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TRANSMISSION AND RECIPROCAL TRANSMISSION TOPOLOGICAL INDICES AND CO-INDICES OF INDU-BALA PRODUCTS OF GRAPHS

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Abstract. The transmission of a vertex u in a connected graph G is denoted by $\sigma(u)$ and defined by $\sigma(u) = \sum_{v \in V(G)} d(u,v)$; that is, the sum of the distances between u and all other vertices of a graph G. The reciprocal transmission of a vertex u in a connected graph G is denoted by $rs(u)$ and defined by $rs(u) = \sum_{v \in V(G)} \frac{1}{d(u,v)}$; that is, the sum of the reciprocal of distances between u and all other vertices of a graph G. In this paper we obtain explicit formulae for various transmission and reciprocal transmission based topological indices and co-indices of Indu-Bala product of graphs.

1. Introduction

In the literature several degree based topological indices have been introduced and studied, [3]. Probably, the most intensively studied topological indices based on the vertex degrees are Zagreb indices. [5, 9]. The first and second Zagreb indices of a graph G are defined by

$$M_1(G) = \sum_{uv \in E(G)} [d(u) + d(v)] \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} d(u)d(v),$$

see [6].

The transmission (or status) of a vertex $u \in V(G)$, [7, 15], denoted by $\sigma(u)$ is defined by

$$\sigma(u) = \sum_{v \in V(G)} d(u,v).$$

The reciprocal transmission (or reciprocal status) of a vertex $u \in V(G)$, [13], denoted by $rs(u)$ is defined by

$$rs(u) = \sum_{v \in V(G)} \frac{1}{d(u,v)}.$$

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The oldest transmission based topological index is the Wiener index, [17], defined by

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v) = \frac{1}{2} \sum_{u \in V(G)} \sigma(u).$$

The Wiener index is also called as gross status, [7].

The first transmission (status) connectivity index $S_1(G)$ and second transmission (status) connectivity index $S_2(G)$ are respectively defined by

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] \quad \text{and} \quad S_2(G) = \sum_{uv \in E(G)} \sigma(u)\sigma(v),$$

see [15].

The harmonic reciprocal status index of a connected graph G, [10], is defined by

$$HRS(G) = \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)}.$$

The harmonic reciprocal status co-index of a graph G, [10], is defined by

$$\overline{HRS}(G) = \sum_{uv \notin E(G)} \frac{2}{rs(u) + rs(v)}.$$

It is worth to note that the first transmission connectivity index $S_1(G)$ coincides with the degree distance $DD(G)$ of a graph, [2, 4], which was defined by

$$DD(G) = \sum_{u \in V(G)} d(u)\sigma(u) = \sum_{\{u,v\} \subseteq V(G)} [d(u) + d(v)]d(u,v).$$

The transmission sum-connectivity index of a graph G, [16], is defined by

$$T_{SC}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u) + \sigma(v)}}.$$

The transmission geometric-arithmetic index of a graph G, [8], denoted by $TGA(G)$ is defined by

$$T_{GA}(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\sigma(u)\sigma(v)}}{\sigma(u) + \sigma(v)}.$$

The transmission arithmetic-geometric index of a graph G, [12], denoted by $T_{AG}(G)$ is defined by

$$T_{AG}(G) = \sum_{uv \in E(G)} \frac{\sigma(u) + \sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}.$$

The transmission atom-bond connectivity index of a graph G, [12], denoted by $T_{ABC}(G)$, is defined by

$$T_{ABC}(G) = \sum_{uv \in E(G)} \sqrt{\frac{\sigma(u) + \sigma(v) - 2}{\sigma(u)\sigma(v)}}.$$

The transmission augmented Zagreb index of a graph G, [12], denoted by $T_{AZ}(G)$ is defined by

$$T_{AZ}(G) = \sum_{uv \in E(G)} \left[\frac{\sigma(u)\sigma(v)}{\sigma(u) + \sigma(v) - 2}\right]^3.$$

