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A PROBABILISTIC PROOF FOR THE SYRACUSE CONJECTURE

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Abstract. We prove the veracity of the Syracuse conjecture by establishing that from an arbitrary positive integer different from 1 and 4, the Syracuse process will never return (after $i \geq 1$ steps) to any positive integer already reached, and we conclude using a probabilistic approach.

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1. Introduction

The Syracuse conjecture is an idea introduced by Lothar Collatz in 1937. It is also known as the $3n + 1$ problem. The Syracuse conjecture has been studied by several mathematicians, and significant results have been established. We mention as examples the results of Steiner [1] and Tao [2].
We consider the following operation on an arbitrary positive integer l:

- If l is even, divide it by two.
- If l is odd, triple it and add one.

The Collatz (or Syracuse) conjecture is: This process will eventually reach the number 1, regardless of which positive integer is chosen initially.

We can also understand this process as follows:
If l is a positive even integer (when l is a positive odd integer we get to the even case by tripling l and adding one to the result of the last multiplication) we divide it by 2 until we get an odd number, this last one we triple it and we add one, or we continue dividing l by two, until we get to 1. This last case is possible when l is of the form $l = 2^n$ with $n \in \mathbb{N}^*$. In fact the process starting from l will reach an odd positive integer, by tripling the latter and adding one, we will reach a positive even integer of the form 2^n (n a positive even integer). The idea is based on the fact that half of the numbers of the form 2^n can be written $3k+1$, k been a positive odd integer, the other half is of course of the form $3k - 1$.

We begin by showing that the Syracuse process does not admit any loop except $1 \leftrightarrows 4$, we then demonstrate the veracity of the Syracuse conjecture by means of Bernoulli measures carried out at each odd number reached, including the starting point

2. Main results

Let l be a strictly positive integer:

a) If l is an odd integer, then the next odd integer will be reached after these two operations:

- triple l and add one.
- divide $3l+1$ by 2 until we have the second odd integer.

The passage from l to the next positive odd integer reached by the two previous operations and so on, will be called a step in the set of positive odd integers.

b) If l is an even integer, then the next even integer will be reached after these two operations:

- divide l by 2 until we have the first odd number.
- triple the odd integer resulting from the first operation and add one.

The passage from l to the next positive even integer reached by the two previous operations and so on, will be called a step in the set of positive even integers.

Lemma 2.1. *For every positive integer l strictly superior to 1 and different of 4, the Syracuse process starting from l will never return to l after $i \geq 1$ steps, $i \in \mathbb{N}$.*

Proof. We first suppose that l is a positive odd integer strictly superior to 1.
Let m_j, $j \in \{1, ..., i\}$, be the number of divisions by 2 after the $j-$ieth step, thus $m_j \geq 1$.

After i steps, we have l_i the $i-$ieth odd number reached :

$$l_i = \frac{1}{2^{m_i}}(\frac{3}{2^{m_{i-1}}}(\frac{3}{2^{m_{i-2}}}(...(\frac{3}{2^{m_2}}(\frac{3}{2^{m_1}}(3l+1)+1)+1)...)+1)+1). \quad (I)$$

If the process returns for the first time to l, after i_0 steps, $i_0 \in \mathbb{N}^*$, then we have:

$$l \times 3^{i_0} = l \times 2^{\sum_{j=1}^{i_0} m_j} - 2^{\sum_{j=1}^{i_0-1} m_j} - 3 \times 2^{\sum_{j=1}^{i_0-2} m_j} - ... - 3^{i_0-2} \times 2^{m_1} - 3^{i_0-1}.$$

If $i_0 = 1$, then since l is the first odd positive integer reached (after one step) we have :

$$3l = 2^{m_1}l - 1$$

this leads to the equality:

$$l(2^{m_1} - 3) = 1$$

The last equality is meaningful in $\mathbb{N}$ if and only if $l = 1$ and $m_1 = 2$ which is absurd because l is supposed to be strictly superior to 1.

