Proceedings of the Jangjeon Mathematical Society
27 (2024), No. 3, pp. 431 - 449

www.jangjeon.or.kr
http://dx.doi.org/10.17777/pjms2024.27.3.431

THE GENERALIZED STATUS SCHULTZ AND STATUS GUTMAN INDICES

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Abstract. Generalization of indices is a major concept of graph theory wherein many different relative graphical indices are represented in the form of a single index with varying powers. In this paper, we initiate the study on generalized status Schultz index and generalized Gutman index. Here, we have obtained exact value of some specific families of graphs and inequalities of these indices in terms of order, size, degree's, radius and diameter. Further, we have done comparative analysis of various status related indices using a graphical plot considering the molecular graphs of some monocarboxylic acids to which the indices are calculated.

2020 Mathematics Subject Classification. 05C09, 05C12, 05C92.

Keywords and phrases. Graphical indices, Chemical graphs, distance, status Schultz index, status Gutman index.

1. Introduction

A graph $G = (V, E)$ is a mathematical structure consisting of a set of vertices $V = V(G)$ and a set of edges $E = E(G)$. Each edge connects a pair of vertices, representing a relationship or connection between them. The graphs considered in this article are non-pseudo undirected graphs with p-vertices and q-edges. The number of vertices at a distance one from v_i where $i = 1, 2, \cdots, p$ is said to be the degree of vertex v_i and is represented as $d_G(v_i)$. The minimum degree is represented by $\delta = \delta(G)$ and the maximum degree is represented by $\Delta = \Delta(G)$. The minimum length of the path between two vertices v_i and v_j is said to be the distance between them and is represented as $d(v_i, v_j)$. The eccentricity $e(v_i)$ of a vertex v_i is the maximum among the distances from vertex v_i to all the other vertices in G. The maximum eccentricity is said to be the diameter of a graph G and is denoted by $diam(G) = D(G) = D$ and the minimum eccentricity is said to be the radius of a graph G and is denoted by $rad(G) = rad$. For the terminology not defined here, we refer to [12] and [13].

The status[14](or transmission [1]) of a vertex $v_i \in V(G)$ of a graph G is defined as

$$\sigma_G(v_i) = \sum_{v_j \in V(G)} d(v_i, v_j).$$

In other words, $\sigma_G(v_i)$ is the sum of its distance from every other vertex in $V(G)$ of a graph G. This quantity serves as a pivotal measure of a vertex's connectedness within G.

Additionally, the Weiner index of a graph G, introduced by Harry Wiener in [31], encapsulates the pairwise distances between vertices in a graph which is defined as follows:

$$W(G)=\sum_{\{v_i,v_j\}\subseteq V(G)} d(v_i,v_j)=\frac{1}{2}\sum_{v_i\in V(G)}\sigma_G(v_i).$$

In the context of chemistry, the Wiener index has been used to model various physicochemical properties of molecules, including boiling points. The rationale behind this is that molecules with similar Wiener indices are expected to have similar boiling points.
For more details on graphical indices, we refer to [4, 5, 6, 7, 8, 11, 20, 21, 22, 23, 28, 30].

Molecular graph theory is a branch of graph theory that focuses on the study of molecular structures using graph theoretical concepts. In this context, a molecule is represented as a graph, where atoms are represented as vertices, and chemical bonds are represented as edges. Graphical indices are numerical values associated with a graph, often used in the field of mathematical chemistry and chemical informatics to describe the topological structure of molecules.

Being motivated by the generalized Schultz and Gutman indices introduced by Ameer et al.[3] and the status distance indices introduced by Sumithra and Adiga [29], we generalize the status distance indices which are defined as follows:

For any two real numbers a and b, the (a,b)- status Schultz index of a graph G is given by

$$SS_{(a,b)}(G)=\sum_{\{v_i,v_j\}\subseteq V(G)}[(\sigma_G(v_i)+\sigma_G(v_j))^a d(v_i,v_j)^b],$$

and the (a,b)- status Gutman index of a graph G is given by

$$SG_{(a,b)}(G)=\sum_{\{v_i,v_j\}\subseteq V(G)}[(\sigma_G(v_i).\sigma_G(v_j))^a d(v_i,v_j)^b].$$

For more details on status related graphical indices, we refer to [2, 9, 10, 15, 24, 25, 27, 32].

2. The particular values of a and b in (a,b)-status distance indices

Most of the status based indices are special cases of (a,b)-Status distance Indices, for particular values of real numbers a and b as follows:

(i) $SS_{(1,0)}(G)=S_1(G)$, the first status connectivity index, [26].
(ii) $SG_{(1,0)}(G)=S_2(G)$, the second status connectivity index, [26].
(iii) $SS_{(-\frac{1}{2},0)}(G)=SS(G)$, the sum connectivity status index, [19].
(iv) $SG_{(-\frac{1}{2},0)}(G)=PS(G)$, the product connectivity status index, [19].

(v) $SS_{(\frac{1}{2},0)}(G) = SN(G)$, the status Nirmala index, [18].
(vi) $SG_{(\frac{1}{2},0)}(G) = RPS(G)$, the reciprocal product connectivity status index, [19].
(vii) $SS_{(2,0)}(G) = HS_1(G)$, the first hyper status index, [19].
(viii) $SG_{(2,0)}(G) = HS_2(G)$, the second hyper status index, [19].
(i) $2SS_{(-1,0)}(G) = HS(G)$, the harmonic status index, [25].
(xi) $SS_{(1,0)}(G) + SG_{(1,0)}(G) = SGO_1(G)$, the first status Gourava index, [17].
(x) $SS_{(1,0)}(G)SG_{(1,0)}(G) = SGO_2(G)$, the second status Gourava index, [17].
(xi) $SS_{(1,1)}(G) = SS(G)$, the distance status connectivity index, [29].
(xii) $SG_{(1,1)}(G) = SG(G)$, the distance status product connectivity index.

Theorem 2.1. *Let G be a non-trivial connected graph with the real numbers a and b. Then,*

$$\frac{SG_{(\frac{1}{2},0)}(G)}{SS_{(1,0)}(G)} \leq GAS(G),$$

where $GAS(G)$ is the geometric arithmetic status index [16] *of G.*

Proof. By the definitions of $SG_{(\frac{1}{2},0)}(G)$, $SS_{(1,0)}(G)$ and $GAS(G)$, we have the desired result. □

3. The (a, b)- status distance indices of certain class of graphs

Proposition 3.1. *Let K_p be a complete graph with $p \geq 2$. Then*

(i) $SS_{(a,b)}(K_p) = 2^{a-1}p(p-1)^{a+1}$.

(ii) $SG_{(a,b)}(K_p) = \dfrac{p}{2}(p-1)^{2a+1}$.

Proof. Let K_p be a complete graph with $p \geq 2$. Then each vertex has status $p-1$ and since each vertex is neighbor of every other vertex, distance between any two vertices is also one. Substituting the above information in the respective definitions of (a, b)-status distance indices and solving, we obtain the proposition as mentioned above. □

Proposition 3.2. *Let C_p be a cycle with $p \geq 3$. Then*

$$\text{(i) } SS_{(a,b)}(C_p) = \begin{cases} p\left(\dfrac{p^2}{2}\right)^a [1^b + 2^b + \cdots + (\dfrac{p}{2} - 1)^b] \\ \qquad + \dfrac{p^{2a+b+1}}{2^{a+b+1}}, & if\ p\ is\ even. \\ \\ p\left(\dfrac{p^2-1}{2}\right)^a [1^b + 2^b + \cdots + (\dfrac{p-1}{2})^b], & if\ p\ is\ odd. \end{cases}$$

$$\text{(ii) } SG_{(a,b)}(C_p) = \begin{cases} p\left(\frac{p^4}{16}\right)^a [1^b + 2^b + \cdots + (\frac{p}{2}-1)^b] \\ \quad + \left(\frac{p}{2}\right)^{4a+b+1}, & if\ p\ is\ even. \\ p\left(\frac{p^2-1}{4}\right)^{2a} [1^b + 2^b + \cdots + (\frac{p-1}{2})^b], & if\ p\ is\ odd. \end{cases}$$

Proof. Let C_p be a cycle with $p \geq 3$. Then each vertex in an even cycle has status $\frac{p^2}{4}$ and each vertex in an odd cycle has status $\frac{p^2-1}{4}$. The distance between any two vertices in an even cycle ranges between 1 and $\frac{p}{2}$ and in an odd cycle it ranges between 1 and $\frac{p-1}{2}$. Substituting the above information in the respective definitions of (a,b)-status distance indices and solving, we obtain the proposition as mentioned above. □

Proposition 3.3. *Let $K_{m,n}$ be a complete bipartite graph with $m \leq n$ and $m, n \geq 1$. Then*

(i) $SS_{(a,b)}(K_{m,n}) = mn(3m+3n-4)^a + [2n+4(m-1)]^a m(m-1)2^{b-1} + n(n-1)[2m+4(n-1)]^a 2^{b-1}$.

