Proceedings of the Jangjeon Mathematical Society
26 (2023), No. 3, pp. 269 – 290

www.jangjeon.or.kr
http://dx.doi.org/10.17777/pjms2023.26.3.269

A FUZZY EOQ MODEL FOR DETERIORATING ITEMS HAVING TIME DEPENDENT DEMAND UNDER PARTIALLY BACKLOGGED SHORTAGES

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Abstract. The EOQ inventory models have a wide range of applications in real-world business. In particular, these models assist the retailers who deal with deteriorating items to determine optimal ordering quantity minimizing the total inventory cost. Moreover, due to the lack of precise existing data and ambiguity caused by the qualitative judgment of decision-makers and rapid changes in market conditions, the inventory parameters become imprecise. As the classical inventory models do not consider this impreciseness, their optimal strategies become inaccurate which leads to economic loss for the retailers. Thus, many researchers adopted fuzzy set theory to deal with the impreciseness of parameters and obtained optimal strategies for their inventory problems. However, none of the researchers considers the inventory problem of impreciseness in demand, deterioration, and backlogging rate. Thus, the objective of the present investigation is to determine the optimal strategy for such retailers' inventory problems and to compare it with the results of the corresponding crisp model. Hence, we develop the model in both crisp and fuzzy environments. Moreover, the applicability of the proposed model is discussed numerically by examining several inventory constraints in both environments. Finally, the sensitivity analysis of different parameters has been conducted to draw managerial insights.

2010 Mathematics Subject Classification. 90B05, 03E72.

Keywords and phrases. Inventory model, Fuzzy EOQ model, Fuzzy demand and deterioration, Shortages, Partial backlogging, Graded mean integration method.

1. Introduction

1.1. **Inventory Models.** The objective of inventory models is to find the time and quantity of orders to be placed for the smooth functioning of the retailer's business. Then, the research problem is to optimize these decision strategies by considering all the costs and constraints associated with the inventory. Basically, there are two types of inventory models:

(i) EOQ Model. The Economic Order Quantity (EOQ) models are used to forecast the appropriate reorder point with an optimal ordering quantity that minimizes the total costs of inventory. Thus, it becomes a valuable tool for most retailers who need to decide on the number of units and when to order is to be placed to minimize the total cost.

(ii) EPQ Model. The Economic Production Quantity (EPQ) models are used to determine the optimum lot size that can be manufactured in a production unit under the constraints of available resources. Also, it avoids excess inventory of raw materials, excess inventory of finished goods, the ideal time of machines, etc.

1.2. **Crisp Inventory Models.** In classical inventory models, the researchers assume fixed values for different parameters that describe the inventory constraints. Such types of inventory models are known as crisp inventory models. These models are suitable for retailers' business when the inventory constraints and costs are not undergoing any external influences. From the past century, various inventory models developed by many researchers in a crisp environment. Among all types of products, deteriorating products drew significant attention from researchers. Since the inventory management of deteriorating items becomes a challenging task for retailers' or inventory managers due to the increase in inventory cost and a partial loss in sales that occurs with the deteriorating nature of the product preventing them from their original utility. For example, the retailers dealing with agricultural products, vegetables, fruits, medicines, packed foods, beverages, chemicals, pesticides, electronic items, etc. always need to employ an optimal strategy to minimize the cost and maximize the profit. Thus, the inventory managers dealing with deteriorating products always investigate the economic ordering quantity, which minimizes the inventory cost and fulfills the demand of the business cycle. In this direction, the first attempt was made by Ghare and Schrader [6]. Next, Covert and Philip [4] extended the work of Ghare and Schrader [6]. Subsequently, many inventory models for deteriorating items have been developed by researchers in this direction.

1.3. **Fuzzy Inventory Models.** In the present global scenario, the costs and constraints associated with inventory are imprecise for several reasons. The impreciseness of parameters can be measured by using various fuzzy numbers. Also, there are fuzzy techniques to deal with the inventory models with impreciseness. Thus, the inventory models which adopt the fuzzy set theory are called fuzzy inventory models. Initially, Lee and Yao [12] considered impreciseness of demand and production quantity in their economic production quantity model and adopted the fuzzy techniques to obtain the optimal strategy. Subsequently, Kao and Hsu [8], and Dutta *et al.* [5] took the account of impreciseness in demand, and used the fuzzy concepts in their inventory models.

1.4. **Preliminaries of Fuzzy Set Theory.**

Fuzzy set. A fuzzy set $\tilde{A}$ in the universal set X is described by its membership function denoted by $\mu_{\tilde{A}}$. That is, $\mu_{\tilde{A}} : X \to [0,1]$ and $\mu_{\tilde{A}}$ understand as the "membership function" of element x in the fuzzy set $\tilde{A}$, $\forall x \in X$.

$\boldsymbol{\alpha}$ *-cut.* The set of elements belonging to the fuzzy set $\tilde{A}$ whose membership grade is not less than 'α' is called the α-level set or α-cut. That is,

$$A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}.$$

Fuzzy Numbers. A fuzzy subset $\tilde{A}$ in the real line R having the membership function $\mu_{\tilde{A}} : R \to [0,1]$ is called a fuzzy number if it fulfills the the accompanying properties as below:

(i) $\tilde{A}$ is normal. That is, $\exists$ an element x_0 for which $\mu_{\tilde{A}}(x_0) = 1$.

(ii) $\tilde{A}$ is convex. That is

$$\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2), x_1, x_2 \in R, \forall \lambda \in [0,1].$$

(iii) $\mu_{\tilde{A}}$ is upper continuous.

(iv) Supp $\tilde{A}$ is bounded, where Supp $\tilde{A} = \{x \in X : \mu_{\tilde{A}}(x) > 0\}$.

Triangular Fuzzy Number. Let a, b, $c \in R$ such that $a < b < c$. Then the fuzzy number $\tilde{A} = (a,b,c)$ is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{Otherwise.} \end{cases}$$

Arithmetic Operations. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be the two triangular fuzzy numbers, and a_1, a_2, a_3, b_1,b_2, and b_3 are real numbers. Then, the arithmetic operations on triangular-fuzzy numbers are defined as follow:

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is denoted by $\tilde{A} + \tilde{B}$ and is given by

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3),$$

where a_1, a_2, a_3, b_1, b_2 and b_3 are real numbers.

(ii) The product of $\tilde{A}$ and $\tilde{B}$ is denoted by $\tilde{A} \times \tilde{B}$ and is given by

$$\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$$

such that $c_1 = \min T$, $c_2 = a_2 b_2$ and $c_3 = \max T$. Here, $T = \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$.

(iii) If a_1, a_2, a_3, b_1, b_2 and b_3 all non-zero positive real numbers, then

$$\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$$

(iv) $-\tilde{B} = (-b_3, -b_2, -b_1)$.

