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CORDIALNESS OF CORONA PRODUCT OF STAR AND PATH

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Abstract. The corona $G \odot H$ of two graphs G with $V(G) = \{v_1, \ldots, v_{p_1}\}$ and H with p_2 vertices is a graph obtained by taking one copy of G and p_1 copies of H , and then for each $v_i \in V(G)$, joining the vertex v_i with an edge to every vertex in the i^{th} copy of H. Cordial labeling is an assignment of labels to the vertices of the graph G from the set $\{0,1\}$ which induces the edge labeling for each edges as the sum of their end vertices congruent modulo 2 such that the difference between the number of vertices labelled 0 and 1 is at most one as well as the difference between the number of edges receiving labels 0 and 1 is at most 1. Deciding whether a given graph is cordial is an NP-complete problem. In this paper we prove that the corona product of Star and Path admits a cordial labeling.

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1. Introduction

Throughout this paper we consider a graph to be simple and connected. Labeling of a graph is an assignment of numbers (labels) to the vertices or edges, or both, of the graph subject to certain conditions.

In 1967, Rosa [11] introduced a hierarchical series of labeling called ρ, σ, β and α labelings as a tool to settle a famous conjecture on graph decomposition, posed by Ringel [10], which states that "A complete graph on $2m+1$ vertices can be decomposed into $2m+1$ copies of any tree with m edges". The conjecture still remains open due to the difficulty of labeling trees. Later on, many weaker versions of the above labelings were introduced. In 1987, Cahit [2] introduced cordial labeling as a weaker version of β labeling (which is also famously called graceful labeling). Let G be a graph and let f be a labeling defined on the vertices of G, that is, $f : V(G) \to \{0,1\}$ which induces the edge labeling of G as $g : E(G) \to \{0,1\}$ by $g(uv) = |f(u) - f(v)|$. Let $\mathcal{V}_i$ denotes the number of vertices labeled i, $i = 0,1$ in G and $\mathcal{E}_i$ denotes the number of edges labeled i, $i = 0,1$ in G. The above labeling f to G is said to be cordial if $|\mathcal{V}_0 - \mathcal{V}_1| \leq 1$ and $|\mathcal{E}_0 - \mathcal{E}_1| \leq 1$. A graph G is said to be cordial if it admits a cordial labeling.

It is interesting to note that, even though cordial labeling is a weaker version of the graceful labeling, some graphs are graceful but not cordial. For example, K_4 is graceful but not cordial. Conversely, there are some

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graphs are cordial but not graceful, for example, K_5^2 (one vertex union of two copies of K_5) is cordial, but not graceful. Thus, understanding the structural property of cordial graphs is an independently interesting and difficult problem. Cairnie and Edwards [9] proved that deciding whether a given graph is cordial is an NP-Complete problem. We refer to [4] for a brief survey on cordial graphs.

1.1. **The Corona between Star and Path.** Let P_n, S_n and C_n denote a path with n vertices, a star with $n+1$ vertices, and a cycle with n vertices respectively.

Let G be a graph with $V(G) = \{v_1, \ldots, v_{p_1}\}$ and q_1 edges, called the center graph, and H be another graph with p_2 vertices and q_2 edges, called the outer graph. The corona between the graphs G and H, denoted as $G \odot H$, is defined as the graph obtained by taking p_1 copies of H and for each i, $1 \leq i \leq p_1$, the vertex v_i of G is made adjacent to all the vertices of the i^{th} copy of H.

Observe that, $|V(G \odot H)| = p_1(1+p_2)$ and $|E(G \odot H)| = q_1 + p_1(p_2+q_2)$ In general, $G \odot H \ncong H \odot G$.