If $i_0 \geq 2$, as the Syracuse process performs a loop $l \to l_1 \to l_2 \to ... \to l_{i_0-1} \to l$ which reaches i_0 different positive odd integers $l_j \neq 1$ for $1 \leq j \leq i_0 - 1$ plus l, then according to equality (I) we have,

$$l \times (2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0}) = 2^{\sum_{j=1}^{i_0-1} m_j} + 3 \times 2^{\sum_{j=1}^{i_0-2} m_j} + ... + 3^{i_0-2} \times 2^{m_1} + 3^{i_0-1}. \quad (II)$$

We can construct a similar equality for each l_j, $1 \leq j \leq i_0 - 1$.
We first remark three trivial cases:

a) If $m_j = 1$ for $1 \leq j \leq i_0$, then $2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0} < 0$ which is absurd because $l > 1$ and from equality (II) we have $l \times (2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0}) > 0$.

b) If $m_j = 2$ for $1 \leq j \leq i_0$, then $l \times (2^{2i_0} - 3^{i_0}) = 2^{2i_0-2} + 3 \times 2^{2i_0-4} + ... + 3^{i_0-2} \times 4 + 3^{i_0-1}$ and therefore $l = 1$.

c) If $m_j \geq 2$ for $1 \leq j \leq i_0$, then since $2^{m_1} l_1 - 3l = 1$ and since

$$l \times (2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0}) = 2^{\sum_{j=1}^{i_0-1} m_j} + 3 \times 2^{\sum_{j=1}^{i_0-2} m_j} + ... + 3^{i_0-2} \times 2^{m_1} + 3^{i_0-1},$$

and

$$l_1 \times (2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0}) = 2^{\sum_{j=2}^{i_0} m_j} + 3 \times 2^{\sum_{j=2}^{i_0-1} m_j} + ... + 3^{i_0-2} \times 2^{m_2} + 3^{i_0-1},$$

then,

$$(2^{m_1} l_1 - 3l) \times (2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0}) = 2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0}$$

and thus

$$2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0} = (2^{m_{i_0}} - 3) \times 2^{\sum_{j=1}^{i_0-1} m_j} + (2^{m_{i_0-1}} - 3) \times 3 \times 2^{\sum_{j=1}^{i_0-2} m_j} + ... + (2^{m_2} - 3) \times 3^{i_0-2} \times 2^{m_1} + (2^{m_1} - 3) \times 3^{i_0-1}. \quad (III)$$

As $m_j \geq 2$ for $1 \leq j \leq i_0$, then

$$2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0} \geq l(2^{\sum_{j=1}^{i_0} m_j} - 3^{i_0})$$

whence $l \leq 1$, which is absurd because l is supposed to be strictly superior to 1.

The following case is when some of the positive integers m_j are superior or equal to 2 and the others (at least one) are equal to 1. Let's suppose that $m_1 \geq 3$, since $l_1 2^{m_1} - 1 = 3l$, then $l_1 2^{m_1} - 4 = 3l - 3$. It follows that $l_1 2^{m_1-2} - 1 = 3 \times \frac{l-1}{4}$ and $l_1 2^{m_1-2} - 1 > 0$ because $l_1 > 1$ and $m_1 \geq 3$. Therefore, there exists a positive odd integer $f = \frac{l-1}{4}$ such that $l_1 2^{m_1-2} = 3f + 1$ which is equivalent to say that f is in the loop and this is absurd. Indeed, to perform a loop, the Syracuse process starting from l needs to satisfy the following condition: $(m_1, l) = \min\{(x, y) \in \mathbb{N}^* \times 2\mathbb{N}^* + 1 : 2^x \times l_1 = 3y + 1\}$, because if we consider the reverse process which consists of finding a couple $x \in \mathbb{N}^*$ and $y \in 2\mathbb{N}^* + 1$ such that $2^x \times l_1 = 3y + 1$, then both couples $(m_1 - 2, f)$ and (m_1, l) satisfy the equality $2^x \times l_1 = 3y + 1$ and since $m_1 - 2 < m_1$ and $f < l$, then the reverse process is not a loop. We can

