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New $\exp(Q(\xi))$-Expansion Method for Solving New Model of Extension System Nonlinear Partial Differential Equations

M.S.AL-Amry[1], E.F.AL-Abdali[2]

[1,2]Department of Mathematics, Faculty of Education Aden, Aden University, Yemen

[1]salehm251251@gmail.com

[2]emanfadhel328@ gmail.com

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Abstract

In this paper, we presented new method, namley, the $\exp(Q(\xi))$-expansion method is proposed for constructing more general and a rich class of new exact traveling wave solutions of nonlinear partial differential equations (NLPDEs). This method has been applied to new the extension system Boussinesq equations (esB). Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions and rational functions.

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Key Words: $\exp(Q(\xi))$- expansion method, extension system Boussinesq equations (esB), traveling wave solutions.

1 Introduction

In recent years, a wide range of straightforward and effective methods have been introduced to obtain traveling wave solutions of nonlinear (PDEs). Such methods, the sine-cosine method [1, 2], the tanh-coth method [3, 4], the multiple-soliton solutions [5]-[7], the $\left(\frac{1}{G'}\right)$-expansion method [8], the mapping method [9], the improved F-expansion scheme [10], the Adomain decomposition approach [11, 12], the modified trial equation method [13], Darboux transformation [14], the $\left(\frac{G'}{G}\right)$-expansion method [15, 16], the improved and generalized $\left(\frac{G'}{G}\right)$ expansion methods [17], the $\left(\frac{G'}{G^2}\right)$-expansion method [18, 19], the extended hyperbolic function method [20], the Residual power series method [21], the Jacobi elliptic function expansion method [22], the $\left(\frac{G'}{G}/\frac{1}{G}\right)$-expansion method [23]-[25], the improved tan $(\varphi(\xi)/2)$-expansion method [26]. Sometimes, numerical techniques are good alternatives to analytical ones, for instance, see [27, 28].

The importance of our present work is, in order to generate many exact traveling wave solutions, new approach $\exp(Q(\xi))$-expansion method. For illustration and to depict of the proposed method, new the extension system Boussinesq equations (esB) has been studied and generated abundant and more types of new travelling wave solutions.

2 Description of new method

Consider the general nonlinear partial differential equations (PDEs), say, in two variables,

$$\Psi\left(v, v_t, v_x, v_{tt}, v_{xt}, v_{xx}, \ldots\right) = 0, \tag{1}$$

where $v = v(x,t)$ is an unknown function, Ψ is a polynomial in $v(x,t)$ and the subscripts stand for the partial derivatives.

We suppose that the combination of real variables x and t by a complex variable ξ

$$v(x,t) = v(\xi), \quad \xi = ax - ct, \tag{2}$$

where a is the wave number and c is the speed of the traveling wave. Now using Eq. (2), Eq. (1) is converted into an ordinary differential equation for $v = v(\xi)$:

$$\Phi\left(v, -cv', av', c^2v'', -cav'', a^2v'', \ldots\right) = 0,' \equiv \frac{d}{d\xi}. \tag{3}$$

Suppose the traveling wave solution of Eq. (3) can be expressed by polynomial in $\exp(Q(\xi))$ as:

$$v(\xi) = \sum_{i=0}^{N} a_i \left(\exp\left(Q(\xi)\right)\right)^i, \tag{4}$$

where the coefficients a_i $(0 \leq i \leq N)$, a and c are constants to be determined, such that $a_N \neq 0$ and $Q = Q(\xi)$ satisfies the following ordinary differential equation:

$$Q'(\xi) = \sqrt{\mu \exp(Q(\xi)) + \lambda}, \tag{5}$$

Eq. (5) gives the following solutions:
Case 1. where $\lambda > 0$ and $\mu \neq 0$,

$$Q(\xi) = \ln\left(\frac{-\lambda}{\mu}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}(\xi + k)\right)\right), \tag{6}$$

$$Q(\xi) = \ln\left(\frac{\lambda}{\mu}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}(\xi + k)\right)\right), \tag{7}$$

where k is a constant of integration.
Case 2. where $\lambda < 0$ and $\mu \neq 0$,

$$Q(\xi) = \ln\left(\frac{-\lambda}{\mu}\sec^2\left(\frac{\sqrt{-\lambda}}{2}(\xi + k)\right)\right), \tag{8}$$

$$Q(\xi) = \ln\left(\frac{-\lambda}{\mu}\csc^2\left(\frac{\sqrt{-\lambda}}{2}(\xi + k)\right)\right). \tag{9}$$