Nada et. al. [5] proved that $P_n \odot C_m$ is cordial if and only if $(n, m) \equiv (0, 2) \pmod 4$. Nada and et. al. [6] proved that $C_n \odot P_m$ is cordial for all $n \geq 3, m \geq 1$. Nada and et. al. [7] gave the necessary and sufficient conditions for cordial labeling of the corona product between paths and fourth power of paths be cordial. Ashraf Elrokh and et. al. [1] proved that cone and lemniscate graphs to be cordial. Nada and et. al. [8] introduced the concept of Total Product cordial labeling and they have proved that the corona product of paths and second power of fan graphs to be total cordial. Elmshtaye and et. al. [3] proved that total cordial for the corona product of paths and the third power of double fans-generalized fans. Motivated from the above results, In this paper, we prove that, $S_n \odot P_m$ admits cordial labeling for all $n, m \geq 1$.

From the above observation, we have $|V(S_n \odot P_m)| = (n+1)(1+m)$ and $|E(S_n \odot P_m)| = n + (n+1)(m+m-1) = 2m(n+1) - 1$. Hence the number of vertices in $S_n \odot P_m$ is odd only when n and m are even, otherwise, it is even and the number of edges of $S_n \odot P_m$ is always odd. Hence if $S_n \odot P_m$ admits a cordial labeling then $|\mathcal{E}_0 - \mathcal{E}_1| = 1$ and

$$|\mathcal{V}_0 - \mathcal{V}_1| = \begin{cases} 1, & \text{if } n \text{ and } m \text{ even} \\ 0, & \text{otherwise .} \end{cases}$$

2. Cordialness of Corona between Star and Path

In this section the cordial labeling of corona product of Star and Path is established. Before proving the main result, we prove few lemmas which will support our main result.

Lemma 2.1. *The corona product graph $S_n \odot P_1$, $n \geq 1$ admits cordial labeling.*

Proof. Consider the graph $S_n \odot P_1, n \geq 1$. Let the vertices of S_n be named as $v_0, v_1, v_2, ..., v_n$ where v_0 is the center of S_n and let $P^i : u_1^i$ be the i^{th} copy of the path P_1 which is connected to the vertex v_i of S_n, for $i, 0 \leq i \leq n$.

Let us define the labeling f on the vertices of $S_n \odot P_1$ by first labeling the vertices of S_n as $f(v_i) = 1$, for $i, 0 \leq i \leq n$.

Then label the vertices of the i^{th} copy of P_1, for all i, $0 \leq i \leq n$ as $f(u_1^i) = 0$.

From the above labeling, we observe that $\mathcal{V}_0 = n+1 = \mathcal{V}_1$ and $\mathcal{E}_0 = n, \mathcal{E}_1 = n+1$. Hence $|\mathcal{V}_0 - \mathcal{V}_1| = 0$ and $|\mathcal{E}_0 - \mathcal{E}_1| = 1$. Thus the above labeling is cordial for the graph $S_n \odot P_1$, $n \geq 1$. □

An example of cordially labeled corona product graph $S_n \odot P_1$ for $n = 4$ and $n = 5$ is given in Figure 1.

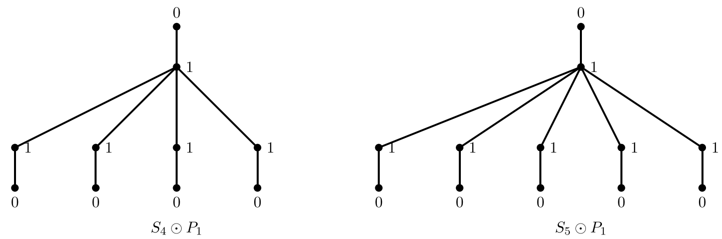

FIGURE 1. The graph $S_n \odot P_1, n = 4, 5$ with cordial labeling

Lemma 2.2. *The corona product graph $S_n \odot P_2$, $n \geq 1$ admits cordial labeling.*

Proof. Consider the graph $S_n \odot P_2, n \geq 1$. Let the vertices of S_n be named as $v_0, v_1, v_2, ..., v_n$ where v_0 is the center of S_n and let $P_2^i : u_1^i u_2^i$ be the i^{th} copy of the path P_2 which is connected to the vertex v_i of S_n, for $i, 0 \leq i \leq n$.

Let us define the labeling f on the vertices of $S_n \odot P_2$ by first labeling the vertices of S_n as $f(v_i) = 1$, for $i, 0 \leq i \leq n$.

