Advanced Studies in Contemporary Mathematics
36 (2026), No. 1, pp. 1 – 3

www.jangjeon.or.kr
http://dx.doi.org/10.17777/ascm2026.36.1.001

PSEUDOCHARACTERS ON TOPOLOGICAL GROUPS DEFINED BY PSEUDOCHARACTERS ON DISCRETE QUOTIENT GROUPS

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Abstract. We indicate necessary and sufficient conditions under which a continuous pseudocharacter on a separated topological group is defined by a pseudocharacter on a discrete quotient group of this group.

§ 1. Introduction

All topological groups under consideration satisfy the axiom T_2.

For the information concerning pseudocharacters on groups, see [1]–[3]. Let G be a topological group, and let f be a continuous pseudocharacter on G. Recall than the set $\ker f = \{g \in G : f(g) = 0\}$ is called the kernel of the pseudocharacter f and the set of all elements $n \in \ker f$ such that

$$f(gn) = f(g) \quad \text{for all} \quad g \in G$$

is called the center of the pseudocharacter f and is denoted by Z_f. This set is a closed normal subgroup of G [4].

2010 *Mathematics Subject Classification.* Primary 22A25
Submitted November 29, 2025.
Key words and phrases. topological group, normal subgroup, pseudocharacter, kernel of a pseudocharacter, center of a pseudocharacter.

§ 2. Preliminaries

Lemma 1. *Let G be a topological group and let f be a continuous pseudocharacter on G. The following conditions are equivalent.*

(i) The pseudocharacter f vanishes on an open normal subgroup of G.

(ii) The center Z_f of f is open.

Proof. (i) $\Longrightarrow$ (ii) It follows from an obvious generalization of the assertion in Theorem 4.1 d) of [4] (where locally compact groups are mentioned instead of arbitrary separated topological groups) that, if a continuous pseudocharacter f vanishes on a closed normal subgroup N of G, then this pseudocharacter is defined by a pseudocharacter F on the quotient group G/N by the rule $f = F \circ \pi$, where π stands for the cfnonical quotient mapping $\pi\colon G \to G/N$. Therefore, $f(gn) = f(g)$ for all $g \in G$, which means that $N \subset Z_f$, which immediately implies that Z_f is open, since Z_f is open by assumption.

The converse assertion is obvious.

§ 3. Main Theorem

Theorem 1. *Let G be a topological group and let f be a continuous pseudocharacter on G. The following conditions are equivalent.*

(i) There is a discrete quotient group G/N of G with the canonical mapping $\pi\colon G \to G/N$ and a continuous pseudocharacter F on G/N such that $f = F \circ \pi$.

(ii) The pseudocharacter F vanishes on sone open normal subgroup N of G.

(iii) The center Z_f of the pseudocharacter f is open.

Proof.

(i) $\Longrightarrow$ (ii)

If G/N is discrete, then the identity element $e_{G/N}$ is open in G/N, and by the continuity of π, the kernel N of π is an open normal subgroup of G. This means that (ii) holds.

(ii) $\Longrightarrow$ (iii)

This follows from the lemma.

(iii) $\Longrightarrow$ (i)

This follows from the obvious generalized version, for the separated topological groups, of the assertion in Theorem 4.1 d) of [4], as in Lemma 1.

§ 4. Discussion

The values of every ordinary continuous additive real character on a totally disconnected locally compact group admitting a compact open normal subgroup are bounded on some compact open normal subgroup, and hence the restriction of this character to the chosen compact open normal subgroup vanishes. Therefore, the kernel of this character is open, and thus the character is defined by a character of the quotient group of the original group by the kernel of this character. The above theorem shows that, for continuous pseudocharacters, the situation is similar.

Acknowledgments

I thank Professor Taekyun Kim for the invitation to publish this paper in the Advanced Studies of Contemporary Mathematics.

Funding

The research was supported by the Scientific Research Institute for System Analysis of the National Research Centre "Kurchatov Institute" according to the project FNEF-2024-0001 (Reg. no. 1023032100070-3-1.2.1).

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