In [13], the first reciprocal transmission (reciprocal status) connectivity index $RS_1(G)$ and second reciprocal transmission (reciprocal status) connectivity index $RS_2(G)$ are defined by

$$RS_1(G) = \sum_{uv \in E(G)} [rs(u) + rs(v)] \quad \text{and} \quad RS_2(G) = \sum_{uv \in E(G)} rs(u)rs(v).$$

The reciprocal transmission arithmetic-geometric index of a graph G is denoted by $RT_{AG}(G)$ and defined by

$$RT_{AG}(G) = \sum_{uv \in E(G)} \frac{rs(u) + rs(v)}{2\sqrt{rs(u)rs(v)}}, \tag{1.1}$$

in [11].

The reciprocal transmission geometric-arithmetic index of a graph G, [11], is denoted by $RT_{GA}(G)$ and defined by

$$RT_{GA}(G) = \sum_{uv \in E(G)} \frac{2\sqrt{rs(u)rs(v)}}{rs(u) + rs(v)}. \tag{1.2}$$

The reciprocal transmission sum-connectivity index of a graph G, [11], is denoted by $RT_{SC}(G)$ and defined by

$$RT_{SC}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{rs(u) + rs(v)}}. \tag{1.3}$$

The reciprocal transmission atom-bond connectivity index of a graph G, [11], is denoted by $RT_{ABC}(G)$ and defined by

$$RT_{ABC}(G) = \sum_{uv \in E(G)} \sqrt{\frac{rs(u) + rs(v) - 2}{rs(u)rs(v)}}. \tag{1.4}$$

The reciprocal transmission augmented Zagreb index of a graph G, [11], is denoted by $RT_{AZ}(G)$ and defined by

$$RT_{AZ}(G) = \sum_{uv \in E(G)} \left[\frac{rs(u)rs(v)}{rs(u) + rs(v) - 2} \right]^3. \tag{1.5}$$

The transmission sum-connectivity co-index of a graph G, [14], denoted by $\overline{T}_{SC}(G)$, is defined by

$$\overline{T}_{SC}(G) = \sum_{uv \notin E(G)} \frac{1}{\sqrt{\sigma(u) + \sigma(v)}}.$$

The transmission geometric-arithmetic co-index of a graph G, [14], denoted by $\overline{T}_{GA}(G)$, is defined by

$$\overline{T}_{GA}(G) = \sum_{uv \notin E(G)} \frac{2\sqrt{\sigma(u)\sigma(v)}}{\sigma(u) + \sigma(v)}.$$

The transmission arithmetic-geometric co-index of a graph G, denoted by $\overline{T}_{AG}(G)$, is defined as [14]

$$\overline{T}_{AG}(G) = \sum_{uv \notin E(G)} \frac{\sigma(u) + \sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}.$$

In [14], the transmission atom-bond connectivity co-index of a graph G denoted by $\overline{T}_{ABC}(G)$ is defined by

$$\overline{T}_{ABC}(G) = \sum_{uv \notin E(G)} \sqrt{\frac{\sigma(u) + \sigma(v) - 2}{\sigma(u)\sigma(v)}}.$$

The transmission augmented Zagreb co-index of a graph G, [14], denoted by $\overline{T}_{AZ}(G)$, is defined by

$$\overline{T}_{AZ}(G) = \sum_{uv \notin E(G)} \left[\frac{\sigma(u)\sigma(v)}{\sigma(u) + \sigma(v) - 2} \right]^3.$$

In [14], the reciprocal transmission arithmetic-geometric co-index of a graph G is denoted by $\overline{RT}_{AG}(G)$ and defined by

$$\overline{RT}_{AG}(G) = \sum_{uv \notin E(G)} \frac{rs(u) + rs(v)}{2\sqrt{rs(u)rs(v)}}. \tag{1.6}$$

The reciprocal transmission geometric-arithmetic co-index of a graph G is denoted by $\overline{RT}_{GA}(G)$ and defined by

$$\overline{RT}_{GA}(G) = \sum_{uv \notin E(G)} \frac{2\sqrt{rs(u)rs(v)}}{rs(u) + rs(v)}, \tag{1.7}$$