conclude that $m_1 = 2$ or $m_1 = 1$.
As we can repeat the last argument to each $m_j \geq 3$, then we have $m_j = 1$ or $m_j = 2$ for $1 \leq j \leq i_0$. However, this case is also absurd according to the following argument. We note that 2^{2i_0} is the smallest power (on the left side of equality (II)) of 2 generated by a loop (if it exists) of the Syracuse process after i_0 steps. Indeed, according to equality (II) this case corresponds to the Syracuse process starting from 1 and performing a loop of i_0 steps, which is equivalent to a repetition i_0 times of the loop $1 \leftrightarrows 4$.

We can conclude that the equality (II) is absurd for all positive odd integers strictly superior to 1.

If l is a positive even integer, let $r = \frac{l}{2^{m_1}}$, $m_1 \in \mathbb{N}^*$, be the first positive odd integer reached. If we suppose that the process returns to l after i steps (in the set of positive even integers), then it will reach r again after i steps (in the set of positive odd integers), which is absurd according to what precedes except for $r = 1$ and in this case $l = 4$.

□

Remark 2.2. a- *The lemma 2.1 confirms that the unique loop performed by the Syracuse process, after one step, is* $1 \leftrightarrows 4$. *Any other loop performed after* $i \in \mathbb{N}^*$ *steps is the repetition* i *times of the loop* $1 \leftrightarrows 4$.

b- *If* l *is a positive odd integer multiple of* 3*, then there is no pair* $x \in \mathbb{N}^*$ *and* $y \in 2 \times \mathbb{N}^* + 1$ *such that* $2^x \times l = 3y + 1$*, if not* 1 *would be divisible by* 3.

c- *If the process returns (after* $i \geq 2$ *steps) to* l*, then we have:*

$$l \times 3^i = l \times 2^{\sum_{j=1}^{i} m_j} - 2^{\sum_{j=1}^{i-1} m_j} - 3 \times 2^{\sum_{j=1}^{i-2} m_j} - ... - 3^{i-2} \times 2^{m_1} - 3^{i-1}. \quad (II)$$

We remark that l *cannot be a multiple of* 3 *otherwise* $2^{\sum_{j=1}^{i-1} m_j}$ *would be divisible by* 3*. Since we can generate a similar equality to* (II) *for every* l_j*,* $j \geq 1$*, then none of the positive odd integers* l_j *is a multiple of* 3*. Recall that* $3 \times l + 1 = 2^{m_1} l_1$ *and* $3 \times l_{i-1} + 1 = 2^{m_i} l$*, thus* $2^{m_i} l - 2^{m_1} l_1$ *is a multiple of* 3.

If $2^{m_i} > 2^{m_1}$ *and* $l_1 - 1 = 0 \mod 3$*, then* $l \times 2^{m_i - m_1} - 1$ *is a whole number multiple of* 3*. Thus there exists* f_1 *a positive odd integer such that* $l \times 2^{m_i - m_1} = 3 \times f_1 + 1$*. In order for the Syracuse process starting from* l *to perform a loop it needs that* $(m_i, l_{i-1}) = \min\{(x, y) \in \mathbb{N}^* \times 2\mathbb{N}^* + 1 : 2^x \times l = 3y + 1\}$*. If we consider the reverse process which consists of finding a couple* $x \in \mathbb{N}^*$ *and* y *such that* $2^x \times l = 3y + 1$*, where* y *is a positive odd integer, then in our situation, we know that the couples* (m_i, l_{i-1}) *and* $(m_i - m_1, f_1)$ *satisfy both the equality* $2^x \times l = 3y + 1$*, therefore the reverse process is not a loop. The loop made by the Syracuse process from* 1 *illustrates the last claim. It is clear that the Syracuse process from* 1 *consists of finding a couple* $a \in \mathbb{N}^*$ *and* q *such that* $2^a \times q = 3 \times 1 + 1$*, where*