(ii) $SG_{(a,b)}(K_{m,n}) = mn[5mn+2(m^2+n^2)-6(m+n)+4]^a + m(m-1)[n^2+4(m-1)^2+4n(m-1)]^a 2^{b-1} + n(n-1)[m^2+4(n-1)^2+4m(n-1)]^a 2^{b-1}$.

Proof. Let $K_{m,n}$ be a complete bipartite graph with $m, n \geq 1$ and $m \leq n$. Then the vertices with degree n has status $n+2(m-1)$ and the vertices with degree m has status $m+2(n-1)$. Since the distance between vertices of different sets is one and the vertices of same set is two, there exists mn set of vertices with distance one and $[\binom{m}{2}+\binom{n}{2}]$ set of vertices with distance two where $\binom{m}{2}$ set of vertices are joined by an edge whose end vertices have status $n+2(m-1)$ and $\binom{n}{2}$ set of vertices are joined by an edge whose end vertices have status $m+2(n-1)$. Substituting the above information in the respective definitions of (a,b)-status distance indices and solving, we obtain the proposition as mentioned above. □

Corollary 3.4. *Let $S_p = K_{1,p-1}$ be a star with $p \geq 2$. Then*

(i) $SS_{(a,b)}(S_p) = (p-1)(3p-4)^a + (p^2-3p+2)(2p-3)^a 2^{a+b-1}$.

(ii) $SG_{(a,b)}(S_p) = (p-1)(2p^2-5p+3)^a + (p^2-3p+2)(4p^2-12p+9)^a 2^{b-1}$.

Proposition 3.5. *Let $W_p = K_1 + C_{p-1}$ be a wheel graph with $p \geq 4$. Then*

(i) $SS_{(a,b)}(W_p) = (p-1)(3p-6)^a + (p-1)(4p-10)^a + (p^2-5p+4)(4p-10)^a.2^{b-1}$.

(ii) $SG_{(a,b)}(W_p) = (p-1)(2p^2-7p+5)^a + (4p^2-20p+25)^a (p-1) + (p^2-5p+4)(4p^2-20p+25)^a 2^{b-1}$.

Proof. Let W_p be a wheel graph with $p \geq 4$. The central vertex has status $p-1$ and all the other vertices have status $2p-5$. There exists $2(p-1)$ set of vertices with distance one where $p-1$ sets of vertices are joined by an edge whose both end vertices have staus $2p-5$ and the other $p-1$ set of vertices are joined by edges whose end vertices have status $p-1$ and $2p-5$, and $\frac{p^2-5p+4}{2}$ set of vertices with distance two. Substituting the above information in the respective definitions of (a,b)-status distance indices and solving, we obtain the proposition as mentioned above. □

Proposition 3.6. *Let F_n be a friendship graph with $n \geq 2$, where F_n is a graph with n copies of C_3 with one common vertex to all. Then*

(i) $SS_{(a,b)}(F_n) = 2n(6n-2)^a + n(8n-4)a + (4n^2-4n)(8n-4)^a 2^{b-1}$.

(ii) $SG_{(a,b)}(F_n) = 2n(8n^2-4n)^a + n(16n^2-16n+4)^a + (4n^2-4n)(16n^2-16n+4)^a 2^{b-1}$.

Proof. Let F_n be a friendship graph with $n \geq 2$. Then the central vertex has status $2n$ and the other vertices have status $4n-2$. There exists $3n$ set of vertices with distance one where n set of vertices are joined by an edge whose end vertices have status $4n-2$ each and $2n$ set of vertices are joined by an edge whose end vertices have status $2n$ and $4n-2$, and $\left[\binom{2n}{2} - n\right]$ set of vertices with distance two. Substituting the above information in the respective definitions of (a,b)-status distance indices and solving, we obtain the proposition as mentioned above. □

4. Inequalities of the (a,b)- status distance indices

We here obtain the inequalities of the (a,b)- status distance indices in terms of order, size, diameter, radius, minimum and maximum degree.

4.1. Inequality in terms of order and size of G.

Theorem 4.1. *Let G be a non-trivial graph with the real numbers a and b both being positive. Then,*

$$p(p-1)2^{a-1} \leq SS_{(a,b)}(G) \leq 2^{a-1}p(p-1)^{2a+b+1}. \tag{1}$$

$$\frac{p(p-1)}{2} \leq SG_{(a,b)}(G) \leq \frac{p}{2}(p-1)^{4a+b+1}. \tag{2}$$

Proof. Let G be a non-trivial graph with p vertices. Then,

$$1 \leq \sigma_G(v_i) \leq (p-1)^2 \quad and \quad 1 \leq \sigma_G(v_j) \leq (p-1)^2. \tag{3}$$

Summing up the inequalities (3) and raising it to the power a, we have

$$1 \leq (\sigma_G(v_i) + \sigma_G(v_j))^a \leq 2^a(p-1)^{2a}. \tag{4}$$

Also, we have

$$1 \leq d(u,v)^b \leq (p-1)^b. \tag{5}$$

Multiplying the inequalities (4) and (5) and taking summation over the set $\{v_i, v_j\} \subseteq V(G)$, we have

$$\sum_{\{v_i,v_j\} \subseteq V(G)} 2^a \leq \sum_{\{v_i,v_j\} \subseteq V(G)} [(\sigma_G(v_i) + \sigma_G(v_j))^a (d(u,v))^b]$$

$$\leq \sum_{\{v_i,v_j\}\subseteq V(G)} 2^a(p-1)^{2a}(p-1)^b.$$

$$\frac{p(p-1)}{2}2^a \leq SS_{(a,b)}(G) \leq \frac{p(p-1)}{2}2^a(p-1)^{2a}(p-1)^b.$$

Thus we obtain the required inequality (1).
Now, to prove the inequality (2), we multiply the inequalities in (3) and raise it to the power a. Thus we obtain,

$$(\sigma_G(v_i).\sigma_G(v_j))^a \leq (p-1)^{4a}. \tag{6}$$

Multiplying the inequalities (5) and (6) and taking summation over the sets $\{v_i, v_j\} \subseteq V(G)$, we have

$$\sum_{\{v_i,v_j\}\subseteq V(G)} 1 \leq \sum_{\{v_i,v_j\}\subseteq V(G)} [(\sigma_G(v_i).\sigma_G(v_j))^a d(u,v)^b]$$

$$\leq \sum_{\{v_i,v_j\}\subseteq V(G)} (p-1)^{4a}(p-1)^b.$$

$$\frac{p(p-1)}{2} \leq SG_{(a,b)}(G) \leq \frac{p(p-1)}{2}(p-1)^{4a}(p-1)^b.$$

Thus we obtain the required inequality (2). □

4.2. **Inequality in terms of order, diameter and degree's of G.**

Theorem 4.2. *Let G be a non-trivial graph with the real numbers a and b both being positive. Then,*

$$\begin{aligned} p(p-1)2^{a-1}(2p-2-\Delta)^a &\leq SS_{(a,b)}(G) \\ &\leq p(p-1)2^{a-1}[D(p-1)-(D-1)\delta]^b D^b. \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{p(p-1)}{2}[4(p-1)^2 - 4\Delta(p-1) + \delta^2]^a \leq SG_{(a,b)}(G) \leq \\ &\frac{p(p-1)}{2}D^b[D^2(p-1)^2 - 2\delta D(D-1)(p-1) + (D-1)^2\Delta^2]^a. \end{aligned} \tag{8}$$

Proof. Let G be a non-trivial graph with the real numbers a and b both being positive. Then,

$$2p-2-d_G(v_i) \leq \sigma_G(v_i) \leq D(p-1)-(D-1)d_G(v_i). \tag{9}$$

$$2p-2-d_G(v_j) \leq \sigma_G(v_j) \leq D(p-1)-(D-1)d_G(v_j). \tag{10}$$

Summing up the inequalities (9) and (10) and raising it to the power a, we have

$$\begin{aligned} [2(2p-2)-(d_G(v_i)+d_G(v_j))]^a &\leq [\sigma_G(v_i)+\sigma_G(v_j)]^a \\ &\leq [2D(p-1)-(D-1)(d_G(v_i)+d_G(v_j))]^a. \end{aligned} \tag{11}$$

Multiplying inequalities (17) and (11) and taking summation over the set $\{v_i, v_j\} \subseteq V(G)$, we have

$$\sum_{\{v_i,v_j\}\subseteq V(G)} [2(2p-2)-(d_G(v_i)+d_G(v_j))]^a$$

$$\leq \sum_{\{v_i,v_j\}\subseteq V(G)} [(\sigma_G(v_i)+\sigma_G(v_j))^a(d(u,v))^b]$$

$$\leq \sum_{\{v_i,v_j\}\subseteq V(G)} [2D(p-1)-(D-1)(d_G(v_i)+d_G(v_j))]^a D^b.$$

Also, substituting $\delta \leq d_G(v_i) \leq \Delta$, the above inequality becomes,

$$\frac{p(p-1)}{2}2^a[2(2p-2)-\Delta]^a \leq SS_{(a,b)}(G) \leq \frac{p(p-1)}{2}2^a D^b[D(p-1)-(D-1)\delta]^a.$$

Thus we obtain the required inequality (7).