Then label the vertices of the i^{th} copy of P_2, for all i, $0 \leq i \leq n$ as follows.

If i is even (including 0), then $f(u_1^i) = 1$ and $f(u_2^i) = 0$

If i is odd, then $f(u_1^i) = 0$ and $f(u_2^i) = 0$.

From the above labeling, we observe that,

$$\mathcal{V}_0 = \begin{cases} \frac{3n+2}{2}, & n \text{ is even} \\ \frac{3n+3}{2}, & n \text{ is odd} \end{cases} \quad ; \quad \mathcal{V}_1 = \begin{cases} \frac{3n+4}{2}, & n \text{ is even} \\ \frac{3n+3}{2}, & n \text{ is odd} \end{cases}$$

and $\mathcal{E}_0 = 2n+1$, $\mathcal{E}_1 = 2n+2$. Hence

$$|\mathcal{V}_0 - \mathcal{V}_1| = \begin{cases} 1, & n \text{ is even} \\ 0, & n \text{ is odd} \end{cases} \quad \text{and} \quad |\mathcal{E}_0 - \mathcal{E}_1| = 1$$

Thus the above labeling yield a cordial labeling for the graph $S_n \odot P_2$, $n \geq 1$. □

An example of cordially labeled corona product graph $S_n \odot P_2$ for $n = 4$ and $n = 5$ is given in Figure 2.

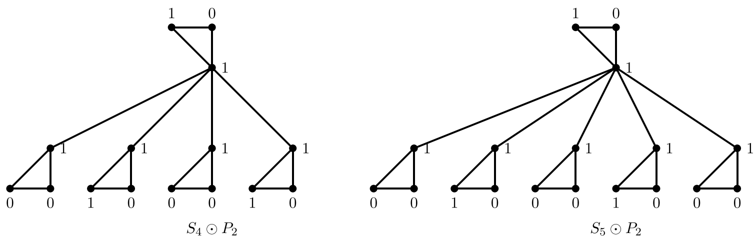

FIGURE 2. The graph $S_n \odot P_2, n = 4, 5$ with cordial labeling

Lemma 2.3. *The corona product graph $S_n \odot P_3$, $n \geq 1$ admits cordial labeling.*

Proof. Consider the graph $S_n \odot P_3$. Let the vertices of S_n be named as $v_0, v_1, v_2, ..., v_n$ where v_0 is the center of S_n and let $P_3^i : u_1^i u_2^i u_3^i$ be the i^{th} copy of the path P_3 which is connected to the vertex v_i of S_n, for $i, 0 \leq i \leq n$.

Let us define the labeling f on the vertices of $S_n \odot P_3$ by first labeling the vertices of S_n as $f(v_i) = i + 1 \pmod 2$, for $i, 0 \leq i \leq n$.

Then label the vertices of the i^{th} copy of P_3, for i, $0 \leq i \leq n$ as follows:
If i is even , then $f(u_1^i) = 1, f(u_2^i) = 1$ and $f(u_3^i) = 0$
If i is odd, then $f(u_1^i) = 0, f(u_2^i) = 1$ and $f(u_3^i) = 0$.
When $i = 0$ and n is even, $f(u_1^0) = 0, f(u_2^0) = 0$ and $f(u_3^0) = 1$.
When $i = 0$ and n is odd, $f(u_1^0) = 1, f(u_2^0) = 1$ and $f(u_3^0) = 0$.
From the above labeling, we observe that $\mathcal{V}_0 = 2n + 2 = \mathcal{V}_1$ and $\mathcal{E}_0 = 3n + 2, \mathcal{E}_1 = 3n + 3$. Hence $|\mathcal{V}_0 - \mathcal{V}_1| = 0$ and $|\mathcal{E}_0 - \mathcal{E}_1| = 1$. Thus the above labeling is cordial for the graph $S_n \odot P_3$, $n \geq 1$. □

An example of cordially labeled corona product graph $S_n \odot P_3$ for $n = 4$ and $n = 5$ is given in Figure 3.