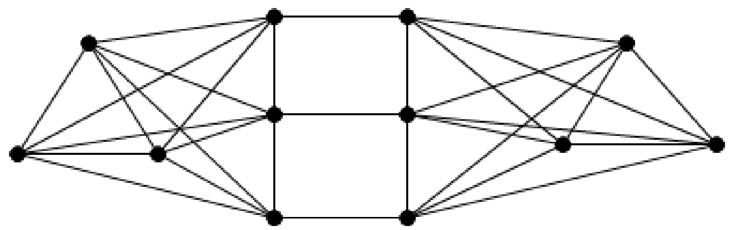

FIGURE 1. The Indu-Bala product graph $K_3 \blacktriangledown P3$.

see [14].

The reciprocal transmission sum-connectivity co-index of a graph G is denoted by $\overline{RT}_{SC}(G)$ and it is defined by

$$\overline{RT}_{SC}(G) = \sum_{uv \notin E(G)} \frac{1}{\sqrt{rs(u) + rs(v)}}. \tag{1.8}$$

The reciprocal transmission atom-bond connectivity co-index of a graph G, [14], is denoted by $\overline{RT}_{ABC}(G)$ and it is defined by

$$\overline{RT}_{ABC}(G) = \sum_{uv \notin E(G)} \sqrt{\frac{rs(u) + rs(v) - 2}{rs(u)rs(v)}}. \tag{1.9}$$

The reciprocal transmission augmented Zagreb co-index of a graph G, [14], is denoted by $\overline{RT}_{AZ}(G)$ and it is defined by

$$\overline{RT}_{AZ}(G) = \sum_{uv \notin E(G)} \left[\frac{rs(u)rs(v)}{rs(u) + rs(v) - 2}\right]^3. \tag{1.10}$$

2. Transmission based topological indices of Indu-Bala graph

Let G_1 and G_2 be two regular graphs. Let $G_1 \vee G_2$ denotes the join of G_1 and G_2. The Indu–Bala product of G_1 and G_2 is denoted by $G_1 \blacktriangledown G_2$ and is obtained from two disjoint copies of the join $G_1 \vee G_2$ of G_1 and G_2 by joining the corresponding vertices in the two copies of G_2. Fig. 1 displays $K_3 \blacktriangledown P_3$. Note that $K_{n,n+1}$ is just $\overline{K_n} \blacktriangledown \overline{K_{n+1}}$ where the bar symbol denotes complement.

Theorem 2.1. *Let G be the Indu-Bala product graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2, respectively denote the numbers of vertices and edges of G_1 and G_2. Then*

$$S_1(G) = 2m_1 \left[2(5n_1 + 3n_2 - 2 - d_{G_1}(u))\right] + (2m_2 + n_2) \left[2(3n_1 + 4n_2 - 3 - d_{G_2}(v))\right]$$

$$+2n_1n_2\left[8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)\right].$$

Proof. Let G be a simple graph with n vertices and m edges. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of a graph G. We partition the edge set $E(G)$ of a graph G into three subsets as follows:

$E_1(G) = \{uv \in E(G) | u, v \in V(G_1)\}$,

$E_2(G) = \{uv \in E(G) | u, v \in V(G_2)\}$,

$E_3(G) = \{uv \in E(G) | u \in V(G_1) \text{ and } v \in V(G_2)\}$.

We have the following possibilities:

Case(i): If $u \in V(G_1)$, then

$$\begin{aligned}\sigma(u) &= d_{G_1}(u)+2(n_1-1-d_{G_1}(u))+n_2+2n_2+3n_1\\ &= 5n_1+3n_2-d_{G_1}(u)-2.\end{aligned}$$

Case(ii): If $v \in V(G_2)$,

$$\begin{aligned}\sigma(v) &= n_1+d_{G_2}(v)+2(n_2-1-d_{G_2}(v))+1+2(n_2-1)+2n_1\\ &= 3n_1+4n_2-3-d_{G_2}(v).\end{aligned}$$