Case 3. where $\lambda = 0$ and $\mu \neq 0$,

$$Q(\xi) = \ln\left(\frac{4}{\mu(\xi + k)^2}\right). \tag{10}$$

Case 4. where $\lambda = \theta^2$ and $\mu \neq 0$,

$$Q(\xi) = \ln\left(\frac{\theta^2}{\mu}\left(\left(\frac{1 + \exp(\theta(\xi + k))}{1 - \exp(\theta(\xi + k))}\right)^2 - 1\right)\right). \tag{11}$$

Case 5. where $\lambda \neq 0$ and $\mu = 0$,

$$Q(\xi) = \sqrt{\lambda}(\xi + k). \tag{12}$$

Where $\theta, a_i(i = 0, 1, 2, ..., N), a, c, \mu$ and λ are constants. The positive integer N can be determined by using homogeneous balance between the highest order derivatives and the nonlinear terms appearing in ODE (3). Substituting Eq. (4) into Eq. (3), using Eq. (5) repeatedly, and setting the coefficients of the each order of $(\exp(Q(\xi)))^i$, $(\exp(Q(\xi)))^i\sqrt{\mu\exp(Q(\xi)) + \lambda}$ to zero, we obtain a set of nonlinear algebraic equations for $a_i(i = 0, 1, 2, ..., N)$, a, c, μ and λ. With the aid of the computer program Maple, we can solve the set of nonlinear algebraic equations and obtain all the constants $a_i(i =$

$0, 1, 2, ..., N), a, c$. Substituting the values of $a_i (i = 0, 1, 2, ..., N), a, c$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

3 The esB equation

Consider the extension system Boussinesq equations (esB) that reads:

$$\begin{cases} v_t + A\left(u^2\right)_x - Eu_{xxx} = 0 \\ u_t - Pv_x - Lw_x = 0 \\ w_t - Bu_x = 0. \end{cases} \tag{13}$$

Where A, B, E, P and L are constants.
Applying the following wave transformation

$$u(x,t) = u(\xi), v(x,t) = v(\xi) \text{ and } w(x,t) = w(\xi), \text{ where } \xi = ax - ct. \tag{14}$$

On equation Eq. (13), we get

$$\begin{cases} -cv' + aA\left(u^2\right)' - a^3Eu''' = 0 \\ -cu' - aPv' - aLw' = 0 \\ -cw' - aBu' = 0. \end{cases} \tag{15}$$

Integrating Eq. (15) once with respect to ξ, yields

$$\begin{cases} -cv + aAu^2 - a^3Eu'' = 0 \\ -cu - aPv - aLw = 0 \\ -cw - aBu = 0. \end{cases} \tag{16}$$

For finding the value N, we use balancing principle on Eq. (16), and get $N = 2$.
From Eq. (4), we have

$$u(\xi) = a_0 + a_1 \exp(Q(\xi)) + a_2 (\exp(Q(\xi)))^2, v(\xi) = b_0 + b_1 \exp(Q(\xi)) \\ +b_2 (\exp(Q(\xi)))^2, w(\xi) = d_0 + d_1 \exp(Q(\xi)) + d_2 (\exp(Q(\xi)))^2, \tag{17}$$

where $a_0, a_1, a_2, b_0, b_1, b_2, d_0, d_1$ and d_2 are arbitrary constants.