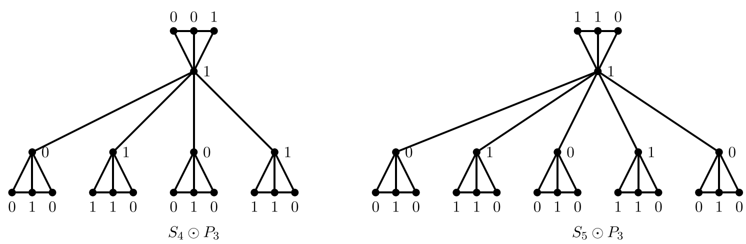

FIGURE 3. The graph $S_n \odot P_3, n = 4, 5$ with cordial labeling

2.1. **Cordial labeling of $S_n \odot P_m$, $m \geq 4$.** In the labeling process of $S_n \odot P_m$, $m \geq 4$, we first label the vertices of S_n ($v_0, v_1, v_2, ..., v_n$, where v_0 is the center of star). And then labeling the copies of P_m. We denote $P_m^i : u_1^i u_2^i \cdots u_m^i$ be the i^{th} copy of P_m which is connected to the vertex v_i of S_n, for $i, 0 \leq i \leq n$.

Let f_m^i be the labeling that is defined on the vertices of P_m^i, for $i, 0 \leq i \leq n$.
When $m \equiv 0 \pmod 4$, that is, $m = 4s$, for some $s \in N, s \geq 1$, we define the labeling f_{4s}^i as $\overline{xyzt}$ ($xyzt$ repeated s times).

That is,

$$f(u_j^i) = \begin{cases} x, & j \equiv 1 \pmod 4 \\ y, & j \equiv 2 \pmod 4 \\ z, & j \equiv 3 \pmod 4 \\ t, & j \equiv 0 \pmod 4 \end{cases}$$

Similarly, when $m \equiv 1, 2$ and $3 \pmod 4$, We define the labeling f_m^i as $\overline{xyzt}a$, $\overline{xyzt}ab$ and $\overline{xyzt}abc$ respectively. Here $xyzt$ repeated s times.

More precisely, the labeling f_m^i is given in Table 1.

TABLE 1. The labeling of the i^{th} copy of $P_m, m \equiv 1, 2$ and $3 \pmod 4$

	If $m = 4s + 1, s \geq 1$	If $m = 4s + 2, s \geq 1$	If $m = 4s + 3, s \geq 1$
$f(u_j^i) =$	$\begin{cases} x, & j \equiv 1 \pmod 4 \\ y, & j \equiv 2 \pmod 4 \\ z, & j \equiv 3 \pmod 4 \\ t, & j \equiv 0 \pmod 4 \\ a, & j = 4s + 1 \end{cases}$	$\begin{cases} x, & j \equiv 1 \pmod 4 \\ y, & j \equiv 2 \pmod 4 \\ z, & j \equiv 3 \pmod 4 \\ t, & j \equiv 0 \pmod 4 \\ a, & j = 4s + 1 \\ b, & j = 4s + 2 \end{cases}$	$\begin{cases} x, & j \equiv 1 \pmod 4 \\ y, & j \equiv 2 \pmod 4 \\ z, & j \equiv 3 \pmod 4 \\ t, & j \equiv 0 \pmod 4 \\ a, & j = 4s + 1 \\ b, & j = 4s + 2 \\ c, & j = 4s + 3 \end{cases}$

For a given labeling of $S_n \odot P_m$, we denote x_i and y_i to be the number of vertices labeled i of S_n and the number of edges labeled i of S_n respectively. Let a_i^j be the number of vertices labeled i of the copies of P_m which are connected to the vertices labeled j of S_n and let b_i^j be the number of edges labeled i of the edges copies of P_m which are connected to the vertices labeled j of S_n. Let c_i^j be the number of edges labeled i of the edges which are connected to the vertices labeled j of S_n to the copies of P_m.