Now we prove the inequality (8). Multiplying the inequalities (9) and (10) and raising it to the power 'a', we have

$$[4(p-1)^2-2(p-1)(d_G(v_i)+d_G(v_j))]^a \leq (\sigma_G(v_i).\sigma_G(v_j))^a \leq [D^2(p-1)^2-D(D-1)(p-1)(d_G(v_i)+d_G(v_j))+(D-1)^2 d_G(v_i)d_G(v_j)]^a. \tag{12}$$

Multiplying inequalities (17) and (12) and taking summation over the set $\{v_i, v_j\} \subseteq V(G)$, we have

$$\sum_{\{v_i,v_j\}\subseteq V(G)} [4(p-1)^2-2(p-1)(d_G(v_i)+d_G(v_j))]^a \leq \sum_{\{v_i,v_j\}\subseteq V(G)} [(\sigma_G(v_i).\sigma_G(v_j))^a(d(u,v))^b] \leq \sum_{\{v_i,v_j\}\subseteq V(G)} [D^2(p-1)^2-D(D-1)(p-1)(d_G(v_i)+d_G(v_j))+(D-1)^2 d_G(v_i)d_G(v_j)]^a D^b.$$

Also, substituting $\delta \leq d_G(v_i) \leq \Delta$ in the above inequality, we obtain the required inequality (7). □

4.3. **Inequality in terms of order, size and diameter of G.**

Theorem 4.3. *Let G be a non-trivial graph with the real numbers a and b both being positive. Then,*

$$p(p-1)2^{a-1} \leq SS_{(a,b)}(G) \leq 2^{a-1}p(p-1)^{a+1}D^{a+b}. \tag{13}$$

$$\frac{p(p-1)}{2} \leq SG_{(a,b)}(G) \leq \frac{p}{2}(p-1)^{2a+1}D^{2a+b}. \tag{14}$$

Proof. Let G be a non-trivial graph with the real numbers a and b both being positive. Then,

$$1 \leq \sigma_G(v_i) \leq (p-1)D \quad \textit{and} \quad 1 \leq \sigma_G(v_j) \leq (p-1)D. \tag{15}$$

Adding up the above inequalities and raising it to the power 'a', a positive real, we have

$$2^a \leq [\sigma_G(v_i)+\sigma_G(v_j)]^a \leq 2^a(p-1)^a D^a. \tag{16}$$

Also, we have the known inequality, $1 \leq d(u,v) \leq D$.

Raising the above inequality to the power 'b', a positive real, we have

$$1 \leq [d(u,v)]^b \leq D^b. \tag{17}$$

Multiplying the inequalities (16) and (17) and taking summation over the sets $\{v_i, v_j\} \subseteq V(G)$, we have

$$\sum_{\{v_i,v_j\}\subseteq V(G)} 2^a \leq \sum_{\{v_i,v_j\}\subseteq V(G)} [(\sigma_G(v_i)+\sigma_G(v_j))^a (d(u,v))^b]$$

$$\leq \sum_{\{v_i,v_j\}\{v_i,v_j\}\subseteq V(G)} 2^a(p-1)^a D^{a+b}.$$

$$i.e., \qquad \frac{p(p-1)}{2}2^a \leq SS_{(a,b)}(G) \leq \frac{p(p-1)}{2}2^a(p-1)^a D^{a+b}.$$

Thus we obtain the required inequality (13).
Now, to prove the inequality (14), we multiply the inequalities in (15) and raise it to the power a. Thus we obtain,

$$1 \leq [\sigma_G(v_i).\sigma_G(v_j)]^a \leq (p-1)^{2a} D^{2a}. \tag{18}$$

Multiplying the inequalities (17) and (18) and taking summation over the sets $\{v_i, v_j\} \subseteq V(G)$, we have

$$\sum_{\{v_i,v_j\}\subseteq V(G)} 1 \leq \sum_{\{v_i,v_j\}\subseteq V(G)} [(\sigma_G(v_i).\sigma_G(v_j))^a (d(u,v))^b]$$

$$\leq \sum_{\{v_i,v_j\}\subseteq V(G)} (p-1)^{2a} D^{2a+b}.$$

$$\frac{p(p-1)}{2} \leq SG_{(a,b)}(G) \leq \frac{p(p-1)}{2}(p-1)^{2a} D^{2a+b}.$$

Thus we obtain the required inequality (14). □

Corollary 4.4. *Let G be a non-trivial graph with p vertices. Then,*

$$p(p-1)2^{a-1} \leq SS_{(a,b)}(G) \leq 2^{2a+b-1}p(p-1)^{a+1}(rad)^{a+b}.$$

$$\frac{p(p-1)}{2} \leq SG_{(a,b)}(G) \leq 2^{2a+b-1}p(p-1)^{2a+1}(rad)^{2a+b}.$$

Observation 4.5. *All the inequalities proved in above theorems hold true for any real numbers a and b having same sign. If the real numbers a and b are of opposite signs, then the inequalities reverse.*

5. Chemical applicabilities of status distance indices

Monocarboxylic acids are chemical compounds that have only one carboxyl group in the molecule. Carboxylic acids are widespread in nature. The aliphatic monocarboxylic acids with C_1 to C_4 carbon atoms are relatively volatile with a different pungency, while the acids with the more carbon atoms are relatively oily materials and have a little solubility in water.

All the vertices in molecular graph of monocarboxylic acids (see figure 1) are numbered from left to right and status of each vertex is represented in table 1. The calculated graphical indices of butyric acid (G_1), caprylic acid(G_2), capric acid(G_3), undecylicacid(G_4), lauric acid(G_5), myristic acid(G_6), margaric acid(G_7) and arachidic acid(G_8) are represented in table 2 and the graph corresponding to these indices is plotted as in figure 2.

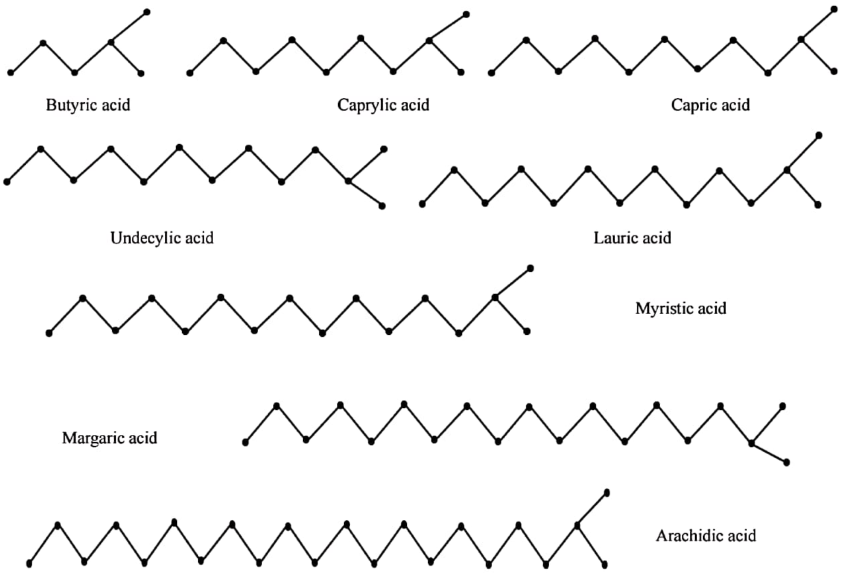

FIGURE 1. Some monocarboxylic acids

TABLE 1. Status of each vertex in G_i

Status	$\sigma_{G_1}(v_i)$	$\sigma_{G_2}(v_i)$	$\sigma_{G_3}(v_i)$	$\sigma_{G_4}(v_i)$	$\sigma_{G_5}(v_i)$	$\sigma_{G_6}(v_i)$	$\sigma_{G_7}(v_i)$	$\sigma_{G_8}(v_i)$
v_1	14	44	65	77	90	119	170	230
v_2	10	36	55	66	78	105	153	210
v_3	8	30	47	57	68	93	138	192
v_4	8	26	41	50	60	83	125	176
v_5	12	24	37	45	54	75	114	162
v_6	12	24	35	42	50	69	105	150
v_7	-	26	35	41	48	65	98	140
v_8	-	30	37	42	48	63	93	132
v_9	-	38	41	45	50	63	90	126
v_{10}	-	38	47	50	54	65	89	122
v_{11}	-	-	57	57	60	69	90	120
v_{12}	-	-	57	68	68	75	93	120
v_{13}	-	-	-	68	80	83	98	122
v_{14}	-	-	-	-	80	93	105	126
v_{15}	-	-	-	-	-	107	114	132
v_{16}	-	-	-	-	-	107	125	140
v_{17}	-	-	-	-	-	-	138	150
v_{18}	-	-	-	-	-	-	155	162
v_{19}	-	-	-	-	-	-	155	176
v_{20}	-	-	-	-	-	-	-	192
v_{21}	-	-	-	-	-	-	-	212
v_{22}	-	-	-	-	-	-	-	212