(v) Then the subtraction of $\tilde{B}$ and $\tilde{A}$ is denoted by $\tilde{A} - \tilde{B}$ and is given by

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1),$$

where a_1, a_2, a_3, b_1, b_2 and b_3 all real numbers.

(vi) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$, where b_1, b_2 and b_3 are all non-zero real numbers, then $\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$.

(vii) Let $\alpha \in R$, then $\alpha\tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3), & if\ \alpha \geq 0 \\ (\alpha a_3, \alpha a_2, \alpha a_1), & if\ \alpha < 0. \end{cases}$

Defuzzification. Defuzzification is the process of producing quantifiable result in the fuzzy logic for a given fuzzy set and corresponding membership degrees. One of such defuzzification process is "**Graded Mean Integration Representation (GMIR)**".

The graded mean integration representation of a fuzzy number $\tilde{A}$ is usually denoted by $P(\tilde{A})$ and is defined as

$$P(\tilde{A}) = \frac{\int_0^{W_A} \frac{h}{2}(L^{-1}(h) + R^{-1}(h))\, dh}{\int_0^{W_A} h\, dh},$$

$0 \leq h \leq W_A$ and $0 \leq W_A \leq 1$. Then by using the formula for triangular fuzzy number $\tilde{A} = (a, b, c)$, we get

$$P(\tilde{A}) = \frac{1}{6}\Big(a + 4b + c\Big).$$

1.5. **Literature Review.** The inventory management playing a vital role in almost all business affairs, and the decision strategies are differ from inventory to inventory depending upon their constraints. The different constraints are demand, deterioration, inflation, shortages, space scarcity, various inventory costs and many others. Recently, Barik *et al.* [1], Mishra *et al.* [15], Routray *et al.* [19], Chakraborty *et al.* [2], Singh and Kumar [24], Wu *et al.* [25], Chen *et al.* [3], Shaikh *et al.* [23], Shaikh *et al.* [21], Yu [26] developed the inventory models with different demand, deterioration and shortages with or without backlogging in crisp environment.

The researchers cited earlier were assumed that fixed parametric values and deterministic constraints while developing the model. However, in the present scenario, most of the parameters are imprecised. Also, the inventory constraints getting influenced by the rapid change in the market. The rapid changes in the present days are mainly due to the introduction of new products, advancement of preservation technology, the quality of the product, the amendment of tax price by the government, etc. which causes fluctuations in demand, deterioration, backlogging rate, and inventory costs from cycle to cycle. As a result, the optimal values obtained in the crisp model are inaccurate or inappropriate in the real inventory problems. Now, the following research questions arise:

(i) How to represent the impreciseness of parametric values?
(ii) How to determine the optimal strategies of inventory models when the constraints and costs are imprecised?

The best answer for these questions is fuzzy set theory and fuzzy inventory models. Since, the fuzzy set theory is primarily concerned with how to quantitatively deal with imprecision and uncertainty. Further, it offers the decision-makers a tool for modeling real-world problems. Also, over the years, there have been successful applications and implementations of fuzzy set theory in various dimensions of research. Similarly, fuzzy inventory models are used to obtain the optimal strategies of inventory models when the constraints and costs are imprecised. The researchers, Lee and Yao [12] are the first among all who applied fuzzy set theory in the inventory models. They used concepts of fuzzy to deal with the impreciseness in demand and production quantity. Later on, Maiti and Maiti [13], and Rong *et al.* [17] considered fuzzy lead time in their inventory models. Shabani *et al.* [20] considered both demand and deterioration as fuzzy numbers in their inventory problems. Furthermore, for more recent results in fuzzy environment see the works of Indrajitsingha *et al.* [7], Kumar *et al.* [9, 10, 11], Mishra *et al.* [14], Nayak *et al.* [16] and Routray *et al.* [18].

Table 1. Summary of Literature Review

Author(s)	Year	Demand Type	Deterioration Type	Backlogging Type	Fuzzy Model
Chakraborty *et al.* [2]	2018	Ramp-type	Weibull	Partial	No
Shaikh *et al.* [22]	2018	Advertisement selling price	Constant	Partial	Yes
Singh and Kumar [24]	2018	Selling price	Constant	Partial	No
Wu *et al.* [25]	2018	Trapezoidal	Time dependent	Partial	No
Chen *et al.* [3]	2019	Stock, time, Price dependent	Constant	No Shortages	No
Indrajitsingha *et al.* [7]	2019	Selling price	Constant	Partial	Yes
Shaikh *et al.* [23]	2019	Stock Dependent	Constant	Partial	No
Shaikh *et al.* [21]	2019	Price and Advertisement cost	Weibull	Partial	No
Yu [26]	2019	Constant	Constant	Completely	No
Kumar *et al.* [9]	2020	Fuzzy Exponential	Fuzzy Constant	No Shortages	Yes
Kumar *et al.* [11]	2020	Decreasing	Negligible	No Shortages	Yes
Nayak *et al.* [16]	2021	Linear	Weibull	Completely	Yes
This Paper		Time Varying	Exponential Decay	Partial	Yes

To the best of my knowledge, from the literature discussed above and existing literature, none of the researchers developed the inventory model for exponentially decaying items having time-varying demand with partial backlogged shortages under an imprecised environment. For quick insight into the literature, we draw the attention of the readers to Table 1.

Motivated essentially by the above mentioned facts, we carried the present investigation in the fuzzy environment that aims to obtain the optimal economic ordering quantity of said inventory model that minimizes the total inventory cost. The triangular-fuzzy numbers are used to quantify the impreciseness in demand, deterioration, and backlogging rate, and the GMIR method is applied to defuzzify the model. The main objective of the proposed work is to obtain an ideal result in the fuzzy environment that helps the retailer reduce the economic loss in the competitive business market.

Further, the crisp model is developed by taking fixed values for the imprecised parameters to compare the optimal strategies in both environments. Moreover, various numerical examples have been presented to show how the optimal strategy differ in both crisp and fuzzy environments. Finally, the effect of changes in different parameters on optimal results is studied and are presented in the form of tables and figures.

2. Assumptions and Notations

2.1. Assumptions.

The present model assumes that,

(i) all the items in the inventory are homogeneous;
(ii) there is a single warehouse;
(iii) replenishment occurs instantaneously;
(iv) $\mathcal{D}(t) = \mu t^{-\beta}$ be the time varying demand rate, where $\mu > 0$ and $0 < \beta < 1$;
(v) $\vartheta(t) = \begin{cases} \theta e^{\theta t}, & t > 0 \\ 0, & \text{otherwise} \end{cases}$ be the exponential deterioration, where $0 < \theta < 1$ is called deterioration rate;
(vi) the deteriorated items have no repair or replacement;
(vii) shortages occur in the cycle and are partially backlogged with backlogging rate $\pi(t) = \frac{1}{1+\delta(T-t)}$, $0 < \delta < 1$, where δ is the backlogging parameter;
(viii) the size of the inventory is finite but the replenishment rate is infinite;
(ix) the time horizon is finite;
(x) the backlogged items delivered to the customer at the beginning of the next cycle.