Therefore

$$\begin{aligned}S_1(G) &= \sum_{uv\in E(G)}(\sigma(u)+\sigma(v))\\ &= \sum_{uv\in E_1(G)}(\sigma(u)+\sigma(v))+\sum_{uv\in E_2(G)}(\sigma(u)+\sigma(v))+\sum_{uv\in E_3(G)}(\sigma(u)+\sigma(v))\\ &= \sum_{uv\in E_1(G)}\left[2(5n_1+3n_2-2-d_{G_1}(u))\right]\\ &\quad+\sum_{uv\in E_2(G)}\left[2(3n_1+4n_2-3-d_{G_2}(v))\right]\\ &\quad+\sum_{uv\in E_3(G)}\left[8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)\right]\end{aligned}$$

giving the result. □

The following results can be proven similarly and therefore the proofs are omitted:

Theorem 2.2. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$S_2(G) = 2m_1\left(5n_1+3n_2-2-d_{G_1}(u)\right)^2+(2m_2+n_2)(3n_1+4n_2-3-d_{G_2}(u))^2$$

$$+2n_1n_2(15n_1^2+12n_2^2+29n_1n_2-21n_1-d_{G_2}(v))(5n_1+3n_2-2)-17n_2$$
$$-d_{G_1}(u)(3n_1+4n_2-3)+6+d_{G_1}(u)d_{G_2}(u).$$

Theorem 2.3. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} HS(G) &= 2m_1\left(\frac{2}{2(5n_1+3n_2-2-d_{G_1}(u))}\right)+(2m_2+n_2)\left(\frac{2}{2(3n_1+4n_2-3-d_{G_2}(v))}\right)\\ &\quad+2n_1n_2\left(\frac{2}{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)}\right). \end{aligned}$$

Theorem 2.4. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} T_{SC} &= 2m_1\left(\frac{1}{\sqrt{2(5n_1+3n_2-2-d_{G_1}(u))}}\right)+(2m_2+n_2)\left(\frac{1}{\sqrt{2(3n_1+4n_2-3-d_{G_2}(v))}}\right)\\ &\quad+2n_1n_2\left(\frac{1}{\sqrt{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)}}\right) \end{aligned}$$

Theorem 2.5. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} T_{ABC}(G) &= 2m_1\sqrt{\frac{2(5n_1+3n_2-2-d_{G_1}(u))-2}{(5n_1+3n_2-2-d_{G_1}(u))^2}}\\ &\quad+(2m_2+n_2)\sqrt{\frac{2(3n_1+4n_2-3-d_{G_2}(v))-2}{(3n_1+4n_2-3-d_{G_2}(v))^2}}\\ &\quad+2n_1n_2\sqrt{\frac{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)-2}{A(5n_1+3n_2-2)-17n_2-d_{G_1}(u)(3n_1+4n_2-3)+6+d_{G_1}(u)d_{G_2}(u)}} \end{aligned}$$

where $A=15n_1^2+12n_2^2+29n_1n_2-21n_1-d_{G_2}(u)$.

Theorem 2.6. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} T_{AZ} &= 2m_1\left(\frac{(5n_1+3n_2-2-d_{G_1}(u))^2}{2(5n_1+3n_2-2-d_{G_1}(u))-2}\right)^3\\ &\quad+(2m_2+n_2)\left(\frac{(3n_1+4n_2-3-d_{G_2}(v))^2}{2(3n_1+4n_2-3-d_{G_2}(v))-2}\right)^3\\ &\quad+2n_1n_2\left(\frac{(5n_1+3n_2-2-d_{G_1}(u))(3n_1+4n_2-3-d_{G_2}(v))}{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)-2}\right)^3. \end{aligned}$$

Theorem 2.7. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$T_{AG} = 2m_1 + 2m_2 + n_2 + n_1 n_2 \left(\frac{8n_1 + 7n_2 - 5 - d_{G_1}(u) - d_{G_2}(v)}{\sqrt{(5n_1 + 3n_2 - 2 - d_{G_1}(u))(3n_1 + 4n_2 - 3 - d_{G_2}(v))}} \right).$$