The $(a,b)-$status distance indices of these acids are as follows:

$$\begin{aligned}
SS_{(a,b)}(G_1) &= 3^a2^{3a+b} + 5^a2^{2a+b+1} + 9^a2^{a+b} + 11^a2^{a+b} + 13^a2^{a+2b+1}\\
&\quad + 22^a3^{b+1} + 2^{2a+1}5^a + 16^a + 18^a + 24^a.\\
SG_{(a,b)}(G_1) &= 3^a2^{5a+b+1} + 5^a2^{4a+b} + 2^{3a+1}5^a3^{a+b} + 7^a2^{4a+b} + 9^a2^{4a+b}\\
&\quad + 21^a2^{3a+2b+1} + 112^a3^b + 2^{5a+1}3^a + 64^a + 80^a + 140^a.\\
SS_{(a,b)}(G_2) &= 2^{6a+b+1} + 2^{a+1}3^{3a+b} + 2^{6a+1}5^b + 7^a2^{3a+2b+1} + 11^a6^{a+b}\\
&\quad + 12^a5^{a+b} + 15^a4^{a+b} + 17^a3^b2^{2a+b+1} + 17^a4^{a+b} + 19^a2^{2a+b}\\
&\quad + 20^a3^{a+b} + 25^a2^{a+b+1} + 27^a2^{a+b+1} + 31^a2^{a+b} + 31^a2^{a+2b+1}\\
&\quad + 2^{a+1}31^a3^b + 35^a3^b2^{a+b} + 37^a2^{a+b} + 41^a2^{a+3b+1} + 52^a3^b\\
&\quad + 62^a5^b + 68^a5^b + 70^a3^b + 3\ 74^a7^b + 2^{3a+1}7^a + 2^{2a+1}17^a\\
&\quad + 2^{a+1}25^a + 48^a + 66^a + 80^a.\\
SG_{(a,b)}(G_2) &= 2^{4a+1}5^a3^{2a+b} + 5^a6^{3a+b} + 2^{4a+1}19^a3^{a+b} + 27^a2^{5a+2b}\\
&\quad + 32^a3^{3a+b} + 33^a2^{5a+2b} + 36^a5^{2a+b} + 39^a2^{4a+b+1} + 676^a3^b\\
&\quad + 57^a2^{4a+2b+1} + 95^a2^{2a+b+1}3^{a+b} + 117^a2^{3a+b} + 143^a3^b2^{3a+b}\\
&\quad + 165^a2^{3a+b} + 2^{3a+1}171^a7^b + 195^a2^{2a+2b+1} + 209^a2^{3a+3b+1}\\
&\quad + 247^a2^{2a+b+1} + 2^{2a+1}247^a5^b + 361^a2^{2a+b} + 45^a2^{4a+b+1}\\
&\quad + 936^a5^b + 1056^a5^b + 1144^a3^b + 1320^a7^b + 2^{4a+1}39^a\\
&\quad + 2^{2a+1}195^a + 2^{2a+1}285^a + 576^a + 1080^a + 1584^a.\\
SS_{(a,b)}(G_3) &= 3^a2^{5a+b} + 4^a5^{2a+b} + 7^a2^{4a+b} + 2^{a+1}7^{2a+b} + 9^a2^{3a+b+1}\\
&\quad + 11^a3^b2^{3a+b+1} + 13^a2^{3a+3b+1} + 18^a5^{a+b} + 19^a2^{2a+b+1}\\
&\quad + 2^{2a+1}19^a3^b + 21^a2^{2a+b+1} + 2^{2a+1}21^a5^b + 23^a2^{2a+2b+1}\\
&\quad + 23^a3^b2^{2a+b} + 2^{2a+1}23^a5^b + 25^a3^b2^{2a+b} + 39^a2^{a+2b+1}\\
&\quad + 41^a2^{a+2b+1} + 2^{a+1}41^a3^b + 45^a2^{a+2b} + 2^{a+1}47^a3^b + 102^a\\
&\quad + 49^a2^{a+b+1} + 47^a3^b2^{a+b+1} + 51^a2^{a+2b} + 51^a2^{a+3b} + 120^a\\
&\quad + 61^a5^b2^{a+b+1} + 74^a3^b + 82^a5^b + 92^a3^b + 94^a7^b + 96^a7^b\\
&\quad + 102^a7^b + 106^a3^b + 112^a3^{2b+1} + 2^{3a+1}9^a + 2^{3a+1}11^a\\
&\quad + 57^a2^{a+b} + 53^a2^{a+3b} + 2^{3a+1}13^a + 2^{a+1}39^a + 70^a.\\
SG_{(a,b)}(G_3) &= 77^a5^{2a+b} + 91^a5^{2a+b} + 741^a2^{b+1}5^{a+b} + 1369^a3^b + 1681^a5^b\\
&\quad + 2(399^a5^{a+b}) + 703^a2^{b+1}3^{a+b} + 2(1045^a3^{a+2b}) + 1295^a2^{b+1}\\
&\quad + 2(703^a3^{a+b}) + 1435^a2^{b+1} + 2(1435^a3^b) + 1517^a2^{2b+1}\\
&\quad + 1645^a2^{2b+1} + 2(1645^a3^b) + 1739^a2^{b+1} + 2(1739^a5^b)\\
&\quad + 1925^a4^b + 1927^a2^{b+1}3^b + 1995^a2^{2b+1} + 2035^a3^b + 2035^a6^b\\
&\quad + 2209^a7^b + 2255^a2^b + 2255^a7^b + 2275^a6^b + 2337^a2^{b+1}\\
&\quad + 2(2337^a7^b) + 2405^a4^b + 2679^a2^{3b+1} + 2405^a7^b + 2585^a8^b
\end{aligned}$$