2.2. Notations.

While developing the model, the following notations are considered.

$Q(t)$: Inventory at any time t.
S: Initial inventory, that is, $S = Q(0)$.
$\mathbb{B}$: Amount of items backlogged during the cycle.
W: The ordering quantity of items at the beginning of each cycle, that is, $W = S + \mathbb{B}$.
T: Duration of the cycle.
$t1$: At $t1$, inventory becomes zero.
pc: Purchase cost per unit item.
hc: Holding cost per unit item per unit time.
sc: Shortage cost per unit item.
oc: Opportunity cost per unit item.
dc: Deterioration cost per unit item.
PC: Total purchase cost.
HC: Total inventory holding cost.
SC: Total shortage cost.
OC: Total opportunity cost due to loss of sale.
DC: Total deterioration cost.
TC: Total cost of the inventory per unit time.

3. Mathematical Model

Let at the beginning of each inventory cycle, W items be ordered and received instantaneously (as lead time is zero). Out of W items, let $\mathbb{B}$ items be used to fulfill the backlogging in the previous cycle and the remaining S items are kept in the inventory. Suppose that, in the first interval $(0, t1]$, the inventory $Q(t)$ diminishes due to demand, deterioration, and reaches zero at time $t1$; in the second interval $(t1, T]$, the inventory $Q(t)$ falls under shortages. The level of inventory at any time t is depicted in figure 1. Also, these shortages are partially backlogged with the backlogging rate $\pi(t)$, and are supplied to the customers in the next cycle.

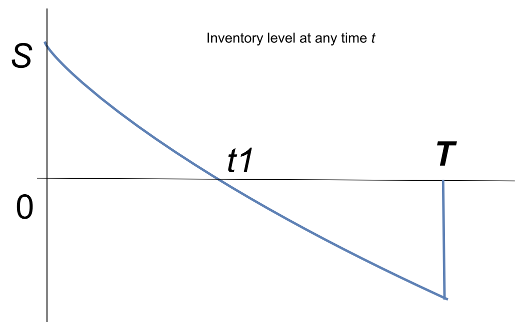

FIGURE 1. Inventory level at any time t

The inventory level $Q(t)$ in the interval $(0, t1]$ is governed by the differential equation

$$\frac{dQ}{dt} + \theta e^{\theta t} Q = -\mu t^{-\beta} \tag{1}$$

under the boundary condition $Q(t1) = 0$.

Similarly, the inventory level $Q(t)$ in the interval $(t1, T]$ is governed by the differential equation

$$\frac{dQ}{dt} = -\frac{\mu t^{-\beta}}{1 + \delta(T - t)} \tag{2}$$

under the boundary condition $Q(t1) = 0$.

On solving the differential equation (1), we get

$$Q(t) = \frac{\mu}{1 + \theta t}\left\{\Big(\frac{t1^{1-\beta} - t^{1-\beta}}{1 - \beta}\Big) + \theta\Big(\frac{t1^{2-\beta} - t^{2-\beta}}{2 - \beta}\Big)\right\}. \tag{3}$$

Using equation (3), we get initial inventory as

$$S = Q(0) = \mu\Big(\frac{t1^{1-\beta}}{1 - \beta} + \theta\frac{t1^{2-\beta}}{2 - \beta}\Big). \tag{4}$$

On solving the differential equation (2), we get

$$Q(t) = \mu\left\{(1 - \delta t)\Big(\frac{t1^{1-\beta} - t^{1-\beta}}{1 - \beta}\Big) + \delta\Big(\frac{t1^{2-\beta} - t^{2-\beta}}{2 - \beta}\Big)\right\}. \tag{5}$$

The amount of items backlogged during the interval $(t1, T]$ is

$$\begin{aligned} \mathbb{B} &= \int_{t1}^{T} \mu t^{-\beta}\Big(\frac{1}{1+\delta(T-t)}\Big)\,dt \\ &= \mu(1-\delta T)\Big(\frac{T^{1-\beta}-t1^{1-\beta}}{1-\beta}\Big) + \delta\mu\Big(\frac{T^{2-\beta}-t1^{2-\beta}}{2-\beta}\Big). \end{aligned} \tag{6}$$

The total amount of inventory during the cycle is

$$\begin{aligned} \mathbb{H} &= \int_{0}^{t1} Q(t)\,dt \\ &= \frac{\mu t1^{2-\beta}}{2}\bigg(\frac{2}{2-\beta} + \theta t1\bigg(\frac{1}{3-\beta} - \frac{\theta t1}{4-\beta}\bigg)\bigg). \end{aligned}$$

The total amount of shortages in the inventory is

$$\begin{aligned} \mathbb{S} &= \int_{t1}^{T} Q(t)\,dt \\ &= \mu\bigg\{\Big(\frac{1-\delta T}{1-\beta}\Big)\Big(t1^{1-\beta}(T-t1) - \Big(\frac{T^{2-\beta}-t1^{2-\beta}}{2-\beta}\Big)\Big) \\ &\quad + \frac{\delta}{2-\beta}\Big(t1^{2-\beta}(T-t1) - \Big(\frac{T^{3-\beta}-t1^{3-\beta}}{3-\beta}\Big)\Big)\bigg\}. \end{aligned}$$

The amount of lost sale during the cycle is

$$\begin{aligned} \mathbb{L} &= \int_{t1}^{T} \mathcal{D}(t)\bigg(1 - \frac{1}{1+\delta(T-t)}\bigg) \\ &= \mu\delta\bigg(\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{Tt1^{1-\beta}}{1-\beta} + \frac{t1^{2-\beta}}{2-\beta}\bigg). \end{aligned}$$

The amount of deteriorating items during the cycle is

$$\begin{aligned} \mathbb{D} &= \int_{0}^{t1} \vartheta(t)Q(t)\,dt \\ &= \mu\theta\bigg(\frac{t1^{2-\beta}}{2-\beta} + \theta\frac{t1^{3-\beta}}{3-\beta}\bigg) - \frac{\mu\theta^3}{3}\bigg(\frac{t1^{4-\beta}}{4-\beta} + \theta\frac{t1^{5-\beta}}{5-\beta}\bigg). \end{aligned}$$