Substituting Eq. (17) in Eq. (16) and using Eq. (5), collecting the coefficients of each power of $(\exp(Q(\xi)))^i$, $0 \leq i \leq 4$, setting each coefficients to zero, and solving the algebraic equations by Maple we get,

set (1).

$$\begin{aligned} a_0 &= 0, a_1 = \frac{3Ea^2\mu}{2A}, a_2 = 0, b_0 = 0, b_1 = \frac{\mp 3E^2a^4\lambda\mu}{2A\sqrt{Ea^2\lambda P + BL}}, b_2 = 0, \\ d_0 &= 0, d_1 = \frac{\mp 3Ea^2\mu B}{2A\sqrt{Ea^2\lambda P + BL}}, d_2 = 0, c = \pm a\sqrt{Ea^2\lambda P + BL}. \end{aligned}$$

set (2).

$$\begin{aligned} a_0 &= \frac{Ea^2\lambda}{A}, a_1 = \frac{3Ea^2\mu}{2A}, a_2 = 0, b_0 = \frac{\pm E^2a^4\lambda^2}{A\sqrt{-Ea^2\lambda P + BL}}, b_1 = \frac{\pm 3E^2a^4\lambda\mu}{2A\sqrt{-Ea^2\lambda P + BL}}, \\ b_2 &= 0, d_0 = \frac{\mp BEa^2\lambda}{A\sqrt{-Ea^2\lambda P + BL}}, d_1 = \frac{\mp 3Ea^2\mu B}{2A\sqrt{-Ea^2\lambda P + BL}}, d_2 = 0, \\ c &= \pm a\sqrt{-Ea^2\lambda P + BL}. \end{aligned}$$

Using Eq. (17), the solutions of Eq. (5) and set (**1**) we get

Case 1. where $\lambda > 0$ and $\mu \neq 0$,

$$u_{1,2} = \frac{-3Ea^2\lambda}{2A}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \mp a\sqrt{Ea^2\lambda P + BL}t + k\right)\right),$$

$$v_{1,2} = \frac{\pm 3E^2a^4\lambda^2}{2A\sqrt{Ea^2\lambda P + BL}}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \mp a\sqrt{Ea^2\lambda P + BL}t + k\right)\right),$$

$$w_{1,2} = \frac{\pm 3Ea^2\lambda B}{2A\sqrt{Ea^2\lambda P + BL}}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \mp a\sqrt{Ea^2\lambda P + BL}t + k\right)\right).$$

$$u_{3,4} = \frac{3Ea^2\lambda}{2A}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \pm a\sqrt{Ea^2\lambda P + BL}t + k\right)\right),$$

$$v_{3,4} = \frac{\pm 3E^2a^4\lambda^2}{2A\sqrt{Ea^2\lambda P + BL}}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \pm a\sqrt{Ea^2\lambda P + BL}t + k\right)\right),$$

$$w_{3,4}=\frac{\pm 3Ea^2\lambda B}{2A\sqrt{Ea^2\lambda P+BL}}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right).$$

Case 2. where $\lambda<0$ and $\mu\neq 0$,

$$u_{5,6}=\frac{-3Ea^2\lambda}{2A}\sec^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right),$$

$$v_{5,6}=\frac{\mp 3E^2a^4\lambda^2}{2A\sqrt{Ea^2\lambda P+BL}}\sec^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right),$$

$$w_{5,6}=\frac{\mp 3Ea^2\lambda B}{2A\sqrt{Ea^2\lambda P+BL}}\sec^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right).$$

$$u_{7,8}=\frac{-3Ea^2\lambda}{2A}\csc^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right),$$

$$v_{7,8}=\frac{\mp 3E^2a^4\lambda^2}{2A\sqrt{Ea^2\lambda P+BL}}\csc^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right),$$

$$w_{7,8}=\frac{\mp 3Ea^2\lambda B}{2A\sqrt{Ea^2\lambda P+BL}}\csc^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax\pm a\sqrt{Ea^2\lambda P+BL}t+k\right)\right).$$

Case 3. where $\lambda=0$ and $\mu\neq 0$,

$$u_{9,10}=\frac{6Ea^2}{A\left(ax\pm a\sqrt{BL}t+k\right)^2},$$

$$w_{9,10}=\frac{6Ea^2B}{A\sqrt{BL}\left(ax\pm a\sqrt{BL}t+k\right)^2},$$

where v is constant.