q is a positive odd integer, of course in this case $a = 2$ and $q = 1$. The reverse process from 1 is to find a couple $b \in \mathbb{N}^*$ and $q^{'}$ such that $2^b \times 1 = 3 \times q^{'} + 1$, where $q^{'}$ is an odd positive integer, in this case $b = 2$ and $q^{'} = 1$ and therefore the couple $(b, q^{'})$ is unique as $\min\{(x, y) \in \mathbb{N}^* \times 2\mathbb{N}^* + 1 : 2^x \times 1 = 3y + 1\}$. *The known loops performed by the $3x - 1$ process ($1 \leftrightarrow 2$ and $5 \to 14 \to 7 \to 20 \to 5$) verify the same properties.*

Let $l \neq 1$ be a positive odd integer, the lemma 2.1 states that starting from l the Syracuse process will never return to l. Let $(l_k)_{k \geq 1}$, $l_k \neq 1$, be the sequence of odd integers reached by the Syracuse process starting from l. Each positive odd integer l_k can be considered as a starting point for the Syracuse process, then according to lemma 2.1, the Syracuse process starting from l_k can never return to l_k. It follows that the Syracuse process starting from l can never return to any l_k, $k \geq 1$.

Theorem 2.3. *Starting from an arbitrary positive integer the Syracuse process will always reach the value* 1*.*

Proof. According to lemma 2.1, starting from an integer l, the Syracuse process will never return to l after $i \geq 1$ steps. Therefore starting from an arbitrary odd positive integer l, the Syracuse process is a walk in the set of positive odd integers without any possibility to return to any of the positive odd integers reached before.

Remark 2.1. *When l is even, then the first positive odd integer reached ($r = \frac{l}{2^{m_1}}$, $m_1 \in \mathbb{N}^*$) will be the starting point of the walk of the Syracuse process.*

Let A be the set of positive odd integers of the forme $\frac{2^n - 1}{3}$ for $n > 2$ such that n is even. Concretely :

$$A := \{\frac{2^n - 1}{3} : n \text{ is even and } > 2\}.$$

Let $l_i \neq 1$, $i \geq 1$, be the sequence of odd numbers, reached from l, we carry out the following random experiment for l and for l_i after each step $i \geq 1$:

We consider the Bernoulli trial with two possible outcomes:

- "Failure" if $l_i \in A$,
- "Success" if $l_i \notin A$.

Let $0 \leq q_0 \leq 1$ be the probability that $l \notin A$, which is equivalent to the event "Success", then $1 - q_0$ is the probability of the event $l \in A$ which is equivalent to the event "Failure". Since the set A is an infinite subset of the set of positive odd integers except 1 and since $(2 \times \mathbb{N}^* + 1 \setminus A) \cap A = \emptyset$ and $(2 \times \mathbb{N}^* + 1 \setminus A) \cup A = 2 \times \mathbb{N}^* + 1$, then $0 < q_0 < 1$.

Let $0 \leq q_1 \leq 1$ be the probability, after one step from l, of the event "Success", then $1 - q_1$ is the probability of the event "Failure". Since the set A is an infinite subset of the set of positive odd integers except 1 and l and since $((2 \times \mathbb{N}^* + 1 \setminus \{l\}) \setminus A) \cap A = \emptyset$ and $((2 \times \mathbb{N}^* + 1 \setminus \{l\}) \setminus A) \cup A = 2 \times \mathbb{N}^* + 1 \setminus \{l\}$, then $0 < q_1 < 1$.