Theorem 2.8. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$T_{GA} = 2m_1 + 2m_2 + n_2 + 4n_1 n_2 \left(\frac{\sqrt{(5n_1 + 3n_2 - 2 - d_{G_1}(u))(3n_1 + 4n_2 - 3 - d_{G_2}(v))}}{8n_1 + 7n_2 - 5 - d_{G_1}(u) - d_{G_2}(v)} \right).$$

3. Transmission based topological co-indices of Indu-Bala graph

Theorem 3.1. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} \overline{S_1}(G) &= 2\left(n_1^2 + 2\left(\tbinom{n_1}{2} - m_1\right)\right)(5n_1 + 3n_2 - 2 - d_{G_1}(u)) \\ &+4\left(\tbinom{n_2}{2} - m_2\right)(3n_1 + 4n_2 - 3 - d_{G_2}(v)) \\ &+n_1 n_2 (8n_1 + 7n_2 - d_{G_1}(u) - d_{G_2}(v) - 7) \end{aligned}$$

Proof. Let us partition the edge set $E(G)$ of G into three sets as follows:

$E_1(G) = \{uv \in E(G) | u, v \in V(G_1)\}$,

$E_2(G) = \{uv \in E(G) | u, v \in V(G_2)\}$,

$E_2(G) = \{uv \in E(G) | u \in V(G_1) \text{and} v \in V(G_2)\}$.

We have the following possibilities:

Case(i): If $u \in V(G_1)$, then

$$\begin{aligned} \sigma(u) &= d_{G_1}(u) + 2(n_1 - 1 - d_{G_1}(u)) + n_2 + 2n_2 + 3n_1 \\ &= 5n_1 + 3n_2 - d_{G_1}(u) - 2. \end{aligned}$$

Case(ii):If $v \in V(G_2)$,

$$\begin{aligned} \sigma(v) &= n_1 + d_{G_2}(v) + 2(n_2 - 1 - d_{G_2}(v)) + 1 + 2(n_2 - 1) + 2n_1 \\ &= 3n_1 + 4n_2 - 3 - d_{G_2}(v). \end{aligned}$$

Therefore

$$\begin{aligned} \overline{S_1}(G) &= \sum_{uv \notin E(G)} (\sigma(u) + \sigma(v)) \\ &= \sum_{uv \notin E_1(G)} (\sigma(u) + \sigma(v)) + \sum_{uv \notin E_2(G)} (\sigma(u) + \sigma(v)) + \sum_{uv \notin E_3(G)} (\sigma(u) + \sigma(v)) \end{aligned}$$

$$= \sum_{uv\notin E_1(G)} 2(5n_1+3n_2-2-d_{G_1}(u)) + \sum_{uv\notin E_2(G)} 2(3n_1+4n_2-3-d_{G_2}(v)) + \sum_{uv\notin E_3(G)} (8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)-7)$$

giving the proof. □

The following results are similarly deduced:

Theorem 3.2. *Let G be the Indu-Bala product graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 denoting the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{S_2}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right)(5n_1+3n_2-d_{G_1}(u)-2)^2 \\ &\quad +2\left(\binom{n_2}{2} - m_2\right)(3n_1+4n_2-d_{G_2}(v)-3)^2 \\ &\quad +n_1n_2(5n_1+3n_2-d_{G_1}(u)-2)(3n_1+4n_2-d_{G_2}(v)-3)\end{aligned}$$

Theorem 3.3. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{HS}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right)\left(\frac{1}{5n_1+3n_2-2-d_{G_1}(u)}\right) \\ &\quad +2\left(\binom{n_2}{2} - m_2\right)\left(\frac{1}{3n_1+4n_2-3-d_{G_2}(v)}\right) \\ &\quad +2n_1n_2\left(\frac{1}{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)}\right)\end{aligned}$$

Theorem 3.4. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{T}_{GA}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right) + 2\left(\binom{n_2}{2} - m_2\right) \\ &\quad +n_1n_2\left(\frac{2\sqrt{(5n_1+3n_2-2-d_{G_1}(u))(3n_1+4n_2-3-d_{G_2}(v))}}{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)}\right).\end{aligned}$$