Case 4. where $\lambda=\theta^2$ and $\mu\neq 0$,

$$u_{11,12}=\frac{3Ea^2\theta^2\left(\left(\frac{1+\exp\left(\theta\left(ax\pm a\sqrt{Ea^2\theta^2P+BL}t+k\right)\right)}{1-\exp\left(\theta\left(ax\pm a\sqrt{Ea^2\theta^2P+BL}t+k\right)\right)}\right)^2-1\right)}{2A},$$

$$v_{11,12}=\frac{\pm 3Ea^4\theta^4\left(\left(\frac{1+\exp\left(\theta\left(ax\pm a\sqrt{Ea^2\theta^2P+BL}t+k\right)\right)}{1-\exp\left(\theta\left(ax\pm a\sqrt{Ea^2\theta^2P+BL}t+k\right)\right)}\right)^2-1\right)}{2A\sqrt{Ea^2\theta^2P+BL}},$$

$$w_{11,12}=\frac{\pm 3BEa^2\theta^2\left(\left(\frac{1+\exp\left(\theta\left(ax\pm a\sqrt{Ea^2\theta^2P+BL}t+k\right)\right)}{1-\exp\left(\theta\left(ax\pm a\sqrt{Ea^2\theta^2P+BL}t+k\right)\right)}\right)^2-1\right)}{2A\sqrt{Ea^2\theta^2P+BL}}.$$

Case 5. when $\lambda \neq 0$ and $\mu = 0$ are constants.

Using Eq. (17), the solutions of Eq. (5) and set (2) we get
Case 1. where $\lambda > 0$ and $\mu \neq 0$,

$$u_{13,14}=\frac{Ea^2\lambda}{A}\left(1-\frac{3}{2}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax\pm a\sqrt{-Ea^2\lambda P+BL}t+k\right)\right)\right),$$

$$v_{13,14}=\frac{\mp E^2a^4\lambda^2}{A\sqrt{-Ea^2\lambda P+BL}}\left(1-\frac{3}{2}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax\pm a\sqrt{-Ea^2\lambda P+BL}t+k\right)\right)\right),$$

$$w_{13,14}=\frac{\pm BEa^2\lambda}{A\sqrt{-Ea^2\lambda P+BL}}\left(1-\frac{3}{2}\operatorname{sech}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax\pm a\sqrt{-Ea^2\lambda P+BL}t+k\right)\right)\right).$$

$$u_{15,16}=\frac{Ea^2\lambda}{A}\left(1+\frac{3}{2}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax\pm a\sqrt{-Ea^2\lambda P+BL}t+k\right)\right)\right),$$

$$v_{15,16} = \frac{\mp E^2a^4\lambda^2}{A\sqrt{-Ea^2\lambda P + BL}} \left(1 + \frac{3}{2}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right),$$

$$w_{15,16} = \frac{\pm BEa^2\lambda}{A\sqrt{-Ea^2\lambda P + BL}} \left(1 + \frac{3}{2}\operatorname{csch}^2\left(\frac{\sqrt{\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right).$$

Case 2. where $\lambda < 0$ and $\mu \neq 0$,

$$u_{17,18} = \frac{Ea^2\lambda}{A}\left(1 - \frac{3}{2}\sec^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right),$$

$$v_{17,18} = \frac{\mp E^2a^4\lambda^2}{A\sqrt{-Ea^2\lambda P + BL}} \left(1 - \frac{3}{2}\sec^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right),$$

$$w_{17,18} = \frac{\pm Ea^2\lambda B}{A\sqrt{-Ea^2\lambda P + BL}} \left(1 - \frac{3}{2}\sec^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right).$$

$$u_{19,20} = \frac{Ea^2\lambda}{A}\left(1 - \frac{3}{2}\csc^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right),$$

$$v_{19,20} = \frac{\mp E^2a^4\lambda^2}{A\sqrt{-Ea^2\lambda P + BL}} \left(1 - \frac{3}{2}\csc^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right),$$

$$w_{19,20} = \frac{\pm Ea^2\lambda B}{A\sqrt{-Ea^2\lambda P + BL}} \left(1 - \frac{3}{2}\csc^2\left(\frac{\sqrt{-\lambda}}{2}\left(ax \pm a\sqrt{-Ea^2\lambda P + BL}t + k\right)\right)\right).$$

Case 3. where $\lambda = 0$ and $\mu \neq 0$, we obtain the same solutions $u_{9,10}, v_{9,10}$ and $w_{9,10}$.