$$
\begin{aligned}
&+2665^a3^b+2665^a8^b+3055^a2^b+3055^a9^b+3249^a2^b\\
&+2(1295^a)+2(1517^a)+2585^a+2(1927^a)+2(2679^a)\\
&+3575^a+1225^a.\\
SS_{(a,b)}(G_4)&=2^{2a+b}3^{3a+b}+2(11^a3^{2a+b})+17^a2^{3a+b}+17^a2^{a+b+1}3^{a+b}\\
&+17^a7^{a+b}+2(19^a5^{a+b})+21^a2^{2a+b}+23^a2^{2a+b+1}+111^a7^b\\
&+25^a3^b2^{2a+b}+27^a4^{a+b}+29^a2^{2a+b}+29^a2^{2a+3b}+107^a5^b\\
&+37^a3^{a+b}+41^a3^{a+2b}+43^a2^{a+b+1}+45^a2^{a+2b}+49^a2^{a+2b+1}\\
&+51^a2^{a+b+1}+55^a2^{a+2b+1}+55^a3^b2^{a+b+1}+23^a2^{2a+2b+1}\\
&+59^a2^{a+b+1}+59^a2^{a+3b+1}+59^a3^b2^{a+b}+57^a2^{a+3b}+67^a2^{a+b}\\
&+61^a2^{a+2b}+61^a2^{a+3b}+3(67^a5^b2^{a+b})+2(91^a3^b)+2(99^a5^b)\\
&+2(109^a5^b)+2(107^a7^b)+2(113^a3^b)+2(113^a7^b)+127^a9^b\\
&+2(125^a9^b)+127^a3^b+2(145^a11^b)+2(83^a)+2(87^a)+143^a\\
&+2(29^a3^{a+b})+2(95^a)+2(107^a)+123^a+2(125^a)+119^a5^b.\\
SG_{(a,b)}(G_4)&=2^{a+1}9^a5^{3a+b}+2^{a+1}35^a3^{3a+b}+66^a7^{2a+b}+77^a6^{2a+b}\\
&+2^{2a+1}85^a3^{2a+b}+95^a2^{b+1}3^{3a+b}+110^a3^{3a+b}+289^a2^{4a+b}\\
&+119^a2^{3a+b+1}3^{a+b}+2^{2a+1}119^a11^{a+b}+2^{a+1}133^a3^{2a+b}\\
&+2^{2a+1}323^a3^{a+2b}+357^a2^{3a+2b+1}+418^a9^{a+b}+425^a2^{3a+b+1}\\
&+425^a2^{3a+3b+1}+441^a2^{2a+b}+525^a2^{2a+b+1}+525^a2^{2a+2b+1}\\
&+561^a5^b2^{3a+b+1}+625^a3^b2^{2a+b}+693^a4^{a+b}+2^{2a+1}697^a5^b\\
&+2^{2a+1}765^a7^b+825^a2^{2a+b}+825^a2^{2a+3b}+2^{a+1}1025^a3^b\\
&+2^{a+1}1197^a5^b+2^{a+1}1425^a7^b+1845^a2^{b+1}+2025^a4^b\\
&+2337^a2^{2b+1}+2565^a2^{b+1}+2706^a5^b+2970^a7^b+3157^a6^b\\
&+3234^a5^b+3249^a8^b+3465^a4^b+3465^a8^b+3850^a3^b\\
&+3850^a9^b+4389^a2^b+4389^a10^b+2^{a+1}861^a+2^{a+1}945^a\\
&+2^{2a+1}969^a+2^{a+1}1125^a+2^{a+1}1425^a+3762^a+5082^a.\\
SS_{(a,b)}(G_5)&=2^{7a+2b}+2^{7a+3b+1}+3^b2^{7a+b+1}+2^{2a+1}3^{3a+b}+5^a2^{5a+b}\\
&+2^{7a+1}5^b+3^a2^{a+b}5^{2a+b}+7^a2^{a+b}3^{2a+b}+9^a4^{2a+b}+16^a9^{a+b}\\
&+2^{a+1}11^a5^{a+b}+13^a2^{3a+2b+1}+2^{a+1}17^a3^{a+b}+28^a5^{a+b}\\
&+19^a2^{a+b+1}3^{a+b}+23^a6^{a+b}+27^a2^{2a+2b+1}+29^a2^{2a+2b+1}\\
&+2^{2a+1}29^a5^b+33^a2^{2a+3b}+35^a2^{2a+b+1}+35^a2^{2a+3b}\\
&+2^{2a+1}35^a9^b+37^a5^b2^{2a+b+1}+49^a2^{a+b+1}+59^a3^b2^{a+b+1}\\
&+46^a3^{a+2b}+50^a3^{a+b}+51^a2^{a+b+1}+55^a2^{a+b+1}+2^{a+1}59^a3^b\\
&+61^a2^{a+b+1}+2^{a+1}61^a7^b+65^a2^{a+2b+1}+2^{a+1}65^a7^b+168^a\\
&+67^a2^{a+3b+1}+2^{a+1}67^a3^b+69^a2^{a+b}+73^a5^b2^{a+b}+79^a2^{a+b}
\end{aligned}
$$

$$
\begin{aligned}
& + 85^a3^b2^{a+2b+1} + 100^a3^b + 108^a5^b + 120^a7^b + 126^a5^b + 96^a \\
& + 128^a7^b + 136^a9^b + 138^a7^b + 44^a3^{a+b} + 3(158^a11^b) + 146^a \\
& + 2^{3a+1}13^a + 2^{2a+1}37^a + 2^{a+1}49^a + 2^{a+1}57^a + 2^{7a+1}. \\
SG_{(a,b)}(G_5) &= 2^{5a+1}3^{4a+b} + 5^a2^{8a+b+1}3^{a+b} + 2^{6a+1}5^a3^{2a+b} + 36^a5^{3a+b} \\
& + 2^{5a+1}5^a3^{3a+b} + 5^a2^{5a+b}3^{3a+b} + 5^a2^{3a+b+1}3^{4a+b} + 7020^a \\
& + 2^{8a+1}3^a5^{a+b} + 2^{3a+1}3^a5^{3a+b} + 13^a2^{5a+b}3^{2a+b} + 52^a3^{4a+b} \\
& + 17^a2^{6a+b+1}5^{a+b} + 20^a3^{5a+2b} + 25^a2^{8a+b} + 25^a2^{5a+2b+1}3^{2a+b} \\
& + 2^{6a+1}25^a3^{a+2b} + 27^a2^{3a+b}5^{2a+b} + 45^a2^{6a+2b+1} + 75^a2^{6a+b+1} \\
& + 51^a2^{6a+2b+1} + 2^{6a+1}51^a5^b + 75^a2^{5a+b+1} + 135^a2^{5a+3b+1} \\
& + 81^a2^{5a+b+1} + 125^a2^{5a+2b+1} + 2^{5a+1}125^a7^b + 2^{5a+1}195^a11^b \\
& + 200^a3^{3a+b} + 255^a2^{4a+3b+1} + 375^a2^{3a+b+1} + 2^{3a+1}425^a3^b \\
& + 425^a3^b2^{3a+b+1} + 459^a2^{3a+b+1} + 2^{3a+1}459^a7^b + 520^a9^{a+b} \\
& + 585^a2^{3a+b} + 663^a5^b2^{3a+b} + 675^a2^{2a+2b+1} + 765^a2^{3a+b} \\
& + 975^a4^{a+b} + 1053^a2^{2a+3b} + 1125^a2^{2a+3b} + 1215^a4^{a+b} \\
& + 2500^a3^b + 2916^a5^b + 3600^a7^b + 3744^a5^b + 3900^a7^b \\
& + 4320^a7^b + 4624^a9^b + 6120^a11^b + 2^{5a+1}75^a + 2^{6a+1}85^a \\
& + 2^{4a+1}255^a + 2^{3a+1}405^a + 2^{2a+1}675^a + 2304^a + 5304^a. \\
SS_{(a,b)}(G_6) &= 2^{7a+1} + 2^{7a+b+1} + 2^{4a+b+1}3^{2a+b} + 2^{a+1}3^{4a+b} + 2^{3a+b}3^{a+b}7^a \\
& + 2^{3a+1}3^{a+2b}7^a + 2^{2a+1}5^{a+b}7^a + 2^{2a+b}5^{a+b}9^a + 2^{2a+1}3^{a+b}11^a \\
& + 2^{4a+2b+1}11^a + 2^{4a+1}11^a + 2^{a+2b}3^{2a+b}11^a + 2^{4a+b+1}5^b11^a \\
& + 2^{4a+1}9^b11^a + 2^{2a+b+1}3^{a+b}13^a + 2^{3a+b+1}19^a + 2^{3a+1}7^b19^a \\
& + 2^{3a+b+1}21^a + 2^{3a+b}3^b23^a + 2^{a+1}3^{a+b}23^a + 2^{3a+2b+1}3^b25^a \\
& + 2^{2a+b+1}33^a + 2^{2a+b+1}35^a + 2^{2a+1}3^b37^a + 2^{2a+b+1}3^b37^a \\
& + 2^{2a+1}5^b39^a + 2^{2a+3b+1}43^a + 2^{2a+1}5^b43^a + 3^{2a+b}20^a + 198^a \\
& + 5^{a+b}34^a + 2^{2a+b}5^b47^a + 7^{a+b}26^a + 2^{2a+b}47^a + 2^{2a+b}53^a \\
& + 2^{4a+1}9^a + 7^{a+b}24^a + 2^{3a+1}25^a + 3^{a+2b}58^a + 2^{a+2b}3^b101^a \\
& + 2^{a+2b+1}67^a + 2^{a+2b+1}69^a + 2^{a+2b+1}73^a + 2^{a+1}5^b73^a + 224^a \\
& + 2^{a+2b+1}79^a + 2^{a+3b+1}79^a + 2^{a+1}7^b79^a + 2^{a+3b+1}81^a + 126^a \\
& + 2^{a+b+1}3^b85^a + 2^{a+1}7^b85^a + 2^{a+2b}87^a + 2^{a+3b}91^a + 2^{a+1}67^a \\
& + 2^{a+3b}85^a + 2^{a+b+1}5^b91^a + 2^{a+b+1}95^a + 2^{a+1}11^b95^a + 7^b150^a \\
& + 2^{a+2b}97^a + 2^{a+b}107^a + 2^{a+b+1}7^b113^a + 3^b130^a + 5^b138^a \\
& + 2^{a+1}3^b91^a + 9^b166^a + 9^b184^a + 11^b186^a + 5^b188^a + 11^b188^a \\
& + 11^b194^a + 3^b202^a + 3(13^b212^a) + 2^{a+1}79^a. \\
SG_{(a,b)}(G_6) &= 2(39^a5^{3a+b}) + 63^a2^b5^{3a+b} + 1085^a4^b3^{2a+b} + 2(713^a3^{2a+b})
\end{aligned}
$$