Theorem 3.5. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{T}_{AG}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right) + 2\left(\binom{n_2}{2} - m_2\right) \\ &\quad +n_1n_2\left(\frac{8n_1+7n_2-5-d_{G_1}(u)-d_{G_2}(v)}{2\sqrt{(5n_1+3n_2-2-d_{G_1}(u))(3n_1+4n_2-3-d_{G_2}(v))}}\right)\end{aligned}$$

Theorem 3.6. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{T}_{SC}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right)\frac{1}{\sqrt{2(5n_1 + 3n_2 - 2 - d_{G_1}(u))}}\\ &+2\left(\binom{n_2}{2} - m_2\right)\frac{1}{\sqrt{2(3n_1 + 4n_2 - 3 - d_{G_2}(v))}}\\ &+n_1n_2\frac{1}{\sqrt{8n_1 + 7n_2 - 5 - d_{G_1}(u) - d_{G_2}(v)}}.\end{aligned}$$

Theorem 3.7. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{T}_{ABC}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right)\sqrt{\frac{2(5n_1 + 3n_2 - 2 - d_{G_1}(u)) - 2}{(5n_1 + 3n_2 - 2 - d_{G_1}(u))^2}}\\ &+2\left(\binom{n_2}{2} - m_2\right)\sqrt{\frac{2(3n_1 + 4n_2 - 3 - d_{G_2}(v)) - 2}{(3n_1 + 4n_2 - 3 - d_{G_2}(v))^2}}\\ &+n_1n_2\sqrt{\frac{8n_1 + 7n_2 - d_{G_1}(u) - d_{G_2}(v) - 7}{(5n_1 + 3n_2 - 2 - d_{G_1}(u))(3n_1 + 4n_2 - 3 - d_{G_2}(v))}}.\end{aligned}$$

Theorem 3.8. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}\overline{T}_{AZ}(G) &= \left(n_1^2 + 2\left(\binom{n_1}{2} - m_1\right)\right)\left(\frac{(5n_1 + 3n_2 - 2 - d_{G_1}(u))^2}{2(5n_1 + 3n_2 - 2 - d_{G_1}(u))) - 2}\right)^3\\ &+2\left(\binom{n_2}{2} - m_2\right)\left(\frac{(3n_1 + 4n_2 - 3 - d_{G_2}(v))^2)}{2(3n_1 + 4n_2 - 3 - d_{G_2}(v)) - 2}\right)^3\\ &+\left(\frac{(3n_1 + 4n_2 - 3 - d_{G_2}(v))(5n_1 + 3n_2 - 2 - d_{G_1}(u))}{8n_1 + 7n_2 - d_{G_1}(u) - d_{G_2}(v) - 7}\right).\end{aligned}$$

4. Reciprocal transmission based topological indices of Indu-Bala graphs

Theorem 4.1. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned}RS_1(G) &= 2m_1\left(2\left(\frac{1}{2}d_{G_1}(u) + \frac{5}{6}n_1 + \frac{3}{2}n_2 - \frac{1}{2}\right)\right)\\ &+(2m_2 + n_2)\left(2\left(\frac{3}{2}n_1 + \frac{5}{6}n_2 + \frac{2}{3}d_{G_2}(v) + \frac{1}{6}\right)\right)\\ &+2n_1n_2\left(\frac{7}{3}(n_1 + n_2) + \frac{1}{2}d_{G_1}(u) + \frac{2}{3}d_{G_2}(v) - \frac{2}{3}\right).\end{aligned}$$

Proof. Let us partition the edge set $E(G)$ of G into three sets as follows:

$E_1(G) = \{uv \in E(G) | u, v \in V(G_1)\},$

$E_2(G) = \{uv \in E(G) | u, v \in V(G_2)\},$

$E_2(G) = \{uv \in E(G) | u \in V(G_1) \text{and} v \in V(G_2)\}.$

Case(i): If $u \in V(G_1)$, then

$$rs(u) = \frac{5}{6}n_1 + \frac{3}{2}n_2 + \frac{1}{2}d_{G_1}(u) - \frac{1}{2}.$$