The different inventory costs are

(i) purchase cost

$$PC = pc * W = pc * (S + \mathbb{B})$$

(ii) holding cost

$$HC = hc * \mathbb{H}$$

(iii) shortage cost

$$SC = -sc * \mathbb{S}$$

(iv) opportunity cost

$$OC = oc * \mathbb{L}$$

(v) deterioration cost

$$DC = dc * \mathbb{D}.$$

By considering all the relevant costs of inventory, the total average cost is

$$TC = \frac{1}{T}\Big(PC + HC + SC + OC + DC\Big).$$

That is,

$$\begin{aligned} TC = \frac{1}{T}\Bigg\{ & pc * \Bigg[\mu\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu(1-\delta T)\Big(\frac{T^{1-\beta} - t1^{1-\beta}}{1-\beta}\Big) \\ & + \delta\mu\Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big)\Bigg] + hc * \Bigg[\frac{\mu t1^{2-\beta}}{2}\Big(\frac{2}{2-\beta} + \frac{\theta t1}{3-\beta} - \frac{(\theta \mathrm{t}1)^2}{4-\beta}\Big)\Bigg] \\ & - sc * \Bigg[\mu\Bigg\{\Big(\frac{1-\delta T}{1-\beta}\Big)\Big(t1^{1-\beta}(T-t1) - \Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big)\Big) \\ & + \frac{\delta}{2-\beta}\Big(t1^{2-\beta}(T-t1) - \Big(\frac{T^{3-\beta} - t1^{3-\beta}}{3-\beta}\Big)\Big)\Bigg\}\Bigg] \\ & + oc * \Bigg[\mu\delta\Big(\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{Tt1^{1-\beta}}{1-\beta} + \frac{t1^{2-\beta}}{2-\beta}\Big)\Bigg] \\ & + dc * \Bigg[\mu\theta\Big(\frac{t1^{2-\beta}}{2-\beta} + \theta\frac{t1^{3-\beta}}{3-\beta}\Big) - \frac{\mu\theta^3}{3}\Big(\frac{t1^{4-\beta}}{4-\beta} + \theta\frac{t1^{5-\beta}}{5-\beta}\Big)\Bigg]\Bigg\}. \end{aligned} \tag{7}$$

The main objective of the model is to find the Economic Ordering Quantity which minimizes the total cost of the inventory.

4. Solution Procedure for Crisp Model

The optimal EOQ can be obtained as follows:

(i) Find $t1$ such that TC becomes minimum. That is, find $t1$ such that $\frac{dTC(t1)}{dt1} = 0$ and $\frac{d^2TC(t1)}{dt1^2} > 0$.
(ii) Use $t1$ in equations (4) and (6) to find the optimal EOQ for $W = S + \mathbb{B}$.
(iii) Use $t1$ in equation (7) to find the minimum total cost TC.

5. Fuzzy Model

While developing classical inventory models, the decision makers find the deterministic demand and deterioration by analyzing the available historical data. But, these data may have vagueness due to lack of accurate information and ground reality. This may cause inaccuracy in the results obtained in classical inventory models, however; the fuzzy inventory model is a better choice to overcome these situations. In the proposed model, we consider the important parameters such as the demand, deterioration and backlogging rate as fuzzy numbers to minimize the inaccuracy in the optimum results. That is, μ, θ and δ are considered as fuzzy parameters $\tilde{\mu}$, $\tilde{\theta}$ and $\tilde{\delta}$ respectively. Thus, by proceeding in the similar lines of crisp model, we obtain the total average inventory cost in fuzzy environment as

$$\begin{aligned}
\widetilde{TC} = \frac{1}{T}\Bigg\{ & pc * \Bigg[\tilde{\mu}\Big(\frac{t1^{1-\beta}}{1-\beta} + \tilde{\theta}\frac{t1^{2-\beta}}{2-\beta}\Big) + \tilde{\mu}(1-\tilde{\delta}T)\Big(\frac{T^{1-\beta}-t1^{1-\beta}}{1-\beta}\Big) \\
& + \tilde{\delta}\tilde{\mu}\Big(\frac{T^{2-\beta}-t1^{2-\beta}}{2-\beta}\Big)\Bigg] + hc * \Bigg[\frac{\tilde{\mu}t1^{2-\beta}}{2}\Bigg(\frac{2}{2-\beta} + \frac{\tilde{\theta}t1}{3-\beta} - \frac{(\tilde{\theta}\mathrm{t}1)^2}{4-\beta}\Bigg)\Bigg] \\
& - sc * \Bigg[\tilde{\mu}\Bigg\{\Big(\frac{1-\tilde{\delta}T}{1-\beta}\Big)\Big(t1^{1-\beta}(T-t1) - \Big(\frac{T^{2-\beta}-t1^{2-\beta}}{2-\beta}\Big)\Big) \\
& + \frac{\tilde{\delta}}{2-\beta}\Big(t1^{2-\beta}(T-t1) - \Big(\frac{T^{3-\beta}-t1^{3-\beta}}{3-\beta}\Big)\Big)\Bigg\}\Bigg] \\
& + oc * \Bigg[\tilde{\mu}\tilde{\delta}\Bigg(\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{Tt1^{1-\beta}}{1-\beta} + \frac{t1^{2-\beta}}{2-\beta}\Bigg)\Bigg] \\
& + dc * \Bigg[\tilde{\mu}\tilde{\theta}\Bigg(\frac{t1^{2-\beta}}{2-\beta} + \tilde{\theta}\frac{t1^{3-\beta}}{3-\beta}\Bigg) - \frac{\tilde{\mu}\tilde{\theta}^3}{3}\Bigg(\frac{t1^{4-\beta}}{4-\beta} + \tilde{\theta}\frac{t1^{5-\beta}}{5-\beta}\Bigg)\Bigg]\Bigg\}. \qquad (8)
\end{aligned}$$

In particular, triangular fuzzy numbers are considered for fuzzy parameters as $\tilde{\mu} = (\mu 1, \mu 2, \mu 3)$, $\tilde{\theta} = (\theta 1, \theta 2, \theta 3)$ and $\tilde{\delta} = (\delta 1, \delta 2, \delta 3)$. Moreover, "**Graded Mean Integration Representation (GMIR)**" method is considered for defuzzification. Thus, the defuzzified total average cost is

$$FTC = \frac{1}{6}\Big[\widetilde{TC1} + 4 * \widetilde{TC2} + \widetilde{TC3}\Big]. \qquad (9)$$