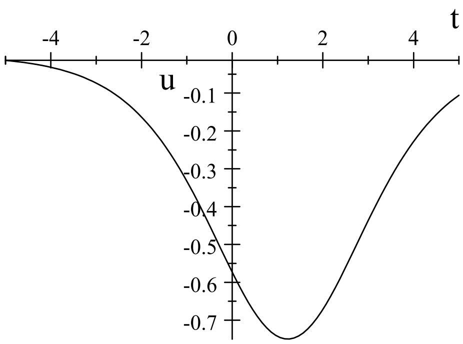
Case 4. where $\lambda = \theta^2$ and $\mu \neq 0$,

$$u_{21,22} = \frac{Ea^2\theta^2}{A}\left(\frac{-1 + 3\left(\frac{1+\exp\left(\theta\left(ax \pm a\sqrt{-Ea^2\theta^2P + BL}t + k\right)\right)}{1-\exp\left(\theta\left(ax \pm a\sqrt{-Ea^2\theta^2P + BL}t + k\right)\right)}\right)^2}{2}\right),$$

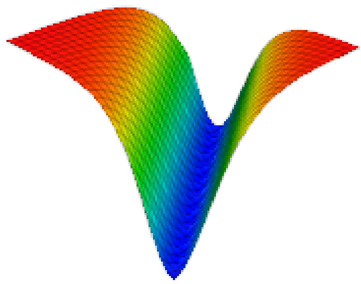
$$v_{21,22}=\frac{\mp E^2a^4\theta^4}{A\sqrt{-Ea^2\theta^2P+BL}}\left(\frac{-1+3\left(\frac{1+\exp\left(\theta\left(ax\pm a\sqrt{-Ea^2\theta^2P+BL}t+k\right)\right)}{1-\exp\left(\theta\left(ax\pm a\sqrt{-Ea^2\theta^2P+BL}t+k\right)\right)}\right)^2}{2}\right),$$

$$w_{21,22}=\frac{\pm BEa^2\theta^2}{A\sqrt{-Ea^2\theta^2P+BL}}\left(\frac{-1+3\left(\frac{1+\exp\left(\theta\left(ax\pm a\sqrt{-Ea^2\theta^2P+BL}t+k\right)\right)}{1-\exp\left(\theta\left(ax\pm a\sqrt{-Ea^2\theta^2P+BL}t+k\right)\right)}\right)^2}{2}\right).$$

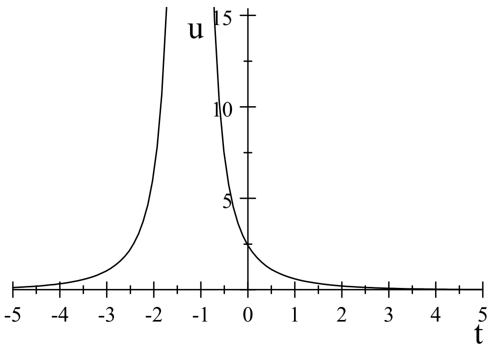
Case 5. when $\lambda \neq 0$ and $\mu = 0$ are constants.