Case(ii): If $v \in V(G_2)$,

$$rs(v) = \frac{3}{2}n_1 + \frac{5}{6}n_2 + \frac{2}{3}d_{G_2}(v) + \frac{1}{6}.$$

Hence, we get

$$\begin{aligned} RS_1(G) &= \sum_{uv \in E(G)} (rs(u) + rs(v)) \\ &= \sum_{uv \in E_1(G)} (rs(u) + rs(v)) + \sum_{uv \in E_2(G)} (rs(u) + rs(v)) + \sum_{uv \in E_3(G)} (rs(u) + rs(v)) \\ &= \sum_{uv \in E_1(G)} \left(2 \left(\frac{1}{2}d_{G_1}(u) + \frac{5}{6}n_1 + \frac{3}{2}n_2 - \frac{1}{2} \right) \right) \\ &\quad + \sum_{uv \in E_2(G)} \left(2 \left(\frac{3}{2}n_1 + \frac{5}{6}n_2 + \frac{2}{3}d_{G_2}(v) + \frac{1}{6} \right) \right) \\ &\quad + \sum_{uv \in E_3(G)} \left(\frac{7}{3}n_1 + \frac{7}{3}n_2 + \frac{1}{2}d_{G_1}(u) + \frac{2}{3}d_{G_2}(v) - \frac{2}{3} \right) \end{aligned}$$

giving the required result. □

The following are similar results which can be proven similarly:

Theorem 4.2. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} RS_2(G) &= 2m_1 \left(\frac{1}{2}d_{G_1}(u) + \frac{5}{6}n_1 + \frac{3}{2}n_2 - \frac{1}{2} \right)^2 \\ &\quad + (2m_2 + n_2) \left(\frac{3}{2}n_1 + \frac{5}{6}n_2 + \frac{2}{3}d_{G_2}(v) + \frac{1}{6} \right)^2 \\ &\quad + 2n_1n_2 \left(\frac{1}{2}d_{G_1}(u) + \frac{5}{6}n_1 + \frac{3}{2}n_2 - \frac{1}{2} \right) \left(\frac{3}{2}n_1 + \frac{5}{6}n_2 + \frac{2}{3}d_{G_2}(v) + \frac{1}{6} \right). \end{aligned}$$

Theorem 4.3. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} HRS(G) &= 2m_1\left(\frac{1}{\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}}\right) \\ &\quad +(2m_2+n_2)\left(\frac{1}{\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}}\right) \\ &\quad +4n_1n_2\left(\frac{1}{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}\right). \end{aligned}$$

Theorem 4.4. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} RT_{SC}(G) &= 2m_1\frac{1}{\sqrt{2\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)}} \\ &\quad +(2m_2+n_2)\frac{1}{\sqrt{2\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}} \\ &\quad +2n_1n_2\frac{1}{\sqrt{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}}. \end{aligned}$$

Theorem 4.5. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} RT_{ABC}(G) &= 2m_1\sqrt{\frac{2\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)-2}{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)^2}} \\ &\quad +(2m_2+n_2)\sqrt{\frac{2\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)-2}{\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)^2}} \\ &\quad +2n_1n_2\sqrt{\frac{\left(\sqrt{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}\right)-2}{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}. \end{aligned}$$

Theorem 4.6. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} RT_{AZ}(G) &= 2m_1\left(\frac{\left(\frac{1}{2}+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)^2}{2\left(\frac{1}{2}+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)-2}\right)^3 \\ &\quad +(2m_2+n_2)\left(\frac{\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)^2}{2\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)-2}\right)^3 \\ &\quad +2n_1n_2\left(\frac{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)\right)}{\left(\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}\right)-2}\right)^3 \end{aligned}$$

Theorem 4.7. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} RT_{AG} &= 2m_1 + 2m_2 + n_2 \\ &+2n_1n_2\left(\frac{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}{2\sqrt{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}\right). \end{aligned}$$

Theorem 4.8. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} RT_{GA} &= 2m_1 + 2m_2 + n_2 \\ &+2n_1n_2\left(\frac{2\sqrt{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}\right). \end{aligned}$$

5. Reciprocal transmission based topological co-indices of Indu-Bala graphs

Now we calculate the reciprocal transmission based topological co-indices of Indu-Bala graphs. The proofs are similar to the ones in previous sections.

