2pq(p+q) + (n-1)(\sigma_1+\sigma_2) - (pq^2 + p^2q) - 2q\sigma_1 - 2p\sigma_2 \\ &= pq(p+q) + (p-q-1)\sigma_1 + (q-p-1)\sigma_2. \end{aligned}$$

□

4. Hamming index of operation of two graphs with self-loops

Theorem 4.1. *Let $(G_1)_{S_1} \cup (G_2)_{S_2}$ be union of two graphs $(G_1)_{S_1}(n_1, m_1)$ and $(G_2)_{S_2}(n_2, m_2)$. Then,*
$$H_A(G_1 \cup G_2)_S = H_{A_1}(G_1)_{S_1} + H_{A_2}(G_2)_{S_2} + 2m_1n_2 + 2m_2n_1 + n_1\sigma_2 + n_2\sigma_1$$

Proof. The vertex set of $(G_1)_{S_1} \cup (G_2)_{S_2}$ is $V(G_1)_{S_1} \cup V(G_2)_{S_2}$, the edge set is $E(G_1)_{S_1} \cup E(G_2)_{S_2}$ and set of self-loops is $S_1 \cup S_2$. Let $|V(G_1)_{S_1}| = n_1$, $|V(G_2)_{S_2}| = n_2$, $|E(G_1)_{S_1}| = m_1$, $|E(G_2)_{S_2}| = m_2$, $|S_1| = \sigma_1$, and $|S_2| = \sigma_2$. Then $|V(G_1)_{S_1} \cup V(G_2)_{S_2}| = n_1 + n_2$, $|E(G_1)_{S_1} \cup E(G_2)_{S_2}| = m_1 + m_2$, and $|S_1 \cup S_2| = \sigma_1 + \sigma_2$. Consider
$$H_A(G_S) = 2mn + (n-1)\sigma - M_1(G) - 2\sum_{v_i \in S} \deg_G(v_i). \tag{7}$$

By substituting $m = m_1 + m_2$, $n = n_1 + n_2$, $\sigma = \sigma_1 + \sigma_2$, $M_1(G) = M_1(G_1) + M_1(G_2)$, and $\sum_{v_i \in S} \deg_G(v_i) = \sum_{v_i \in S_1} \deg_G(v_i) + \sum_{v_i \in S_2} \deg_G(v_i)$ in Equation (7), we get

$$\begin{aligned} H_A(G_S) &= 2(m_1+m_2)(n_1+n_2) + (n_1+n_2-1)(\sigma_1+\sigma_2) - (M_1(G_1)+M_1(G_2)) \\ &\quad - 2\left(\sum_{v_i \in S_1} \deg_G(v_i) + \sum_{v_j \in S_2} \deg_G(v_j)\right) \\ &= 2m_1n_1 + (n_1-1)\sigma_1 - M_1(G_1) - 2\sum_{v_i \in S_1} \deg_G(v_i) \\ &\quad + 2m_2n_2 + (n_2-1)\sigma_2 - M_2(G_2) - 2\sum_{v_i \in S_2} \deg_G(v_i) \\ &\quad + 2m_1n_2 + 2m_2n_1 + n_1\sigma_2 + n_2\sigma_1 \\ &= H_{A_1}(G_1)_{S_1} + H_{A_2}(G_2)_{S_2} + 2m_1n_2 + 2m_2n_1 + n_1\sigma_2 + n_2\sigma_1. \end{aligned}$$

□

Theorem 4.2. *Let $(G_1)_{S_1} + (G_2)_{S_2}$ be join of two graphs $(G_1)_{S_1}(n_1, m_1)$ and $(G_2)_{S_2}(n_2, m_2)$. Then,*

$$\begin{aligned} H_A(G_1+G_2)_S = H_{A_1}(G_1)_{S_1} + H_{A_2}(G_2)_{S_2} + +n_1^2n_2 + n_1n_2^2 - n_1\sigma_2 - n_2\sigma_1 \\ - 2n_1m_2 - 2n_2m_1. \end{aligned}$$

Proof. The vertex set of $(G_1)_{S_1} + (G_2)_{S_2}$ is $V(G_1)_{S_1} \cup V(G_2)_{S_2}$, the edge set is $E(G_1)_{S_1} \cup E(G_2)_{S_2}$ along with the edges joining each vertex of $(G_1)_{S_1}$ with each vertex of $(G_2)_{S_2}$, and set of self-loops is $S_1 \cup S_2$. Let $|V(G_1)_{S_1}| = n_1$, $|V(G_2)_{S_2}| = n_2$, $|E(G_1)_{S_1}| = m_1$, $|E(G_2)_{S_2}| = m_2$, $|S_1| = \sigma_1$, and $|S_2| = \sigma_2$. Then $|V(G_1)_{S_1} \cup V(G_2)_{S_2}| = n_1 + n_2$, $|E(G_1 + G_2)_S| = m_1 + m_2 + n_1n_2$, and $|S_1 \cup S_2| = \sigma_1 + \sigma_2$.

By substituting $m = m_1 + m_2 + n_1n_2$, $n = n_1 + n_2$, $\sigma = \sigma_1 + \sigma_2$, $M_1(G) = M_1(G_1) + M_1(G_2) + n_1^2n_2 + n_1n_2^2 + 4n_1m_2 + 4n_2m_1$, and $\sum_{v_i \in S} \deg_G(v_i) = \sum_{v_i \in S_1} (\deg_G(v_i) + n_2) + \sum_{v_i \in S_2} (\deg_G(v_i) + n_1)$ in Equation (7), we get

$$\begin{aligned} H_A(G_S) &= 2(m_1+m_2+n_1n_2)(n_1+n_2) + (n_1+n_2-1)(\sigma_1+\sigma_2) \\ &\quad - (M_1(G_1) + M_1(G_2) + n_1^2 + n_1^2n_2 + n_1n_2^2 + 4n_1m_2 + 4n_2m_1) \\ &\quad - 2\left(\sum_{v_i \in S_1} (\deg_G(v_i) + n_2) + \sum_{v_i \in S_2} (\deg_G(v_i) + n_1)\right) \\ &= 2m_1n_1 + (n_1-1)\sigma_1 - M_1(G_1) - 2\sum_{v_i \in S_1} \deg_G(v_i) \\ &\quad + 2m_2n_2 + (n_2-1)\sigma_2 - M_2(G_2) - 2\sum_{v_i \in S_2} \deg_G(v_i) \\ &\quad + n_1^2n_2 + n_1n_2^2 - n_1\sigma_2 - n_2\sigma_1 - 2m_1n_2 - 2m_2n_1 \\ &= H_{A_1}(G_1)_{S_1} + H_{A_2}(G_2)_{S_2} + +n_1^2n_2 + n_1n_2^2 - n_1\sigma_2 - n_2\sigma_1 \\ &\quad - 2n_1m_2 - 2n_2m_1. \end{aligned}$$

□

5. Conclusion

In this article, the Hamming distance between two vertices and the Hamming index of graphs with self-loops are studied with respect to the adjacency matrix. An expression for the Hamming distance between two vertices in a graph with self-loops is derived in terms of the Hamming distance between those two vertices in the corresponding simple graph and a variable x The Hamming index of a graph with self-loops is expressed in terms of the number of vertices, number of edges, number of self-loops, the First Zagreb index, and the degrees of the vertices that have self-loops. The Hamming index of regular graphs with self-loops is obtained, and the Hamming indices of several standard graphs with self-loops are also computed.

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