Hence $\mathcal{V}_i = x_i + a_i^0 + a_i^1$ and $\mathcal{E}_i = y_i + b_i^0 + c_i^0 + b_i^1 + c_i^1$, for $i, i = 0, 1$.

Theorem 2.4. *The corona product graph $S_n \odot P_m$, $m \geq 4$ admits cordial labeling.*

Proof. Consider the graph $S_n \odot P_m$. Let $S_n : v_0, v_1, v_2, ..., v_n$ be a star and let $P_m^i : u_1^i u_2^i \cdots u_m^i$ be the i^{th} copy of P_m which is connected to the vertex v_i of S_n, for $i, 0 \leq i \leq n$.
We define the labeling on the vertices of $S_n \odot P_m$ as follows:
We first label the vertices of S_n by $f(v_i) = i + 1 \pmod 2$, for $i, 0 \leq i \leq n$. Then define the labeling f_m^i on the copies of $P_m, m \geq 4$, which is connected with the vertex v_i, where $0 \leq i \leq n$, of S_n as follows,
For $m \equiv 0, 1, 2 \pmod 4$, the labeling f_m^i is given in Table 2.

TABLE 2. The labeling f_m^i defined on the i^{th} copy of P_m, for $m \equiv 0, 1, 2 \pmod 4$

For m	For $0 \le i \le n$	
	$i \ge 0$ and i is even including 0	$i \ge 1$ and i is odd
$m \equiv 0 \pmod 4$	$f_m^i = \overline{0110}$	$f_m^i = \overline{0011}$
$m \equiv 1 \pmod 4$	$f_m^i = \overline{0110}0$	$f_m^i = \overline{0011}1$
$m \equiv 2 \pmod 4$	$f_m^i = \overline{0011}01$	$f_m^i = \overline{0011}10$

When $m \equiv 3 \pmod 4$, the labeling f_m^i is given in Table 3.

TABLE 3. The labeling f_m^i defined on the i^{th} copy of P_m, for $m \equiv 3 \pmod 4$

For all n	For $0 \le i \le n$		
	if $i = 0$	if $i \ge 1$ and i is even	if $i \ge 1$ and i is odd
n is odd	$f_m^i = \overline{0011}011$	$f_m^i = \overline{0011}011$	$f_m^i = \overline{0011}010$
n is even	$f_m^i = \overline{0011}001$	$f_m^i = \overline{0011}011$	$f_m^i = \overline{0011}010$

From the labeling f, that is defined on the vertices of S_n, Table 4 gives the values of x_i and $y_i, i = 0, 1$.

TABLE 4. The values x_i and $y_i, i = 0, 1$ obtained from the labeling f

For all m	x_0	x_1	y_0	y_1
n is even	$\frac{n}{2}$	$\frac{n}{2} + 1$	$\frac{n}{2}$	$\frac{n}{2}$
n is odd	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$

From the labeling f_m^i that is defined on the copies of $P_m, m \ge 4$, which are connected to the vertices v_i, where $0 \le i \le n$, of S_n, Table 5 gives the values of $a_i^j, i = 0, 1; j = 0, 1$.

TABLE 5. The values of $a_i^j, i = 0, 1; j = 0, 1$ which are obtained from the labeling f_m^i

For m	n	$a_i^j, i = 0, 1; j = 0, 1$			
		a_0^0	a_0^1	a_1^0	a_1^1
$m = 4s$	n is even	ns	$ns + 2s$	ns	$ns + 2s$
	n is odd	$ns + s$	$ns + s$	$ns + s$	$ns + s$
$m = 4s + 1$	n is even	ns	$ns + 2s + \frac{n}{2} + 1$	$ns + \frac{n}{2}$	$ns + 2s$
	n is odd	$ns + s$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$	$ns + s$
$m = 4s + 2$	n is even	$ns + \frac{n}{2}$	$ns + 2s + \frac{n}{2} + 1$	$ns + \frac{n}{2}$	$ns + 2s + \frac{n}{2} + 1$
	n is odd	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$
$m = 4s + 3$	n is even	$ns + n$	$ns + \frac{n}{2} + 2s + 2$	$ns + \frac{n}{2}$	$ns + n + 2s + 1$
	n is odd	$ns + n + s + 1$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$	$ns + n + s + 1$