Theorem 5.1. *Let G be the Indu-Bala graph of two graphs G_1 and G_2 with n_1, n_2 and m_1, m_2 are the numbers of vertices and edges of G_1 and G_2, respectively. Then*

$$\begin{aligned} \overline{RS_1}(G) &= 2\left(n_2^2+2\left(\binom{n_1}{2}-m_1\right)\right)\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right) \\ &+4\left(\binom{n_2}{2}-m_2\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right) \\ &+n_1n_2\left(\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}\right). \end{aligned}$$

$$\begin{aligned} \overline{RS_2}(G) &= \left(n_2^2+2\left(\binom{n_1}{2}-m_1\right)\right)\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)^2 \\ &+2\left(\binom{n_2}{2}-m_2\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)^2 \\ &+n_1n_2\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right). \end{aligned}$$

$$\begin{aligned} \overline{HRS}(G) &= \left(n_2^2+2\left(\binom{n_1}{2}-m_1\right)\right)\left(\frac{1}{\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}}\right) \\ &+2\left(\binom{n_2}{2}-m_2\right)\left(\frac{1}{\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}}\right) \\ &+n_1n_2\left(\frac{2}{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}\right) \end{aligned}$$

$$\begin{aligned}\overline{RT}_{SC}(G) &= \left(n_2^2+2\left(\binom{n_1}{2}-m_1\right)\right)\left(\frac{1}{\sqrt{2\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)}}\right)\\ &\quad +2\left(\binom{n_2}{2}-m_2\right)\left(\frac{1}{\sqrt{2\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}\right)\\ &\quad +n_1n_2\left(\frac{1}{\sqrt{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}}\right).\end{aligned}$$

$$\begin{aligned}\overline{RT}_{ABC}(G) &= \left(n_1^2+2\left(\binom{n_1}{2}-m_1\right)\right)\sqrt{\frac{2\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)-2}{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)^2}}\\ &\quad +2\left(\binom{n_2}{2}-m_2\right)\sqrt{\frac{2\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)-2}{\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)^2}}\\ &\quad +n_1n_2\sqrt{\frac{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}-2}{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}\end{aligned}$$

$$\begin{aligned}\overline{RT}_{AZ}(G) &= \left(n_1^2+2\left(\binom{n_1}{2}-m_1\right)\right)\left(\frac{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)^2}{2\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)-2}\right)^3\\ &\quad +2\left(\binom{n_2}{2}-m_2\right)\left(\frac{\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)^2}{2\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)-2}\right)^3\\ &\quad +n_1n_2\left(\frac{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}-2}\right)^3.\end{aligned}$$

$$\begin{aligned}\overline{RT}_{GA}(G) &= \left(n_1^2+2\left(\binom{n_1}{2}-m_1\right)\right)+2\left(\binom{n_2}{2}-m_2\right)\\ &\quad +n_1n_2\left(\frac{2\sqrt{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}\right).\end{aligned}$$

$$\begin{aligned}\overline{RT}_{AG}(G) &= \left(n_1^2+2\left(\binom{n_1}{2}-m_1\right)\right)+2\left(\binom{n_2}{2}-m_2\right)\\ &\quad +n_1n_2\left(\frac{\frac{7}{3}(n_1+n_2)+\frac{1}{2}d_{G_1}(u)+\frac{2}{3}d_{G_2}(v)-\frac{2}{3}}{2\sqrt{\left(\frac{1}{2}d_{G_1}(u)+\frac{5}{6}n_1+\frac{3}{2}n_2-\frac{1}{2}\right)\left(\frac{3}{2}n_1+\frac{5}{6}n_2+\frac{2}{3}d_{G_2}(v)+\frac{1}{6}\right)}}\right)\end{aligned}$$

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