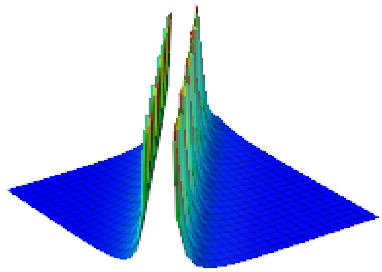
$u_{1,2}$ if $\lambda > 0$



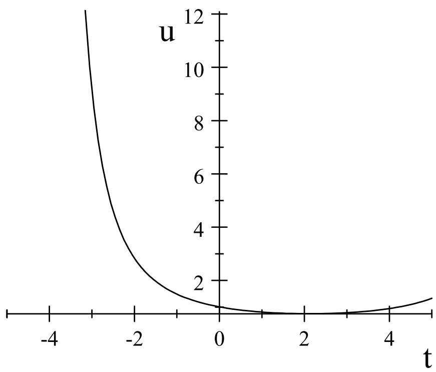
$u_{1,2}$ if $\lambda > 0$



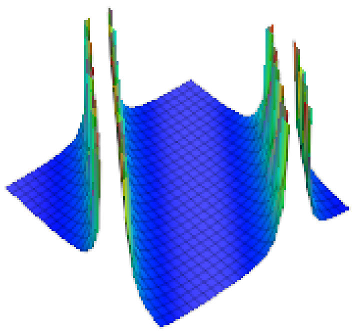
$u_{3,4}$ if $\lambda > 0$



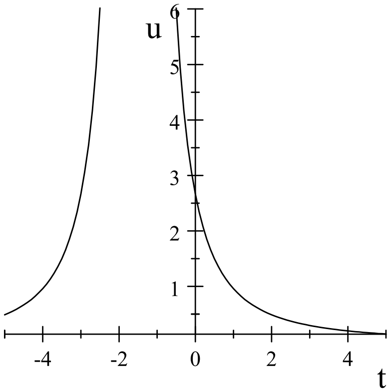
$u_{3,4}$ if $\lambda > 0$



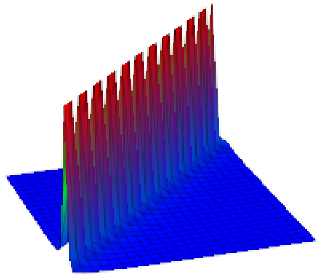
$u_{5,6}$ if $\lambda < 0$



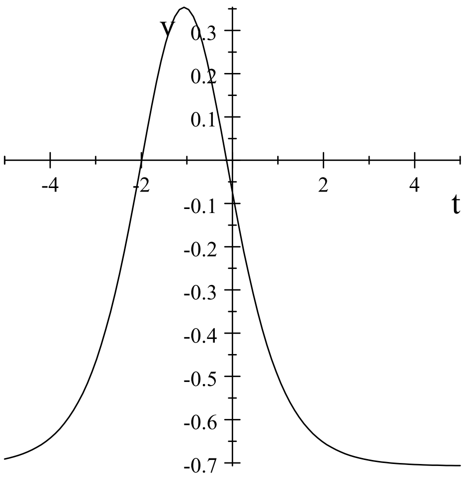
$u_{5,6}$ if $\lambda < 0$



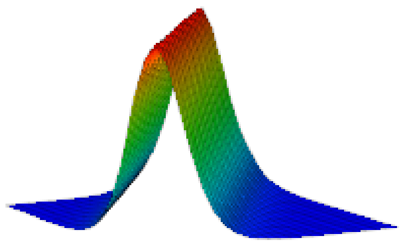
$u_{9,10}$ if $\lambda = 0$



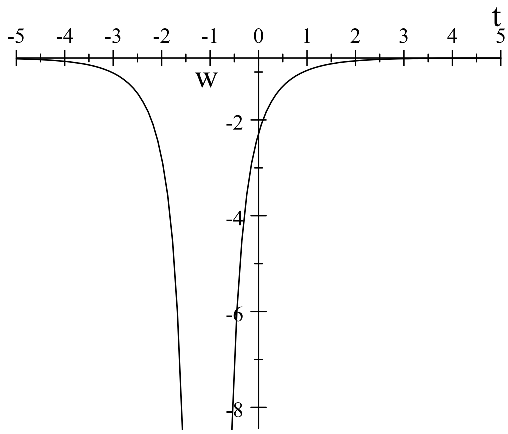
$u_{9,10}$ if $\lambda = 0$



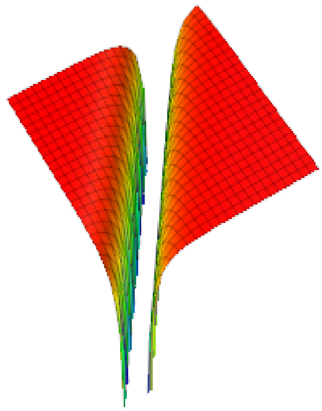
$v_{13,14}$ if $\lambda > 0$



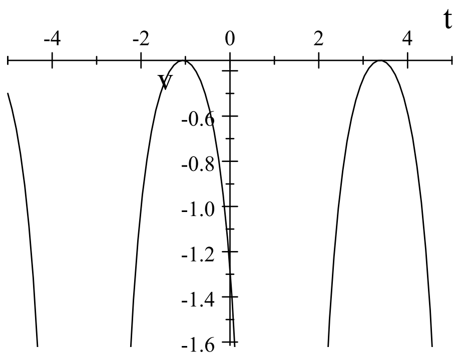
$v_{13,14}$ if $\lambda > 0$



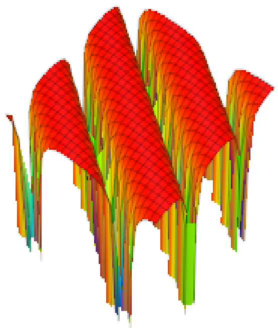
$w_{15,16}$ if $\lambda > 0$



$w_{15,16}$ if $\lambda > 0$



$v_{17,18}$ if $\lambda < 0$



$v_{17,18}$ if $\lambda < 0$

4 Conclusion

In this paper, we presented a new method to solve a new extension system equations (esB). A new method has been successfully implemented to find new traveling wave solutions for same nonlinear partial differential equations. We expressed to some our new solutions graphically.

References

[1] N. Azizi and R. Pourgholi, Applications of Sine–Cosine wavelets method for solving the generalized Hirota–Satsuma coupled KdV equation, Mathematical Sciences., 17.4 (2023), 503-516.

[2] K. R. Raslan, T. S. El-Danaf and K. K. Ali, New exact solutions of coupled generalized regularized long wave equations, Journal of the Egyptian Mathematical Society., 25.4 (2017), 400-405.

[3] S. N. Ananna, T. An, M. Asaduzzaman and M. M. Miah, Solitary wave structures of a family of 3D fractional WBBM equation via the tanh–coth approach, Partial Differential Equations in Applied Mathematics., 5 (2022), 100237.