TABLE 6. The values of $\mathcal{V}_i, i = 0, 1$ which obtained from the labeling that is defined on $S_n \odot P_m$

m	n	$\mathcal{V}_i, i = 0, 1$	
		$\mathcal{V}_0$	$\mathcal{V}_1$
$m = 4s$	n is even	$2ns + 2s + \frac{n}{2}$	$2ns + 2s + \frac{n}{2} + 1$
	n is odd	$2ns + 2s + \frac{n+1}{2}$	$2ns + 2s + \frac{n+1}{2}$
$m = 4s + 1$	n is even	$2ns + 2s + n + 1$	$2ns + 2s + n + 1$
	n is odd	$2ns + 2s + n + 1$	$2ns + 2s + n + 1$
$m = 4s + 2$	n is even	$2ns + 2s + n + \frac{n}{2} + 1$	$2ns + 2s + n + \frac{n}{2} + 2$
	n is odd	$2ns + 2s + 3(\frac{n+1}{2})$	$2ns + 2s + 3(\frac{n+1}{2})$
$m = 4s + 3$	n is even	$2ns + 2s + 2n + 2$	$2ns + 2s + 2n + 1$
	n is odd	$2ns + 2s + 2n + 2$	$2ns + 2s + 2n + 2$

Hence we have $\mathcal{V}_i = x_i + a_i^0 + a_i^1$, for $i = 0, 1$ which is given in Table 6. From the labeling f_m^i that is defined on the copies of $P_m, m \geq 4$, which are connected to the vertices v_i, where $0 \leq i \leq n$, of S_n, Table 7 gives the values of $b_i^j, i = 0, 1; j = 0, 1$.

TABLE 7. The values of $b_i^j, i = 0, 1; j = 0, 1$ which obtained from the labeling f_m^i

m	n	$b_i^j, i = 0, 1; j = 0, 1$			
		b_0^0	b_0^1	b_1^0	b_1^1
$m = 4s$	n is even	ns	$ns + 2s - \frac{n}{2} - 1$	$ns - \frac{n}{2}$	$ns + 2s$
	n is odd	$ns + s$	$ns + s - \frac{n+1}{2}$	$ns + s - \frac{n+1}{2}$	$ns + s$
$m = 4s + 1$	n is even	$ns + \frac{n}{2}$	$ns + 2s$	$ns - \frac{n}{2}$	$ns + 2s$
	n is odd	$ns + s + \frac{n+1}{2}$	$ns + s$	$ns + s - \frac{n+1}{2}$	$ns + s$
$m = 4s + 2$	n is even	$ns + \frac{n}{2}$	$ns + 2s$	ns	$ns + 2s + \frac{n}{2} + 1$
	n is odd	$ns + s + \frac{n+1}{2}$	$ns + s$	$ns + s$	$ns + s + \frac{n+1}{2}$
$m = 4s + 3$	n is even	ns	$ns + 2s + \frac{n}{2} + 1$	$ns + n$	$ns + 2s + \frac{n}{2} + 1$
	n is odd	$ns + s$	$ns + s + \frac{n+1}{2}$	$ns + s + n + 1$	$ns + s + \frac{n+1}{2}$

From the labeling f_m^i that is defined on the copies of $P_m, m \geq 4$, which are connected to the vertices v_i, where $0 \leq i \leq n$, of S_n, Table 8 gives the values of $c_i^j, i = 0, 1; j = 0, 1$.