$$
\begin{aligned}
&+ 2(161^a3^{3a+b}) + 2(175^a3^{3a+b}) + 217^a2^{b+1}3^{3a+b} + 273^a5^{2a+b}\\
&+ 245^a2^b3^{3a+b} + 321^a2^{b+1}5^{2a+b} + 575^a2^{b+1}3^{2a+b} + 135^a7^{2a+b}\\
&+ 749^a2^{b+1}3^{2a+b} + 2(775^a9^{a+b}) + 2(963^a7^{a+b}) + 153^a7^{2a+b}\\
&+ 2(1391^a5^{a+b}) + 1819^a2^{b+1}7^{a+b} + 2(2461^a3^{a+2b}) + 805^a9^{a+b}\\
&+ 875^a3^{2a+b} + 2(2675^a3^{a+b}) + 3317^a2^{2b+1}3^{a+b} + 4095^a2^{b+1}\\
&+ 4225^a3^b + 4347^a2^{b+1} + 4485^a2^{2b+1} + 4725^a2^{2b+1} + 4761^a5^b\\
&+ 4875^a2^{b+1} + 5229^a2^{2b+1} + 2(5395^a3^b) + 5625^a7^b + 12495^a\\
&+ 2(5229^a5^b) + 5395^a2^{b+1}3^b + 2(5727^a7^b) + 2(5859^a5^b)\\
&+ 6045^a2^{2b+1} + 2(6045^a7^b) + 6225^a2^{3b+1} + 6417^a2^{3b+1}\\
&+ 6889^a9^b + 6955^a2^{3b+1} + 6975^a2^{b+1} + 7245^a4^b + 7383^a2^{2b+1}\\
&+ 7497^a8^b + 7719^a2^{b+1}5^b + 7735^a6^b + 7735^a9^b + 8211^a5^b\\
&+ 8211^a10^b + 8649^a11^b + 8715^a2^b + 8715^a11^b + 8881^a2^{b+1}\\
&+ 2(8881^a11^b) + 8925^a4^b + 8925^a11^b + 9877^a3^b + 9877^a12^b\\
&+ 11067^a2^b + 11067^a13^b + 2(11235^a13^b) + 11449^a2^b + 3969^a\\
&+ 2(4095^a) + 2(4485^a) + 2(5175^a) + 2(6225^a) + 2(7719^a)\\
&+ 5727^a2^{b+1} + 9765^a + 6825^a8^b + 2(9951^a).
\end{aligned}
$$

$$
\begin{aligned}
SS_{(a,b)}(G_7) &= 3^{2a+b}4^{a+b}7^a + 2^{3a+b+1}5^a7^{a+b} + 2(5^a7^{2a+b}) + 2^{a+b+1}5^{a+b}23^a\\
&+ 2(3^{5a+b}) + 3^{5a+2b} + 5^{2a+b}11^a + 11^a12^{a+b} + 2^{a+b+1}3^{2a+b}11^a\\
&+ 2^{2a+b}5^{a+b}13^a + 2^{2a+b+1}3^{a+b}17^a + 2^{2a+b+1}3^{a+b}19^a + 323^a\\
&+ 2(3^{2a+b}23^a) + 2(11^{a+b}23^a) + 2(7^{a+b}29^a) + 2^{3a+b+1}3^b31^a\\
&+ 2^{3a+b+1}5^b31^a + 2^{3a+b+1}35^a + 2(5^{a+b}39^a) + 2^{a+2b}3^{a+b}43^a\\
&+ 6^{a+b}41^a + 2(5^{a+b}43^a) + 2^{2a+b}45^a + 2^{2a+b+1}47^a + 2(179^a)\\
&+ 2^{2a+2b+1}47^a + 2^{2a+b}3^b49^a + 2^{2a+2b+1}51^a + 2^{2a+b+1}53^a\\
&+ 2^{2a+3b+1}53^a + 2^{2a+3b+1}57^a + 2^{2a+b}5^b57^a + 2^{2a+2b+1}59^a\\
&+ 2^{2a+b+1}5^b59^a + 3^b5^{a+b}59^a + 2^{2a+3b+1}61^a + 2(3^{a+b}61^a)\\
&+ 2^{2a+2b+1}65^a + 2^{2a+3b}65^a + 2^{2a+2b+1}3^b65^a + 2(3^{a+b}65^a)\\
&+ 2^{2a+b}3^b67^a + 3^b4^{a+b}67^a + 2^{2a+b}7^b69^a + 4^{a+b}71^a + 3^{a+b}89^a\\
&+ 2^{2a+b}7^b71^a + 2(3^{a+2b}73^a) + 2^{2a+b+1}63^a + 2(3^{a+2b}77^a)\\
&+ 2^{a+b+1}91^a + 2^{a+2b}93^a + 2^{a+2b+1}97^a + 3^{a+b}5^b97^a + 5^b251^a\\
&+ 2^{a+3b}105^a + 2^{a+b+1}3^b107^a + 2^{a+2b+1}109^a + 2^{a+3b+1}109^a\\
&+ 2^{a+b+1}115^a + 2^{a+3b}121^a + 2^{a+b}5^b123^a + 2^{a+2b}3^b125^a\\
&+ 3(4^{a+2b}77^a) + 2^{a+2b}129^a + 2^{a+b}139^a + 2^{a+b}7^b139^a\\
&+ 2^{2a+b}77^a + 2(183^a) + 2(3^b187^a) + 2(191^a) + 2(5^b191^a)\\
&+ 2^{a+b+1}99^a + 2(5^b203^a) + 2(7^b207^a) + 2(7^b215^a) + 2(219^a)
\end{aligned}
$$

$$
\begin{aligned}
&+ 2(9^b223^a) + 2(7^b227^a) + 2(5^b231^a) + 2(239^a) + 2(11^b239^a)\\
&+ 7^b243^a + 2(11^b243^a) + 2(9^b245^a) + 11^b251^a + 2(3^b223^a)\\
&+ 2(5^b253^a) + 9^b259^a + 2(263^a) + 7^b263^a + 11^b263^a\\
&+ 2(13^b263^a) + 13^b267^a + 2(3^b269^a) + 2(13^b269^a) + 291^a\\
&+ 2(203^a) + 2(293^a) + 2(15^b293^a) + 3^b295^a + 13^b275^a\\
&+ 2^{a+b}155^a + 2(17^b325^a).
\end{aligned}
$$

$$
\begin{aligned}
SG_{(a,b)}(G_7) &= 2^{a+1}5^{4a+b}9^a + 2^{a+1}7^{3a+b}15^a + 3^{3a+b}4^{a+b}19^a + 3^b5^{4a+b}34^a\\
&+ 2^{b+1}5^{4a+b}21^a + 2^{2a+b+1}3^{3a+b}95^a + 2^{2a+b+1}3^{3a+b}115^a\\
&+ 10^{2a+b}153^a + 2^{a+1}3^{3a+b}155^a + 9^{2a+b}170^a + 2^{a+1}3^{3a+b}175^a\\
&+ 2^{a+1}5^{2a+b}189^a + 3^{2a+b}4^{a+b}437^a + 2^b3^{3a+b}527^a + 9^b15130^a\\
&+ 2^{a+1}3^{2a+b}589^a + 3^{3a+b}4^b595^a + 3^{3a+b}646^a + 2^{a+1}9^{a+b}665^a\\
&+ 2^{a+1}9^{a+b}713^a + 2^{a+1}15^{a+b}713^a + 5^{2a+b}714^a + 3^{3a+b}5^b782^a\\
&+ 2^{a+1}9^{a+b}775^a + 2^{a+1}17^{a+b}775^a + 2^{a+1}3^{2a+b}805^a + 26010^a\\
&+ 2^{b+1}3^{2a+b}1085^a + 2^{a+1}5^{a+b}1519^a + 2^{2a+b}2025^a + 2^b24025^a\\
&+ 2^{2a+b+1}2205^a + 2^{2a+2b+1}2205^a + 2^{2a+b}3^b2401^a + 5^b14994^a\\
&+ 2^{2a+2b+1}2565^a + 2^{2a+b+1}2793^a + 2^{2a+3b+1}2793^a + 3^b21250^a\\
&+ 2^{b+1}5^{a+b}2883^a + 2^{a+1}3^{a+b}2945^a + 2^{2a+3b+1}3105^a + 21114^a\\
&+ 2^{2a+b}5^b3249^a + 2^{2a+2b+1}3381^a + 2^{2a+b+1}5^b3381^a + 4^b8649^a\\
&+ 2^{2a+3b}3825^a + 2^{2a+b+1}3933^a + 2^{a+1}4005^a + 2^{2a+b}3^b4165^a\\
&+ 3^b4^{a+b}4165^a + 2^{a+1}4185^a + 2^{a+1}3^b4361^a + 2^{a+1}4557^a\\
&+ 2^{a+1}5^b4557^a + 2^{2a+b}7^b4761^a + 2^{b+1}3^{a+b}4805^a + 4^{a+b}4845^a\\
&+ 2^{2a+b}7^b4845^a + 2^{a+1}5^b5073^a + 2^{a+1}5145^a + 2^{a+1}7^b5301^a\\
&+ 2^{2b+1}3^{a+b}5425^a + 2^{a+1}7^b5625^a + 2^{2a+b}5865^a + 4^{a+2b}5865^a\\
&+ 2^{a+1}5985^a + 2^{a+1}3^b6125^a + 2^{a+1}9^b6125^a + 2^{a+1}7^b6141^a\\
&+ 2^{a+1}5^b6417^a + 2^{a+1}7^b6975^a + 2^{a+1}7125^a + 2^{a+1}11^b7125^a\\
&+ 2^{a+1}11^b7245^a + 2^{a+1}11^b7595^a + 2^{b+1}8277^a + 2^{a+1}8625^a\\
&+ 2^{a+1}13^b8625^a + 2^{a+1}13^b8835^a + 2^{2b+1}9345^a + 7^b15810^a\\
&+ 2^{b+1}9765^a + 2^{a+1}10695^a + 8^b11025^a + 2^{b+1}3^b11125^a\\
&+ 2^{2b+1}11625^a + 2^{3b+1}11625^a + 2^{b+1}13125^a + 8^b13617^a\\
&+ 7^b13770^a + 2^{3b+1}13795^a + 10^b14229^a + 2^{b+1}7^b19375^a\\
&+ 11^b14994^a + 12^b15625^a + 2^{4b+1}23715^a + 2^b19125^a\\
&+ 11^b15810^a + 4^b16065^a + 2^{2b+1}16275^a + 13^b17442^a\\
&+ 13^b17850^a + 14^b19125^a + 2^{b+1}19375^a.
\end{aligned}
$$