Here,

$$\begin{aligned}
\widetilde{TC1} = \frac{1}{T}\Bigg\{ & pc * \Bigg[\mu 1\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta 1\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu 1(1-\delta 1T)\Big(\frac{T^{1-\beta}-t1^{1-\beta}}{1-\beta}\Big) \\
& + \delta 1\mu 1\Big(\frac{T^{2-\beta}-t1^{2-\beta}}{2-\beta}\Big)\Bigg] + hc * \Bigg[\frac{\mu 1t1^{2-\beta}}{2}\Bigg(\frac{2}{2-\beta} + \frac{\theta 1t1}{3-\beta} - \frac{(\theta 1\mathrm{t}1)^2}{4-\beta}\Bigg)\Bigg] \\
& - sc * \Bigg[\mu 1\Bigg\{\Big(\frac{1-\delta 1T}{1-\beta}\Big)\Big(t1^{1-\beta}(T-t1) - \Big(\frac{T^{2-\beta}-t1^{2-\beta}}{2-\beta}\Big)\Big) \\
& + \frac{\delta 1}{2-\beta}\Big(t1^{2-\beta}(T-t1) - \Big(\frac{T^{3-\beta}-t1^{3-\beta}}{3-\beta}\Big)\Big)\Bigg\}\Bigg] \\
& + oc * \Bigg[\mu 1\delta 1\Bigg(\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{Tt1^{1-\beta}}{1-\beta} + \frac{t1^{2-\beta}}{2-\beta}\Bigg)\Bigg] \\
& + dc * \Bigg[\mu 1\theta 1\Bigg(\frac{t1^{2-\beta}}{2-\beta} + \theta 1\frac{t1^{3-\beta}}{3-\beta}\Bigg) - \frac{\mu 1\theta 1^3}{3}\Bigg(\frac{t1^{4-\beta}}{4-\beta} + \theta 1\frac{t1^{5-\beta}}{5-\beta}\Bigg)\Bigg]\Bigg\},
\end{aligned}$$

$$\widetilde{TC2} = \frac{1}{T}\Bigg\{pc * \Big[\mu 2\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta 2\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu 2(1-\delta 2T)\Big(\frac{T^{1-\beta} - t1^{1-\beta}}{1-\beta}\Big)$$
$$+ \delta 2\mu 2\Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big)\Big] + hc * \Big[\frac{\mu 2t1^{2-\beta}}{2}\Big(\frac{2}{2-\beta} + \frac{\theta 2t1}{3-\beta} - \frac{(\theta 2\mathrm{t}1)^2}{4-\beta}\Big)\Big]$$
$$- sc * \Big[\mu 2\Big\{\Big(\frac{1-\delta 2T}{1-\beta}\Big)\Big(t1^{1-\beta}(T-t1) - \Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big)\Big)$$
$$+ \frac{\delta 2}{2-\beta}\Big(t1^{2-\beta}(T-t1) - \Big(\frac{T^{3-\beta} - t1^{3-\beta}}{3-\beta}\Big)\Big)\Big\}\Big]$$
$$+ oc * \Big[\mu 2\delta 2\Big(\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{Tt1^{1-\beta}}{1-\beta} + \frac{t1^{2-\beta}}{2-\beta}\Big)\Big]$$
$$+ dc * \Big[\mu 2\theta 2\Big(\frac{t1^{2-\beta}}{2-\beta} + \theta 2\frac{t1^{3-\beta}}{3-\beta}\Big) - \frac{\mu 2\theta 2^3}{3}\Big(\frac{t1^{4-\beta}}{4-\beta} + \theta 2\frac{t1^{5-\beta}}{5-\beta}\Big)\Big]\Bigg\},$$

and

$$\widetilde{TC3} = \frac{1}{T}\Bigg\{pc * \Big[\mu 3\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta 3\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu 3(1-\delta 3T)\Big(\frac{T^{1-\beta} - t1^{1-\beta}}{1-\beta}\Big)$$
$$+ \delta 3\mu 3\Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big)\Big] + hc * \Big[\frac{\mu 3t1^{2-\beta}}{2}\Big(\frac{2}{2-\beta} + \frac{\theta 3t1}{3-\beta} - \frac{(\theta 3\mathrm{t}1)^2}{4-\beta}\Big)\Big]$$
$$- sc * \Big[\mu 3\Big\{\Big(\frac{1-\delta 3T}{1-\beta}\Big)\Big(t1^{1-\beta}(T-t1) - \Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big)\Big)$$
$$+ \frac{\delta 3}{2-\beta}\Big(t1^{2-\beta}(T-t1) - \Big(\frac{T^{3-\beta} - t1^{3-\beta}}{3-\beta}\Big)\Big)\Big\}\Big]$$
$$+ oc * \Big[\mu 3\delta 3\Big(\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{Tt1^{1-\beta}}{1-\beta} + \frac{t1^{2-\beta}}{2-\beta}\Big)\Big]$$
$$+ dc * \Big[\mu 3\theta 3\Big(\frac{t1^{2-\beta}}{2-\beta} + \theta 3\frac{t1^{3-\beta}}{3-\beta}\Big) - \frac{\mu 3\theta 3^3}{3}\Big(\frac{t1^{4-\beta}}{4-\beta} + \theta 3\frac{t1^{5-\beta}}{5-\beta}\Big)\Big]\Bigg\}.$$

Consequently, the defuzzified EOQ is given by

$$FW = \frac{1}{6}\Big[W1 + 4 * W2 + W3\Big]. \tag{10}$$

Here,

$$W1 = \mu 1\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta 1\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu 1(1-\delta 1T)\Big(\frac{T^{1-\beta} - t1^{1-\beta}}{1-\beta}\Big)$$
$$+ \delta 1\mu 1\Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big),$$

$$W2 = \mu 2\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta 2\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu 2(1-\delta 2T)\Big(\frac{T^{1-\beta} - t1^{1-\beta}}{1-\beta}\Big)$$
$$+ \delta 2\mu 2\Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big),$$

and

$$W3 = \mu 3\Big(\frac{t1^{1-\beta}}{1-\beta} + \theta 3\frac{t1^{2-\beta}}{2-\beta}\Big) + \mu 3(1-\delta 3T)\Big(\frac{T^{1-\beta} - t1^{1-\beta}}{1-\beta}\Big) + \delta 3\mu 3\Big(\frac{T^{2-\beta} - t1^{2-\beta}}{2-\beta}\Big).$$

6. Solution Procedure for Fuzzy Model

The optimal EOQ can be obtained as follow:

(i) Find $t1$ such that FTC becomes minimum. That is, find $t1$ such that $\frac{dFTC(t1)}{dt1} = 0$ and $\frac{d^2FTC(t1)}{dt1^2} > 0$.
(ii) Use $t1$ in equation (10) to find the optimal EOQ for FW.
(iii) Use $t1$ in equation (9) to find the minimum total cost FTC.

7. Numerical Examples

Example 1

(a) *Crisp Model*

$\theta = 0.2$, $\delta = 0.5$, $\mu = 250$, $T = 3$, $\beta = 0.2$, $hc = 4$, $pc = 15$, $sc = 10$, $oc = 12$ and $dc = 9$.

Solution

The optimal solution is $t1 = 1.31814$, $W = 538.939$ and $TC = 4005.67$.