Table 8. The values of $c_i^j, i = 0,1; j = 0,1$ which obtained from the labeling f_m^i

m	n	$c_i^j, i = 0,1; j = 0,1$			
		c_0^0	c_0^1	c_1^0	c_1^1
$m = 4s$	n is even	ns	$ns + 2s$	ns	$ns + 2s$
	n is odd	$ns + s$	$ns + s$	$ns + s$	$ns + s$
$m = 4s + 1$	n is even	ns	$ns + 2s$	$ns + \frac{n}{2}$	$ns + 2s + \frac{n}{2} + 1$
	n is odd	$ns + s$	$ns + s$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$
$m = 4s + 2$	n is even	$ns + \frac{n}{2}$	$ns + 2s + \frac{n}{2} + 1$	$ns + \frac{n}{2}$	$ns + 2s + \frac{n}{2} + 1$
	n is odd	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$
$m = 4s + 3$	n is even	$ns + n$	$ns + n + 2s + 1$	$ns + \frac{n}{2}$	$ns + \frac{n}{2} + 2s + 2$
	n is odd	$ns + s + n + 1$	$ns + s + n + 1$	$ns + s + \frac{n+1}{2}$	$ns + s + \frac{n+1}{2}$

Hence we have $\mathcal{E}_i = y_i + b_i^0 + c_i^0 + b_i^1 + c_i^1$, for $i, i = 0,1$ which is given in Table 9.

Table 9. The values of $\mathcal{E}_i, i = 0,1$ which obtained from the labeling that defined on $S_n \odot P_m$

m	n	$e_i, i = 0,1$	
		e_0	e_1
$m = 4s$	n is even	$4ns + 4s - 1$	$4ns + 4s$
	n is odd	$4ns + 4s - 1$	$4ns + 4s$
$m = 4s + 1$	n is even	$4ns + 4s + n$	$4ns + 4s + n + 1$
	n is odd	$4ns + 4s + n$	$4ns + 4s + n + 1$
$m = 4s + 2$	n is even	$4ns + 4s + 2n + 1$	$4ns + 4s + 2n + 2$
	n is odd	$4ns + 4s + 2n + 1$	$4ns + 4s + 2n + 2$
$m = 4s + 3$	n is even	$4ns + 4s + 3n + 2$	$4ns + 4s + 3n + 3$
	n is odd	$4ns + 4s + 3n + 2$	$4ns + 4s + 3n + 3$

Therefore, from Table 6 and Table 9, the values of $|\mathcal{V}_0 - \mathcal{V}_1|$ and $|\mathcal{E}_0 - \mathcal{E}_1|$ is given in Table 10.

Table 10. The values of $|\mathcal{V}_0 - \mathcal{V}_1|$ and $|\mathcal{E}_0 - \mathcal{E}_1|$ which obtained from the labeling that defined on $S_n \odot P_m$

m	n	$\|\mathcal{V}_0 - \mathcal{V}_1\|$	$\|\mathcal{E}_0 - \mathcal{E}_1\|$
$m = 4s$	n is even	1	1
	n is odd	0	1
$m = 4s + 1$	n is even	0	1
	n is odd	0	1
$m = 4s + 2$	n is even	1	1
	n is odd	0	1
$m = 4s + 3$	n is even	0	1
	n is odd	0	1

From Table 10, we observe that the labeling defined on $S_n \odot P_m, m \geq 4$, is a cordial labeling. $\square$

The following theorem follows from Lemmas 2.1, Lemma 2.2, Lemma 2.3 and Theorem 2.4.

Theorem 2.5. *The corona product graph* $S_n \odot P_m$*,* $m, n \geq 1$*, admits cordial labeling.*

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3. Appendix

An example of cordially labeled corona product graph $S_n \odot P_m$ for $n = 3, 4$ and $m = 4s$, where $s = 1$ is given in Figure 4.

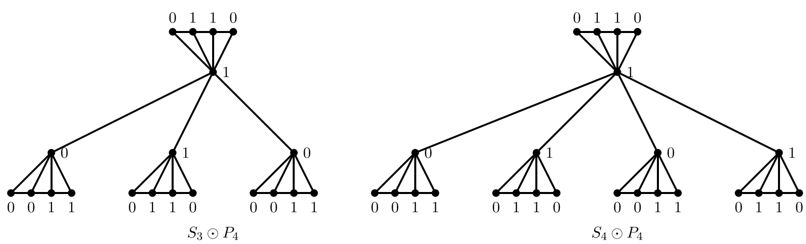

FIGURE 4. The graph $S_n \odot P_4, n = 3, 4$ with cordial labeling

An example of cordially labeled corona product graph $S_n \odot P_m$ for $n = 4$ and $m = 4s + 1$, where $s = 2$ is given in Figure 5.