[4] S. Behera, S. Mohanty and J. P. S. Virdi, Analytical solutions and mathematical simulation of traveling wave solutions to fractional order nonlinear equations, Partial Differential Equations in Applied Mathematics., 8 (2023), 100535.

[5] A. M. Wazwaz, Integrable couplings of the Burgers equation and the Sharma-Tasso-Olver equation: multiple kink solutions, Rom. Rep. Phys., 65.2 (2013), 383-390.

[6] A. M. Wazwaz, Multiple soliton solutions for the integrable couplings of the KdV and the KP equations, Open Physics., 11.3 (2013), 291-295.

[7] A. M. Wazwaz, A two-mode Burgers equation of weak shock waves in a fluid: multiple kink solutions and other exact solutions, International Journal of Applied and Computational Mathematics., 3 (2017), 3977-3985.

[8] H. Durur and A. Yokuş, Exact solutions of (2+1)-Ablowitz-Kaup - Newell-Segur equation, Applied Mathematics and Nonlinear Sciences., 6.2 (2021), 381-386.

[9] A. Maher, H. M. El-Hawary and M. S. Al-Amry, New exact solutions for new model nonlinear partial differential equation, Journal of Applied Mathematics., 2013.1 (2013), 767380.

[10] Z. Xue-Qin and Z. Hong-Yan, An Improved F-Expansion Method and Its Application to Coupled Drinfel'd–Sokolov–Wilson Equation, Communications in Theoretical Physics., 50.2 (2008), 309.

[11] Q. Wang, Numerical solutions for fractional KdV–Burgers equation by Adomian decomposition method, Applied mathematics and computation., 182.2 (2006), 1048-1055.

[12] O. Gonzalez-Gaxiola, A. Biswas, M. Asma and A. K. Alzahrani, Highly dispersive optical solitons with non-local law of refractive index by Laplace-Adomian decomposition, Optical and Quantum Electronics., 53 (2021), 1-12.