(b) *Fuzzy Model*

$\theta 1 = 0.1$, $\theta 2 = 0.2$, $\theta 3 = 0.3$, $\delta 1 = 0.4$, $\delta 2 = 0.5$, $\delta 3 = 0.6$, $\mu 1 = 240$, $\mu 2 = 250$, $\mu 3 = 260$, $T = 3$, $\beta = 0.2$, $hc = 4$, $pc = 15$, $sc = 10$, $oc = 12$ and $dc = 9$.

Solution

The optimal solution is $t1 = 1.32136$, $FW = 539.534$ and $FTC = 4009.99$.

Example 2

(a) *Crisp Model*

$\theta = 0.2$, $\delta = 0.5$, $\mu = 400$, $T = 1.5$, $\beta = 0.4$, $hc = 4$, $pc = 15$, $oc = 12$, $sc = 8$ and $dc = 9$.

Solution

The optimal solution is $t1 = 0.180477$, $W = 625.855$ and $TC = 9426.25$.

(b) ***Fuzzy Model***

$\theta1 = 0.1$, $\theta2 = 0.2$, $\theta3 = 0.3$, $\delta1 = 0.4$, $\delta2 = 0.5$, $\delta3 = 0.6$, $\mu1 = 390$, $\mu2 = 400$, $\mu3 = 410$, $T = 1.5$, $\beta = 0.4$, $hc = 4$, $pc = 15$, $oc = 12$, $sc = 8$ and $dc = 9$.

Solution

The optimal solution is $t1 = 0.176565$, $FW = 623.314$ and $FTC = 9423.82$.

Example 3

(a) ***Crisp Model***

$\theta = 0.2$, $\delta = 0.56$, $\mu = 250$, $T = 2.5$, $\beta = 0.9$, $hc = 4$, $pc = 15$, $oc = 9$, $sc = 8$ and $dc = 3$.

Solution

The optimal solution is $t1 = 1.79399$, $W = 2807.79$ and $TC = 17853$.

(b) ***Fuzzy Model***

$\theta1 = 0.1$, $\theta2 = 0.2$, $\theta3 = 0.3$, $\delta1 = 0.46$, $\delta2 = 0.56$, $\delta3 = 0.66$, $\mu1 = 240$, $\mu2 = 250$, $\mu3 = 260$, $T = 2.5$, $\beta = 0.9$, $hc = 4$, $pc = 15$, $oc = 9$, $sc = 8$ and $dc = 3$.

Solution

The optimal solution is $t1 = 1.80268$, $FW = 2809.29$ and $FTC = 17864.2$.

Example 4

(a) ***Crisp Model***

$\theta = 0.32$, $\delta = 0.15$, $\mu = 410$, $T = 1$, $\beta = 0.2$, $hc = 4$, $pc = 10$, $oc = 12$, $sc = 6$ and $dc = 3$.

Solution

The optimal solution is $t1 = 0.403987$, $W = 514.596$ and $TC = 5980.19$.

(b) ***Fuzzy Model***

$\theta1 = 0.22$, $\theta2 = 0.32$, $\theta3 = 0.42$, $\delta1 = 0.05$, $\delta2 = 0.15$, $\delta3 = 0.25$, $\mu1 = 400$, $\mu2 = 410$, $\mu3 = 420$, $T = 1$, $\beta = 0.2$, $hc = 4$, $pc = 10$, $oc = 12$, $sc = 6$ and $dc = 3$.

Solution

The optimal solution is $t1 = 0.403532$, $FW = 514.517$ and $FTC = 5980.75$.

Example 5

(a) ***Crisp Model***

$\theta = 0.8$, $\delta = 0.6$, $\mu = 560$, $T = 2$, $\beta = 0.43$, $hc = 2$, $pc = 15$, $oc = 120$, $sc = 8$ and $dc = 9$.

Solution

The optimal solution is $t1 = 1.51727$, $W = 1976.2$ and $TC = 20854.5$.

(b) ***Fuzzy Model***

$\theta1 = 0.7$, $\theta2 = 0.8$, $\theta3 = 0.9$, $\delta1 = 0.5$, $\delta2 = 0.6$, $\delta3 = 0.7$, $\mu1 = 550$, $\mu2 = 560$, $\mu3 = 570$, $T = 2$, $\beta = 0.43$, $hc = 2$, $pc = 15$, $oc = 120$, $sc = 8$ and $dc = 9$.

Solution

The optimal solution is $t1 = 1.52065$, $FW = 1978.95$ and $FTC = 20844.9$.

Example 6

(a) ***Crisp Model***

$\theta = 0.3$, $\delta = 0.5$, $\mu = 100$, $T = 1.5$, $\beta = 0.4$, $hc = 4$, $pc = 15$, $oc = 12$, $sc = 8$ and $dc = 9$.

Solution

The optimal solution is $t1 = 0.118359$, $W = 147.257$ and $TC = 2361.28$.

(b) ***Fuzzy Model***

$\theta1 = 0.2$, $\theta2 = 0.3$, $\theta3 = 0.4$, $\delta1 = 0.4$, $\delta2 = 0.5$, $\delta3 = 0.6$, $\mu1 = 90$, $\mu2 = 100$, $\mu3 = 110$, $T = 1.5$, $\beta = 0.4$, $hc = 4$, $pc = 15$, $oc = 12$, $sc = 8$ and $dc = 9$.

Solution

The optimal solution is $t1 = 0.107532$, $FW = 144.927$ and $FTC = 2358.11$.

Effect of imprecise parameters on optimal results. Comparing the crisp and the fuzzy results in above Examples, we notice that

(i) the positive inventory time, economic ordering quantity and total average cost in the fuzzy model are higher than the corresponding values in the crisp model in Example 1 and 3
(ii) the positive inventory time, economic ordering quantity and total average cost in crisp model are higher than the corresponding values in the fuzzy model in Example 2 and 6
(iii) the positive inventory time, economic ordering quantity in the crisp model is higher than the corresponding values in fuzzy model, and the total average cost in the fuzzy model is higher than that of the crisp model in Example 4
(iv) the positive inventory time, economic ordering quantity in the fuzzy model is higher than the corresponding values in the crisp model and the total average cost in the crisp model is higher than that of the fuzzy model in Example 5.

Clearly, the optimal results in fuzzy model are more accurate than that of the crisp results. This is because, the crisp model considers deterministic parameters; whereas, the fuzzy model considers the imprecise parameters.

8. Sensitivity Analysis

Here we consider Example 2 to examine the effect of sensitive behavior of different parameters in the model.