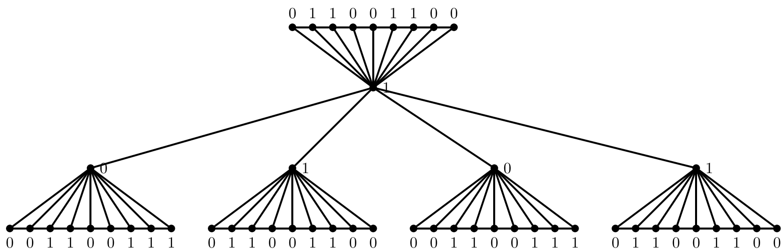

FIGURE 5. The graph $S_4 \odot P_9$ with cordial labeling

An example of cordially labeled corona product graph $S_n \odot P_m$ for $n = 4$ and $m = 4s + 2$ where $s = 2$ is given in Figure 6.

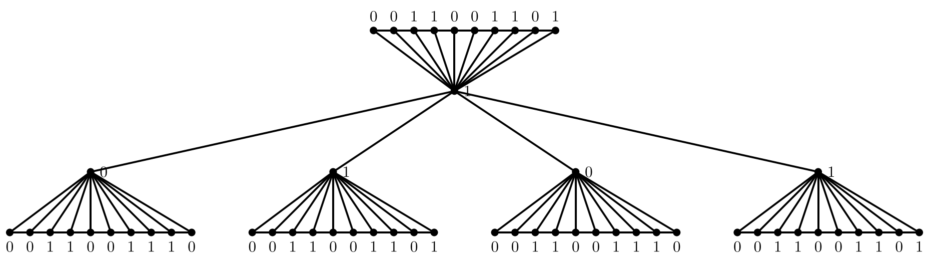

FIGURE 6. The graph $S_4 \odot P_{10}$ with cordial labeling

An example of cordially labeled corona product graph $S_n \odot P_m$ for $n = 5$ and $m = 4s + 3$, where $s = 1$ is given in Figure 7.

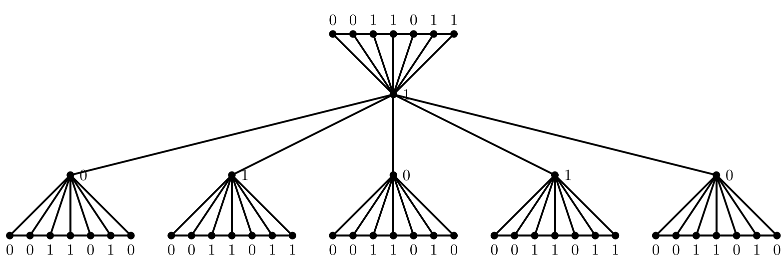

FIGURE 7. The graph $S_5 \odot P_7$ with cordial labeling

An example of cordially labeled corona product graph $S_n \odot P_m$ for $n = 4$ and $m = 4s + 3$, where $s = 2$ is given in Figure 8.

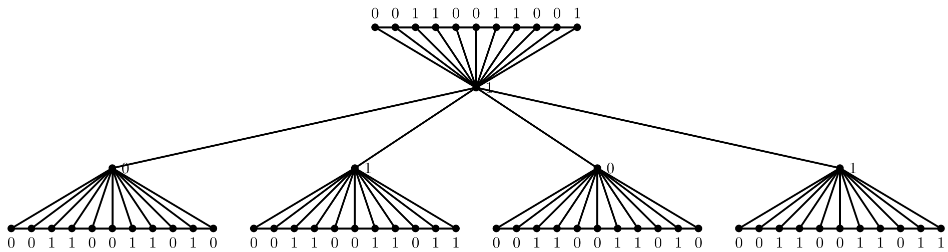

FIGURE 8. The graph $S_4 \odot P_{11}$ with cordial labeling

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