Repeating the same experience after each step $i \geq 1$, by considering the set A and the set of positive odd integers except $\{1, l, l_1, ..., l_{i-1}\}$ one has $0 < q_j < 1$, $1 \leq j \leq i$. We have the diagram below which represents the Syracuse process whose starting point is l:

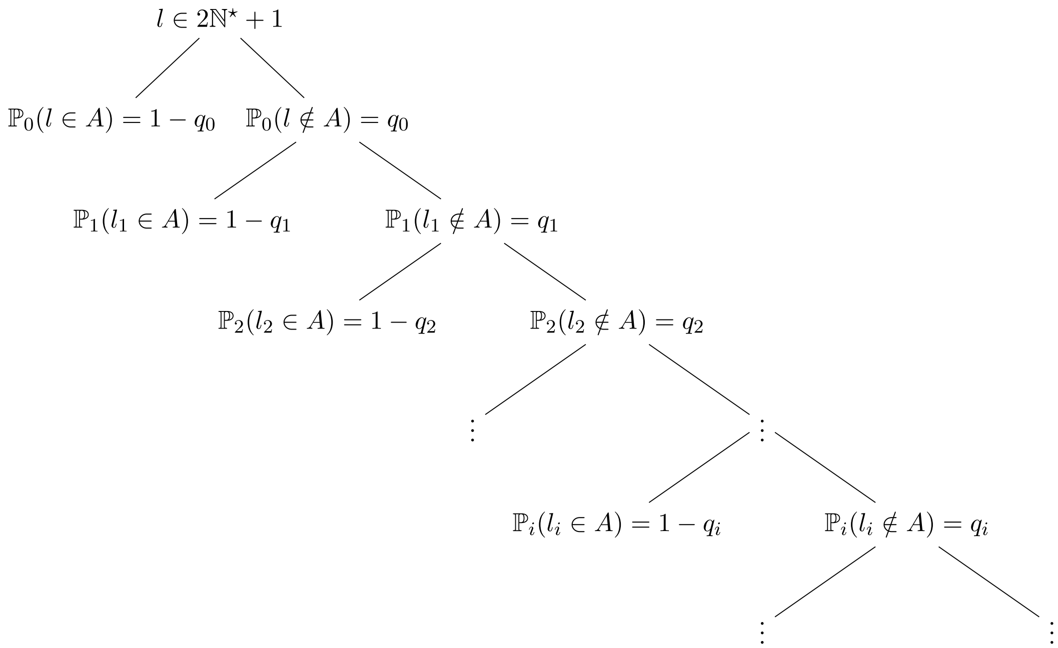

Figure 1. the diagram represents the Syracuse process starting from l, provided with the probability of reaching set A or not, after each step.

From the diagram we have

$$\mathbb{P}_1(l_1 \in A) = \mathbb{P}_1(l_1 \in A/l)$$

$$\mathbb{P}_1(l_1 \in A) = \mathbb{P}_1(l_1 \in A/l \in A) + \mathbb{P}_1(l_1 \in A/l \notin A)$$

as we have $\mathbb{P}_1(l_1 \in A/l \in A) = 0$ because if $l \in A$ then $l_1 = 1$, we therefore have

$$\mathbb{P}_1(l_1 \in A) = \mathbb{P}_1(l_1 \in A/l \notin A)$$

which implies that the event $\{l_1 \in A\}$ is independent of the event $\{l \notin A\}$. Likewise we have,

$$\mathbb{P}_1(l_1 \notin A) = \mathbb{P}_1(l_1 \notin A/l)$$

$$\mathbb{P}_1(l_1 \notin A) = \mathbb{P}_1(l_1 \notin A/l \in A) + \mathbb{P}_1(l_1 \notin A/l \notin A)$$

as we have $\mathbb{P}_1(l_1 \notin A/l \in A) = 0$ because if $l \in A$ then $l_1 = 1$, we therefore have