$$
SS_{(a,b)}(G_8) = 2^{2a+2b+1}3^{4a+b} + 2^{a+b+1}3^{3a+b}5^a + 2^{2a+1}3^{2a+b}7^a + 2^{5a+2b+1}9^a
$$

$$
\begin{aligned}
&+ 3^{a+b}4^{2a+b}7^a + 2^{a+b}5^{2a+b}7^a + 3^{2a+b}5^{a+b}8^a + 2^{5a+1}9^{a+b} \\
&+ 2^{5a+2b}3^b11^a + 2^{5a+1}5^b11^a + 2^{5a+b+1}7^b11^a + 2^{2a+1}7^a11^{a+b} \\
&+ 2^{3a+2b+1}3^{a+b}13^a + 2^{3a+1}3^{a+2b}13^a + 2^{2a+1}5^{a+b}13^a + 440^a \\
&+ 2^{4a+1}17^a + 2^{4a+2b+1}17^a + 2^{4a+3b+1}17^a + 2^{4a+1}7^b17^a \\
&+ 2^{a+1}3^{2a+b}19^a + 2^{a+b}3^{2a+b}19^a + 2^{a+1}7^{a+b}19^a + 2^{4a+1}23^a \\
&+ 2^{4a+4b+1}23^a + 2^{2a+1}3^{a+b}23^a + 14^{a+b}25^a + 2^{a+1}5^{a+b}27^a \\
&+ 2^{a+b+1}5^{a+b}29^a + 2^{3a+1}31^a + 2^{3a+2b+1}31^a + 2^{3a+3b+1}37^a \\
&+ 2^{3a+1}7^b37^a + 2^{3a+3b+1}39^a + 2^{a+1}3^{a+b}41^a + 2^{2a+1}5^b71^a \\
&+ 2^{3a+b+1}3^b43^a + 2^{a+b+1}3^{a+b}43^a + 2^{3a+1}13^b43^a + 2^{3a+2b}45^a \\
&+ 2^{a+b+1}3^{a+b}47^a + 2^{a+1}3^{a+2b}47^a + 2^{3a+2b}49^a + 2^{a+1}3^{a+b}49^a \\
&+ 2^{3a+b}53^a + 2^{a+b+1}3^{a+b}53^a + 2^{a+1}3^{a+b}5^b59^a + 2^{2a+2b+1}63^a \\
&+ 2^{2a+2b+1}65^a + 2^{a+b}3^{a+2b}67^a + 2^{2a+3b+1}69^a + 2^{2a+3b+1}71^a \\
&+ 2^{3a+1}39^a + 3^b5^{a+b}74^a + 5^{a+b}76^a + 10^{a+b}33^a + 2^{2a+2b+1}77^a \\
&+ 2^{2a+1}3^b79^a + 2^{2a+2b+1}3^b79^a + 2^{2a+b+1}9^b101^a + 2^{a+1}121^a \\
&+ 2^{2a+2b+1}83^a + 2^{2a+3b}83^a + 2^{2a+1}9^b83^a + 2^{a+1}5^b127^a \\
&+ 2^{2a+b+1}97^a + 2^{2a+1}13^b83^a + 4^{a+2b}93^a + 4^{a+2b}95^a + 7^{a+b}48^a \\
&+ 2^{2a+3b}89^a + 2^{2a+1}17^b97^a + 2^{2a+1}101^a + 2^{2a+1}5^b81^a \\
&+ 3^{a+2b}110^a + 2^{a+b+1}121^a + 2^{a+b+1}123^a + 3^{a+b}124^a + 5^b252^a \\
&+ 2^{a+b+1}127^a + 2^{a+1}3^b131^a + 2^{a+b+1}3^b131^a + 2^{a+b+1}133^a \\
&+ 2^{a+b+1}5^b147^a + 2^{a+b+1}3^b149^a + 2^{a+1}9^b149^a + 2^{a+1}129^a \\
&+ 2^{a+b+1}141^a + 2^{a+1}7^b141^a + 2^{a+1}5^b151^a + 2^{a+b+1}5^b151^a \\
&+ 2^{a+1}11^b151^a + 2^{a+b+1}5^b157^a + 2^{a+b+1}151^a + 2^{a+1}145^a \\
&+ 2^{a+1}7^b157^a + 2^{a+1}11^b159^a + 2^{a+b+1}163^a + 2^{a+1}13^b163^a \\
&+ 2^{a+3b+1}167^a + 2^{a+1}11^b167^a + 2^{a+1}169^a + 2^{a+2b+1}3^b169^a \\
&+ 2^{a+1}7^b169^a + 2^{a+b+1}7^b169^a + 2^{a+b+1}7^b171^a + 2^{a+b+1}177^a \\
&+ 2^{a+2b+1}181^a + 2^{a+b}7^b181^a + 2^{a+1}15^b181^a + 2^{a+2b+1}5^b221^a \\
&+ 2^{a+4b+1}187^a + 2^{a+1}3^b187^a + 2^{a+b}193^a + 2^{a+b}9^b203^a + 402^a \\
&+ 2^{2a+b+1}5^b83^a + 2^{a+b}3^b185^a + 2^{a+b}211^a + 3^b244^a + 13^b342^a \\
&+ 7^b264^a + 9^b280^a + 11^b300^a + 13^b324^a + 11^b332^a + 240^a \\
&+ 11^b350^a + 9^b352^a + 15^b352^a + 13^b356^a + 7^b362^a + 3^b406^a \\
&+ 17^b384^a + 17^b386^a + 17^b392^a + 3(19^b422^a) + 5^{2a+b}14^a.
\end{aligned}
$$

$$
\begin{aligned}
SG_{(a,b)}(G_8) &= 2^{4a+b+1}3^{5a+b}5^a + 2^{7a+1}3^{5a+b}5^b + 2^{7a+b+1}3^{3a+b}7^a + 48300^a \\
&+ 2^{4a+1}5^{3a+b}9^a + 2^{8a+2b+1}3^{2a+b}11^a + 2^{3a+1}3^{5a+b}11^a + 40320^a \\
&+ 2^{9a+1}5^a9^{a+b} + 2^{2a+1}7^a9^{3a+b} + 2^{6a+1}3^a11^{2a+b} + 2^{8a+1}159^a
\end{aligned}
$$