[13] H. Bulut, H. M. Baskonus and Y. Pandir, The modified trial equation method for fractional wave equation and time fractional generalized Burgers equation, Abstract and applied analysis., Vol. 2013. No. 1. Hindawi Publishing Corporation, 2013.

[14] C. Q. Dai, Y. Y. Wang and J. F. Zhang, Managements of scalar and vector rogue waves in a partially nonlocal nonlinear medium with linear and harmonic potentials, Nonlinear Dynamics., 102.1 (2020), 379-391.

[15] A. Aniqa and J. Ahmad, Soliton solution of fractional Sharma-Tasso-Olever equation via an efficient $\left(\frac{G'}{G}\right)$-expansion method, Ain Shams Engineering Journal., 13.1 (2022), 101528.

[16] S. K. Mohanty, S. Kumar, A. N. Dev, M. K. Deka, D. V. Churikov and O. V. Kravchenko, An efficient technique of $\left(\frac{G'}{G}\right)$–expansion method for modified KdV and Burgers equations with variable coefficients, Results in Physics., 37 (2022), 105504.

[17] S. K. Mohanty, O. V. Kravchenko and A. N. Dev, Exact traveling wave solutions of the Schamel Burgers' equation by using generalized-improved and generalized $\left(\frac{G'}{G}\right)$ expansion methods, Results in Physics., 33 (2022), 105124.

[18] M. S. Al-Amry and E. F. Abdullah, Exact solutions for a new models of nonlinear partial differential equations Using$\left(\frac{G'}{G^2}\right)$-Expansion Method, University of Aden Journal of Natural and Applied Sciences., 23.1 (2019), 189-199.

[19] A. Zafar,A. Saboor, M. Inc,M. Ashraf and S. Ahmad, New fractional solutions for the Clannish Random Walker's Parabolic equation and the Ablowitz-Kaup-Newell-Segur equation, Optical and Quantum Electronics., 56.4 (2024), 581.

[20] M. S. Al-Amry and E. F. Al-Abdali, New Exact Solutions for Generalized of Combined with Negative Calogero-Bogoyavlenskii Schiff and Generalized Yu–Toda–Sassa–Fukuyama Equations, University of Aden Journal of Natural and Applied Sciences., 28.1 (2024), 25-30.

[21] S. C. Unal, Approximate Solutions of the Fractional Clannish Random Walker's Parabolic Equation with the Residual Power Series Method, Journal of New Theory., 45 (2023), 18-29.

[22] J. A. Haider, S. Asghar and S. Nadeem, Travelling wave solutions of the third-order KdV equation using Jacobi elliptic function method, International Journal of Modern Physics., B 37.12 (2023), 2350117.

[23] O. Guner, A. Bekir and O. Unsal, Two reliable methods for solving the time fractional Clannish Random Walker's Parabolic equation, Optik., 127.20 (2016), 9571-9577.

[24] A. A. Al-Shawba, F. A. Abdullah, A. Azmi and M. A. Akbar, An Extension of the Double $(G'/G, 1/G)$-Expansion Method for Conformable Fractional Differential Equations, Complexity., 2020.1 (2020), 7967328.

[25] M. A. Khatun, M. A. Arefin, M. H. Uddin, D. Baleanu, M. A. Akbar and M. İnç, Explicit wave phenomena to the couple type fractional order nonlinear evolution equations, Results in Physics., 28 (2021), 104597.

[26] S. Ampun, S. Sungnul and S. Koonprasert, New exact solutions for the time fractional clannish random walker's parabolic equation by the improved tan $(\varphi(\xi)/2)$-expansion method, (2017).

[27] J. Alzabut, R. Dhineshbabu, A. G. M. Selvam, J. F. Gomez-Aguilar and H. Khan, Existence, uniqueness and synchronization of a fractional tumor growth model in discrete time with numerical results, Results in Physics., 54 (2023), 107030.

[28] H. Khan, J. Alzabut, W. F. Alfwzan and H. Gulzar, Nonlinear dynamics of a piecewise modified ABC fractional-order leukemia model with symmetric numerical simulations, Symmetry., 15.7 (2023), 1338.