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AUTOMATIC CONTINUITY CONDITIONS FOR PSEUDOCHARACTERS ON ALMOST CONNECTED LOCALLY COMPACT GROUPS

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Abstract. We find subgroups of an almost connected locally compact group on which every pseudocharacter on the group is continuous.

§ 1. Introduction

In this note, we find subgroups of an almost connected locally compact group such that the restriction of every pseudocharacter on the group to these subgroups is continuous in the intrinsic Lie topology. For the definition and properties of pseudocharacters on groups, see [1]–[3].

§ 2. Preliminaries

By Yamabe's theorem [4], every almost connected locally compact group G has arbitrarily small compact normal subgroups N for which the quotient group G/N is a Lie group.

We need below the following assertion.

Lemma 1. *Let $S = G$ be a group, let N be a normal subgroup of G, and let π be the canonical epimorphism of G onto G/N. If a pseudocharacter φ on G vanishes on N, then there exists a pseudocharacter ψ on the group G/N such that $\varphi = \psi \circ \pi$. If G is a locally compact group, N is closed, and φ is continuous, then ψ is continuous.*

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For the proof, see, e.g., [1].

§ 3. Main Results

Theorem 1. *Let G be an almost connected locally compact group and let f be a pseudocharacter on G. Let N be the maximal compact normal subgroup of G (see* [5] *and* [6, 7]*). Suppose that f is bounded on N. Then f vanishes on N, and the quotient group G/N is a (possibly disconnected) Lie group. Let R be the radical of G/N (coinciding with the radical of the connected component of the identity element $(G/N)_0$), let D be a Dong Hoon Lee* [8] *finite supplementary subgroup of G/N, and let L be a Levi semisimple subgroup of $(G/N)_0$. Then there is a pseudocharacter F on the group G/N defining f (if π is the canonical mapping of G onto G/N, then $f(g) = F(\pi(g))$ for all $g \in G$), and F vanishes on D and is continuous in the intrinsic group topology of $(G/N)_0$ on L and on the commutator subgroup R' of R.*

Proof. The pseudocharacter f vanishes on N since a bounded pseudocharacter is zero. The factorization result and the formula $f(g) = F(\pi(g))$ for all $g \in G$ follow from Lemma 1. The pseudocharacter F is obviously bounded on D, and hence vanishes on D. The continuity of F on L follows from the continuity of every pseudocharacter on a semisimple Lie group in the intrinsic Lie topology. The restriction of F to R is an ordinary real character (an additive real-valued mapping), since R is solvable, and thus amenable [9]. Therefore, F vanishes on the commutator subgroup R' of R, and thus is continuous on R'.

§ 4. Discussion

As is well known (see [6]), every locally bounded pseudocharacter on a Hausdorff topological group is continuous.

Obviously, F is continuous on L in the topology of G/N if and only if L is closed in G/N, which is equivalent to the condition that the center of L is closed in G/N.

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