Table 2. The results of sensitivity of parameter T

Parameter	Value	$t1$	FW	FTC
T	1.5	0.176565	623.314	9423.82
	1.51	0.168033	618.764	9397.03
	1.52	0.158887	613.635	9370.13
	1.53	0.149058	607.836	9343.07
	1.54	0.138464	601.25	9315.83

Table 3. The results of sensitivity of parameter β

Parameter	Value	$t1$	FW	FTC
β	0.4	0.176565	623.314	9423.82
	0.5	0.176565	735.684	10787.8
	0.6	0.176565	912.868	12831.9
	0.7	0.176565	1221.31	16225.6
	0.8	0.176565	1860.79	22973.1

Table 4. The results of sensitivity of parameter hc

Parameter	Value	$t1$	FW	FTC
hc	4	0.176565	623.314	9423.82
	5	0.145959	605.444	9432.83
	6	0.123656	591.299	9439.6
	7	0.106895	579.891	9444.87
	8	0.0939374	570.514	9449.1

Table 5. The results of sensitivity of parameter pc

Parameter	Value	$t1$	FW	FTC
pc	15	0.176565	623.314	9423.82
	15.1	0.162268	615.164	9465.1
	15.2	0.147369	606.303	9505.82
	15.3	0.131798	596.588	9545.92
	15.4	0.115472	585.822	9585.34

Table 6. The results of sensitivity of parameter oc

Parameter	Value	$t1$	FW	FTC
oc	12	0.176565	623.314	9423.82
	13	0.286958	677.674	9558.54
	14	0.368955	711.181	9667.41
	15	0.434822	735.127	9759.6
	16	0.490031	753.55	9839.77

Table 7. The results of sensitivity of parameter sc

Parameter	Value	$t1$	FW	FTC
sc	8	0.176565	623.314	9423.82
	9	0.263848	667.319	9574.81
	10	0.345131	701.912	9707.66
	11	0.419398	729.724	9825.01
	12	0.486602	752.446	9929.1

Table 8. The results of sensitivity of parameter dc

Parameter	Value	$t1$	FW	FTC
dc	9	0.176565	623.314	9423.82
	10	0.169438	619.292	9425.88
	11	0.1628	615.473	9427.82
	12	0.156609	611.845	9429.63
	13	0.150825	608.395	9431.34