$$\mathbb{P}_1(l_1 \notin A) = \mathbb{P}_1(l_1 \in A/l \notin A)$$

which implies that the event $\{l_1 \notin A\}$ is independent of the event $\{l \notin A\}$. We repeat the same reasoning for each $i \in \mathbb{N}^*$ and we have

$$\mathbb{P}_i(l_i \in A) = \mathbb{P}_i(l_i \in A/l_{i-1})$$
$$\mathbb{P}_i(l_i \in A) = \mathbb{P}_i(l_i \in A/l_{i-1} \in A) + \mathbb{P}_i(l_i \in A/l_{i-1} \notin A)$$

as we have $\mathbb{P}_i(l_i \in A/l_{i-1} \in A) = 0$ because if $l_{i-1} \in A$ then $l_i = 1$, we therefore have,

$$\mathbb{P}_i(l_i \in A) = \mathbb{P}_i(l_i \in A/l_{i-1} \notin A)$$

which implies that the event $\{l_{i-1} \in A\}$ is independent of the event $\{l_{i-1} \notin A\}$. Likewise we have

$$\mathbb{P}_i(l_i \notin A) = \mathbb{P}_i(l_i \notin A/l_{i-1})$$
$$\mathbb{P}_i(l_i \notin A) = \mathbb{P}_i(l_i \notin A/l_{i-1} \in A) + \mathbb{P}_i(l_i \notin A/l_{i-1} \notin A)$$

as we have $\mathbb{P}_i(l_i \notin A/l_{i-1} \in A) = 0$ because if $l_{i-1} \in A$ then $l_i = 1$(see point a- remark 2.1), therefore have

$$\mathbb{P}_i(l_i \notin A) = \mathbb{P}_i(l_i \in A/l_{i-1} \notin A)$$

which implies that the event $\{l_i \notin A\}$ is independent of the event $\{l_{i-1} \notin A\}$.

On the other hand we have

$$\mathbb{P}_i(l_i \in A) = \mathbb{P}_i(l_i \in A/l_{i-1}, \ldots, l_1, l)$$
$$\mathbb{P}_i(l_i \in A) = \mathbb{P}_i(l_i \in A/l_{i-1} \notin A, \ldots, l_1 \notin A, l \notin A)$$

because if $l_j \in A$ for any $j \in [1, \ldots, i-1]$ then $l_{j+1} = 1$ and therefore $l_i = 1$, which implies that the event $\{l_i \notin A\}$ is independent of the event $\{l_j \notin A\}$ for each $j \in [1, \ldots, i-1]$. Likewise we have,

$$\mathbb{P}_i(l_i \notin A) = \mathbb{P}_i(l_i \notin A/l_{i-1}, \ldots, l_1, l)$$
$$\mathbb{P}_i(l_i \notin A) = \mathbb{P}_i(l_i \notin A/l_{i-1} \notin A, \ldots, l_1 \notin A, l \notin A)$$

because if $l_j \in A$ for any $j \in [1, \ldots, i-1]$ then $l_{j+1} = 1$ and therefore $l_i = 1$(see point a- remark 2.1). This implies that the event $\{l_i \notin A\}$ is independent of the event $\{l_j \notin A\}$ for any $j \in [1, \ldots, i-1]$.

From the above we define the probability law $\mathbb{P}$ of the random variable Y_l indicating the number $i \geq 0$ of independent Bernoulli trials of probability of success $q_i \in]0, 1[$ necessary to obtain the first failure and we have:

$$\mathbb{P}(Y_l \leq i) = \mathbb{P}(l \in A) + \mathbb{P}(l_1 \in A, l \notin A) + \mathbb{P}(l_2 \in A, l_1 \notin A, l \notin A) + \ldots$$
$$+\mathbb{P}(l_i \in A, l_{i-1} \notin A, \ldots, l_1 \notin A, l \notin A)$$