$$
\begin{aligned}
&+ 2^{3a+b+1}5^{3a+b}21^{a} + 2^{7a+1}3^{2a+b}25^{a} + 2^{2a+2b+1}3^{5a+b}25^{a} \\
&+ 3^{2a+b}5^{3a+b}28^{a} + 2^{10a+1}33^{a} + 2^{10a+4b+1}33^{a} + 2^{4a+1}3^{3a+b}35^{a} \\
&+ 2^{7a+b}9^{a+b}35^{a} + 2^{9a+3b+1}45^{a} + 2^{3a+1}7^{2a+b}45^{a} + 3^{5a+b}140^{a} \\
&+ 2^{8a+b+1}3^{a+2b}53^{a} + 2^{3a+1}3^{a+b}5^{2a+b}53^{a} + 2^{5a+1}3^{2a+b}55^{a} \\
&+ 2^{4a+b}5^{2a+b}63^{a} + 2^{4a+b}5^{2a+b}69^{a} + 2^{3a+b}7^{2a+b}75^{a} + 14400^{a} \\
&+ 2^{3a+b+1}3^{3a+b}77^{a} + 2^{8a+1}5^{b}99^{a} + 2^{8a+2b+1}105^{a} + 3^{b}40480^{a} \\
&+ 2^{8a+1}13^{b}105^{a} + 2^{4a+b+1}3^{2a+b}125^{a} + 2^{5a+1}5^{2a+b}21^{a} \\
&+ 2^{3a+1}3^{4a+b}53^{a} + 2^{5a+b+1}5^{a+b}159^{a} + 2^{7a+b+1}7^{b}225^{a} \\
&+ 2^{7a+3b+1}165^{a} + 2^{2a+1}3^{3a+b}175^{a} + 2^{7a+b+1}5^{b}183^{a} \\
&+ 2^{7a+1}7^{b}165^{a} + 2^{7a+1}7^{b}183^{a} + 2^{7a+1}11^{b}189^{a} + 2^{6a+1}3^{b}385^{a} \\
&+ 2^{7a+b+1}243^{a} + 2^{5a+1}3^{a+2b}265^{a} + 2^{4a+b+1}3^{a+b}583^{a} \\
&+ 2^{4a+b+1}7^{a+b}265^{a} + 2^{3a+1}9^{a+b}275^{a} + 5^{3a+b}276^{a} + 2^{7a+b}345^{a} \\
&+ 2^{6a+2b+1}363^{a} + 2^{3a+2b+1}3^{2a+b}371^{a} + 2^{4a+1}5^{a+b}371^{a} \\
&+ 2^{6a+2b+1}3^{b}385^{a} + 2^{3a+b}3^{2a+b}385^{a} + 2^{3a+1}7^{a+b}477^{a} \\
&+ 2^{5a+2b+1}525^{a} + 7^{2a+b}540^{a} + 2^{6a+b+1}583^{a} + 3^{3a+b}4^{a+b}245^{a} \\
&+ 2^{6a+1}17^{b}583^{a} + 2^{5a+b+1}3^{b}671^{a} + 2^{5a+1}9^{b}671^{a} + 2^{5a+1}5^{b}693^{a} \\
&+ 2^{5a+2b+1}495^{a} + 2^{5a+b+1}5^{b}693^{a} + 2^{5a+b+1}825^{a} + 19^{b}44160^{a} \\
&+ 2^{5a+1}891^{a} + 2^{5a+b+1}7^{b}891^{a} + 2^{4a+1}915^{a} + 2^{4a+b+1}915^{a} \\
&+ 2^{4a+b+1}945^{a} + 2^{4a+1}1155^{a} + 2^{5a+b}1155^{a} + 2^{4a+3b+1}1155^{a} \\
&+ 5^{2a+b}1176^{a} + 2^{4a+1}7^{b}1215^{a} + 2^{3a+2b+1}5^{a+b}1219^{a} \\
&+ 2^{5a+b}9^{b}1265^{a} + 3^{b}5^{2a+b}1288^{a} + 2^{4a+1}13^{b}1749^{a} + 3^{b}14884^{a} \\
&+ 2^{5a+1}13^{b}825^{a} + 2^{3a+1}5^{b}2013^{a} + 2^{3a+1}3^{b}2135^{a} + 5^{b}15876^{a} \\
&+ 2^{3a+b+1}3^{b}2135^{a} + 2^{3a+b+1}2205^{a} + 2^{3a+b+1}2475^{a} \\
&+ 2^{3a+1}2625^{a} + 2^{3a+b+1}5^{b}2673^{a} + 9^{a+b}2800^{a} + 2^{4a+b}2809^{a} \\
&+ 2^{3a+b+1}2835^{a} + 2^{3a+1}11^{b}2835^{a} + 2^{3a+3b+1}3233^{a} \\
&+ 2^{3a+1}11^{b}3233^{a} + 2^{3a+b}7^{b}3795^{a} + 2^{2a+1}3843^{a} \\
&+ 2^{3a+2b+1}3975^{a} + 2^{3a+b}3^{b}4025^{a} + 2^{3a+4b+1}4293^{a} \\
&+ 2^{2a+2b+1}4575^{a} + 2^{2a+1}7^{b}4575^{a} + 2^{2a+3b+1}4725^{a} \\
&+ 2^{2a+3b+1}4941^{a} + 2^{2a+1}5^{b}4941^{a} + 2^{2a+2b+1}5103^{a} \\
&+ 2^{3a+1}19^{b}5565^{a} + 2^{2a+1}6075^{a} + 2^{2a+3b}6405^{a} \\
&+ 2^{2a+3b}7245^{a} + 4^{a+b}7875^{a} + 4^{a+2b}8505^{a} + 4^{a+2b}8625^{a} \\
&+ 4^{a+b}9315^{a} + 7^{b}17424^{a} + 7^{b}30360^{a} + 3^{b}4^{a+b}7015^{a} \\
&+ 9^{b}19600^{a} + 11^{b}22500^{a} + 11^{b}25620^{a} + 13^{b}26244^{a} \\
&+ 11^{b}27600^{a} + 13^{b}27720^{a} + 9^{b}28060^{a} + 13^{b}28980^{a}
\end{aligned}
$$

$$+ 15^b 30976^a + 17^b 36864^a + 17^b 36960^a + 17^b 37260^a$$
$$+ 2^{3a+b+1} 2013^a + 2^{3a+1} 2079^a + 2^{2a+2b+1} 3843^a.$$

Table 2. Graphical indices of monocarboxylic acids.

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
S_1	320	2844	6094	8496	11544	20010	40344	73584
S_2	1692	44716	140100	230487	364836	831618	2373889	5846796
SN	69.13	356.8	632.59	811.97	1022.2	1545.4	2618.92	4111.4
RPS	157.9	1405	3011	4197.9	5703.9	9886.8	19912	36354.3
SS	712	10424	26716	40290	58872	116320	276926	584424
SG	3932	170624	639207	1137474	1936228	5030965	16930742	48335916

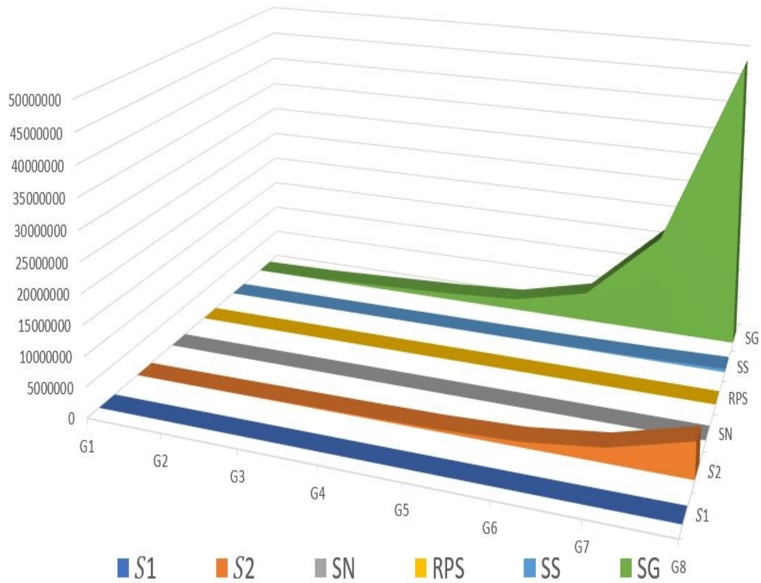

Figure 2. Plotting of graphical indices of monocarboxylic acids.

6. Conclusion and Open problems

In this article, we have generalized the concepts related to status distance which has lead to the easy calculation of existing status related indices and the birth of new indices like F-status distance, sum connectivity status distance, product connectivity status distance, etc., which can be further studied. We have obtained some exact values along with the inequalities of the generalized status indices. Also we have computed few indices for some of the monocarboxylic acids and have plotted those for comparison.We came to the conclusion that the increase in the number of vertices in a linear molecular chain increases in status related graphical index value. Also $SN(G) \leq S_1(G) \leq RPS(G) \leq SS(G) \leq S_2(G) \leq SG(G)$.

Author Contributions

Both the authors contributed equally to the writing of this paper. Both authors read and approved the final manuscript.

Conflict of interest

The authors declare no conflict of interest.

Acknowledgement

The authors would like to thank anonymous referees for their valuable suggestions to improve the paper.

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