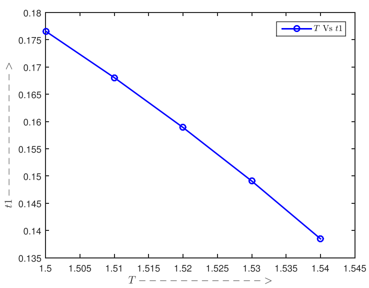

Figure 2. T vs $t1$

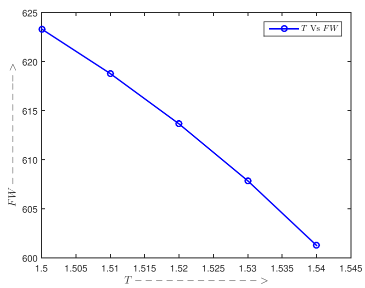

Figure 3. T vs FW

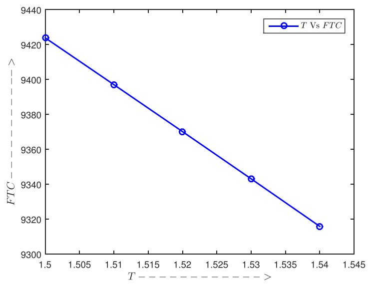

FIGURE 4. T vs FTC

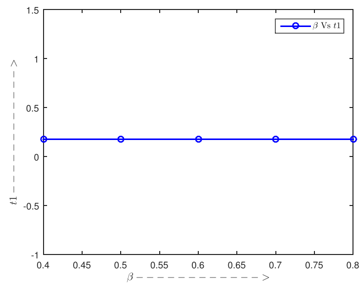

FIGURE 5. β vs $t1$

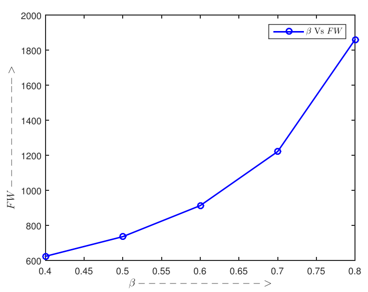

FIGURE 6. β vs FW

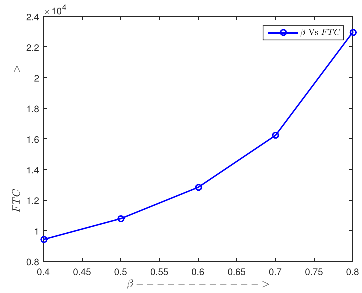

FIGURE 7. β vs FTC

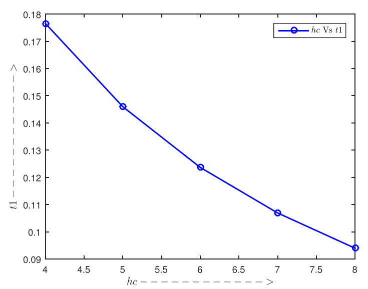

FIGURE 8. hc vs $t1$

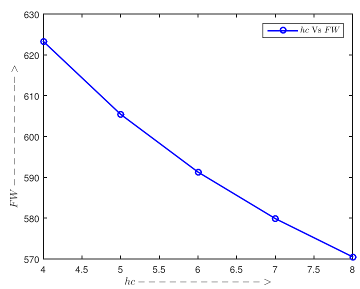

FIGURE 9. hc vs FW

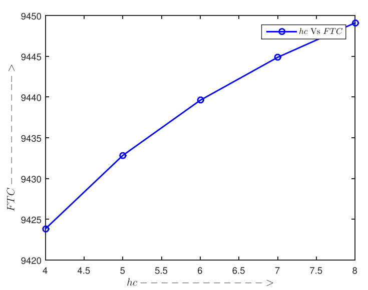

FIGURE 10. hc vs FTC

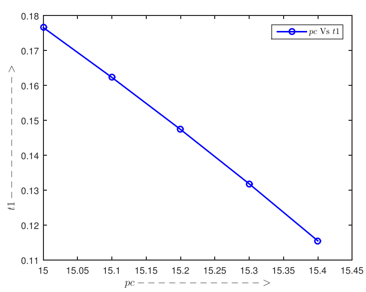

FIGURE 11. pc vs $t1$

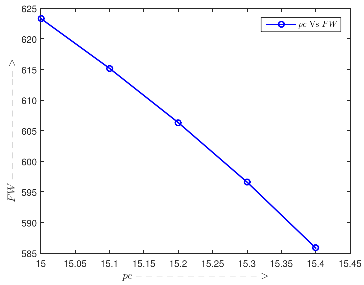

Figure 12. pc vs FW

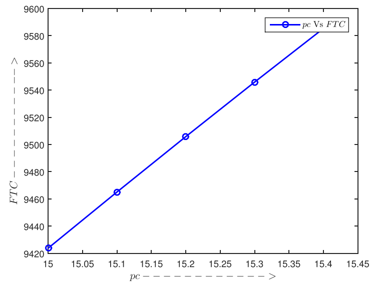

Figure 13. pc vs FTC

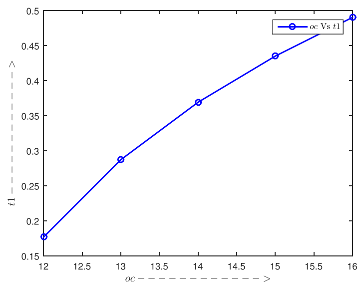

Figure 14. oc vs $t1$

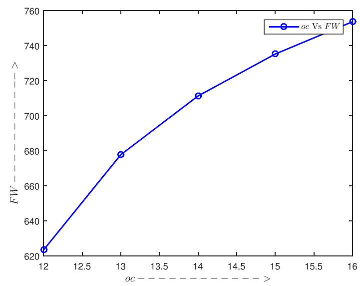

Figure 15. oc vs FW

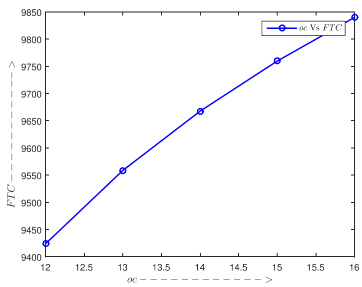

Figure 16. oc vs FTC

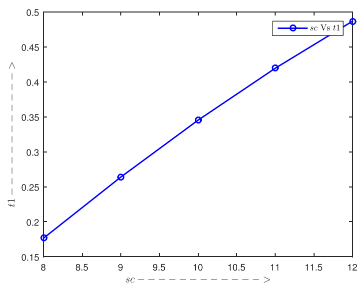

Figure 17. sc vs $t1$

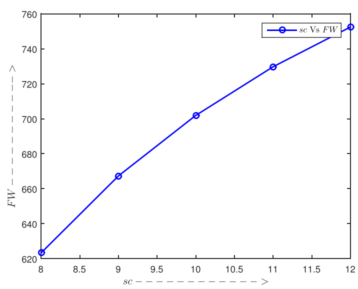

Figure 18. sc vs FW

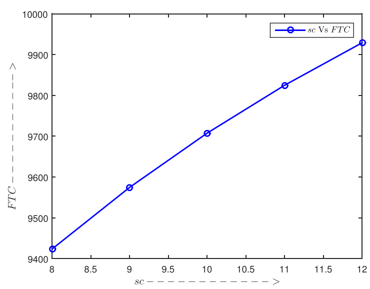

Figure 19. sc vs FTC

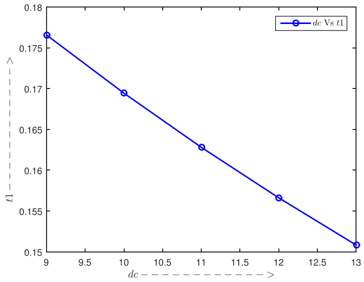

FIGURE 20. dc vs $t1$

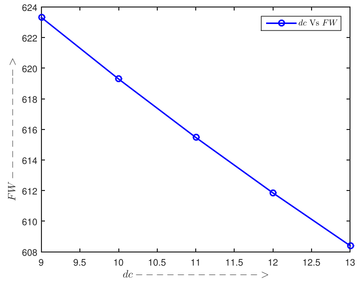

FIGURE 21. dc vs FW

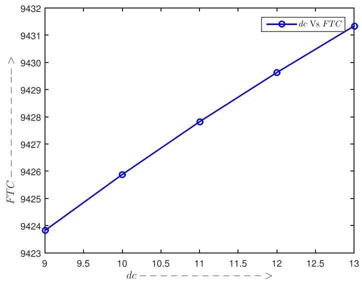

FIGURE 22. dc vs FTC

Based on the results in Table 2 to 8 and Figure 2 to 22, the following conclusions are drawn.

- (T) An increase in the value of the cycle time T results a decrease in the positive inventory time $t1$, economic ordering quantity FW and the total average cost FTC (see Table 2 and Figure 2 to 4).
- (β) An increase in the value of demand parameter β results no change in the positive inventory time $t1$. But, it results a significant increase in both economic ordering quantity FW and the total average cost FTC (see Table 3 and Figure 5 to 7).
- (hc) An increase in the value of the holding cost hc per unit item results a decrease in both positive inventory time and economic ordering quantity FW; whereas, the total average cost FTC increases (see Table 4 and Figure 8 to 10).
- (pc) An increase in the value of the purchase cost pc per unit item results a decrease in both positive inventory time and the economic ordering quantity FW; whereas, the total average cost FTC increases (see Table 5 and Figure 11 to 13).
- (oc) An increase in the value of opportunity cost oc per unit item results in an increase in the positive inventory time, economic ordering quantity FW and the total average cost FTC (see Table 6 and Figure 14 to 16).

(sc) An increase in the value of the shortage cost sc per unit item results in an increase in the positive inventory time, economic ordering quantity FW and the total average cost FTC (see Table 7 and Figure 17 to 19).

(dc) An increase in the value of deteriorating cost dc per unit item results in a decrease in both the positive inventory time and the economic ordering quantity FW; whereas, the total the average cost FTC increases (see Table 8 and Figure 20 to 22).

Managerial Insights. The following managerial insights have been found from the sensitivity examination of various parameters.

(i) The results in Table 2 and Figures 2 to 4 show that the positive-inventory time, economic ordering quantity, and the total average cost of inventory are inversely proportional to the length of the cycle time. So, the retailers need to take care of the planning horizon for their business cycle to prevent the reduction in profit.

(ii) If the value of β increases, then the demand for items increases. Further, the results in Table 3 and Figures 5 to 7 show that the economic ordering quantity and the total average cost of inventory are directly proportional to the demand. But, the positive-inventory time remains constant. Thus, to overcome this scenario, the managers should order more quantities to avoid the loss in sales and smooth functioning of their business.

(iii) The results in the Table 4 to 8 and Figure 8 to 22 shows that the increase in associated inventory costs results the rise in inventory total cost. Hence, inventory managers must pay more attention and take extra precautions to prevent the rise of associated inventory costs to keep their business under control.

9. CONCLUSION

In this article, we have developed two inventory models for exponential decay items having time-varying demand with partially fulfilled shortages by considering deterministic parameters in the crisp environment and imprecised parameters in the fuzzy environment for demand, deterioration, and backlogging rate. The cost optimization procedure for both models was proposed and validated with several numerical examples. Further, it observed that the optimal results in a fuzzy model are more accurate than that of a crisp model. Finally, the sensitivity behavior of various parameters was examined and their managerial insights were suggested.

Future Research Directions. The interested researchers can extend the present work by including more constraints like trade credit financing, two-ware houses for space scarcity, preservation technology for minimizing the deterioration, etc. Further, this article is a reference for different inventory problems with impreciseness.

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