which gives

$$\mathbb{P}(Y_l \leq i) = \mathbb{P}_0(l \in A) + \mathbb{P}_1(l_1 \in A)\mathbb{P}_0(l \notin A) + \mathbb{P}_2(l_2 \in A)\mathbb{P}_1(l_1 \notin A)\mathbb{P}_0(l \notin A) + \ldots$$
$$+\mathbb{P}_i(l_i \in A)\mathbb{P}_{i-1}(l_{i-1} \notin A) \ldots \mathbb{P}_1(l_1 \notin A)\mathbb{P}_0(l \notin A)$$

so we have

$$\mathbb{P}(Y_l \leq i) = 1 - q_0 + (1 - q_1)q_0 + (1 - q_2)q_1q_0 + \ldots$$
$$+(1 - q_i)q_{i-1} \ldots q_1q_0$$

hence we have

$$\mathbb{P}(Y_l \leq i) = 1 - q_0q_1q_2q_3 \ldots q_{i-1}q_i.$$

The probability that the sequence l_i, $i \geq 0$, starting from l, does not reach the set A after $i \geq 0$ steps is equal to $q_0 \times q_1 \times q_2 \times ... \times q_i$, so the probability of the event: the sequence l_i, $i \geq 0$ never reaching the set A, is equal to $\lim_{i \to +\infty} q_0 \times q_1 \times q_2 \times ... \times q_i$. Since $q_0 \times q_1 \times q_2 \times ... \times q_i \leq (\max_{j \geq 0} q_j)^i$ such that $0 \leq j \leq i$ and $\lim_{i \to +\infty} (\max_{j \geq 0} q_j)^i = 0$, then $\lim_{i \to +\infty} q_0 \times q_1 \times q_2 \times ... \times q_i = 0$. It follows that the occurrence of the first "failure" after a finite number of the previously mentioned Bernoulli trials, is a certain event.

This means that the sequence l_i, $i \geq 1$, starting from l, will necessarily reach a positive odd integer belonging to A, after a finite number of steps in the set of positive odd integers (except 1).

Once such a positive odd integer $s = \frac{2^{n_0}-1}{3}$ (for some positive even integer $n_0 > 2$) reached, the next operation in the Syracuse process is to multiply s by 3 and to add 1, then we get to the even integer 2^{n_0}, after n_0 divisions by 2, we get to the value 1.

According to what have been proved before, we deduce that starting from an arbitrary positive integer the Syracuse process will always reach the value 1. □

Corollary 2.4.
Every positive odd integer l can be written as follows:

$$l = \frac{2^{\sum_{j=1}^{i} m_j} - 2^{\sum_{j=1}^{i-1} m_j} - 3 \times 2^{\sum_{j=1}^{i-2} m_j} - ... - 3^{i-2} \times 2^{m_1} - 3^{i-1}}{3^i}$$

where i and m_j, $1 \leq j \leq i$, are positive integers depending on l.

Proof. According to equality (I) and the previous theorem. □

Remark 2.5. *In a future article we will prove that for every positive odd integer $p > 1$, if the $px + 1$ (resp. $px - 1$) problem performs a unique loop containing 1, then the problem $px + 1$ (resp. $px - 1$) reaches 1 and in this case we will show that every positive odd integer l can be written as follows:*

$$l = \frac{2^{\sum_{j=1}^{i} m_j} - 2^{\sum_{j=1}^{i-1} m_j} - p \times 2^{\sum_{j=1}^{i-2} m_j} - ... - p^{i-2} \times 2^{m_1} - p^{i-1}}{p^i}$$

where i and m_j, $1 \leq j \leq i$, are positive integers depending on l and p. And respectively,

$$l = \frac{2^{\sum_{j=1}^{i} m_j} + 2^{\sum_{j=1}^{i-1} m_j} + p \times 2^{\sum_{j=1}^{i-2} m_j} + ... + p^{i-2} \times 2^{m_1} + p^{i-1}}{p^i}$$

where i and m_j, $1 \leq j \leq i$, are positive integers depending on l and p.

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