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PSEUDO PROJECTIVE CURVATURE TENSOR ON GENERALIZED SASAKIAN SPACE FORMS

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Abstract. This paper deals with the certain survey of generalized Sasakian space forms admitting pseudo projective curvature tensor. Firstly, we have considered ϕ-pseudo projectively flat and quasi-pseudo projectively flat generalized Sasakian space form. Later we characterize pseudo projective pseudo-symmetric generalized Sasakian space form. Finally, we have investigated generalized Sasakian space form endowed with $\tilde{P} \cdot \tilde{P} = 0$.

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1. Introduction

In 2004, Blair and Carriazo [1] have introduced and studied the structure of generalized Sasakian space forms. An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be generalized Sasakian space form if there exists three differentiable functions f_1, f_2, f_3 on M^{2n+1} such that the curvature tensor R is given by

$$
\begin{aligned}
(1) \qquad R(X,Y)Z = & f_1\{g(Y,Z)X - g(X,Z)Y\} \\
& + f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\
& + f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\
& + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\},
\end{aligned}
$$

for any vector fields X, Y, Z on M^{2n+1} and is denoted by $M^{2n+1}(f_1, f_2, f_3)$.

In [6], the author studied certain properties of concircular curvature tensor on generalized Sasakian space forms. In [9], Kim studied conformally flat and locally symmetric generalized Sasakian space forms. Later in [7], De and Majhi have analyzed the properties of ϕ-semisymmetric generalized Sasakian space form. Prakasha [12] investigated generalized Sasakian space form with weyl-conformal curvature tensor. Recently, many geometers such as [2, 3, 14, 19] and many others have made an attempt to weakened the notion of generalized Sasakian space forms with different extent.

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On the other hand, in [13] author has defined a tensor field $\tilde{P}$ on a Riemannian manifold of dimension $2n+1$ ($n > 2$) which includes the projective curvature tensor P known as pseudo projective curvature tensor and is given by:

$$\tilde{P}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{(2n+1)}\left(\frac{a}{2n} + b\right)[g(Y,Z)X - g(X,Z)Y], \tag{2}$$

for any vector fields X, Y, Z on M^{2n+1}, where a and b are constants and R, S and r are the Riemannian curvature tensor, the Ricci tensor of type $(0,2)$ and the scalar curvature of the manifold respectively. A manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be pseudo projectively flat if the pseudo projective curvature tensor $\tilde{P} = 0$. It is well known that the pseudo projectively flat manifold is either projectively flat ($a \neq 0$) or an Einstein manifold ($a = 0$ and $b \neq 0$). Some related works can be found in [4, 10, 11, 15–18].

After some preliminaries of generalized Sasakian space forms given in Section 2, in Sections 3 and 4 we characterize ϕ-pseudo projectively flat and quasi-pseudo projectively flat generalized Sasakian space forms and in both the cases we have obtained η-Einstein manifold. Section 5 deals with the pseudo projective pseudo-symmetric generalized Sasakian space forms and it is shown that the manifold reduces to η-Einstein. Finally, in section 6 we consider generalized Sasakian space form satisfying $\tilde{P} \cdot \tilde{P} = 0$.

2. Preliminaries

A $(2n+1)$-dimensional Riemannian manifold is said to be almost contact metric manifold if the following conditions hold [5]:

$$\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \tag{3}$$

$$\eta(\xi) = 1, \quad g(X,\xi) = \eta(X), \quad \eta(\phi X) = 0, \tag{4}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{5}$$

$$g(\phi X, Y) = -g(X, \phi Y), \quad g(\phi X, X) = 0, \tag{6}$$

where ϕ is a $(1,1)$ tensor field, ξ is a vector field, η is a 1-form and g is the Riemannian metric.

Again for a $(2n+1)$-dimensional generalized Sasakian space form $M^{2n+1}(f_1, f_2, f_3)$, the following relations hold [1]:

$$R(X,Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\}, \tag{7}$$

$$\eta(R(X,Y)Z) = (f_1 - f_3)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}, \tag{8}$$

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi, \tag{9}$$

$$Q\xi = 2n(f_1 - f_3)\xi, \tag{10}$$

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y) - (3f_2 + (2n-1)f_3)\,\eta(X)\eta(Y), \tag{11}$$

$$S(X,\xi) = 2n(f_1 - f_3)\,\eta(X), \tag{12}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{13}$$

for any vector fields X, Y, Z on M^{2n+1}, where R, Q, S and r are the Riemannian curvature tensor, Ricci operator, Ricci tensor and scalar curvature of $M^{2n+1}(f_1, f_2, f_3)$, respectively.

We have also, for a $(2n+1)$-dimensional generalized Sasakian space form $M^{2n+1}(f_1, f_2, f_3)$, pseudo projective curvature tensor satisfies the following relations:

$$\tilde{P}(X,Y)\xi = \left[a(f_1 - f_3) - \frac{r}{2n+1}(\frac{a}{2n} + b)\right] \{\eta(Y)X - \eta(X)Y\} + b\{S(Y,\xi)X - S(X,\xi)Y\}, \tag{14}$$

$$\tilde{P}(\xi,Y)Z = \left[a(f_1 - f_3) - \frac{r}{2n+1}(\frac{a}{2n} + b)\right] \{g(Y,Z)\xi - \eta(Z)Y\} + b\{S(Y,Z)\xi - S(\xi,Z)Y\}, \tag{15}$$

$$\tilde{P}(X,\xi)Y = \left[a(f_1 - f_3) - \frac{r}{2n+1}(\frac{a}{2n} + b)\right] \{\eta(Y)X - g(X,Y)\xi\} + b\{S(\xi,Y)X - S(X,Y)\xi\}, \tag{16}$$

$$\eta(\tilde{P}(X,Y)Z) = \left[a(f_1 - f_3) - \frac{r}{2n+1}(\frac{a}{2n} + b)\right] \{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\} + b\{S(Y,Z)\eta(X) - S(X,Z)\eta(Y)\}, \tag{17}$$

$$\eta(\tilde{P}(X,Y)\xi) = 0. \tag{18}$$

We also need the notion of Ricci solitons [8]. It is the generalization of the Einstein metric and is defined on a Riemannian manifold (M^{2n+1}, g). It is a triple (g, V, λ) with g is a Riemannian metric, V is a vector field and λ is a real scalar such as

$$L_v g + 2S + 2\lambda g = 0, \tag{19}$$

where S is a Ricci tensor and L_v denotes the Lie derivative operator along the vector field V. Ricci soliton is said to be shrinking, steady and expanding according to λ is negative, zero and positive.

Throughout this paper we have assumed that the manifold under consideration having solenoidal vector field V (i.e., $divV = 0$).

3. ϕ-Pseudo Projectively Flat Generalized Sasakian Space Forms

Definition 3.1. *A $(2n+1)$-dimensional generalized Sasakian space form M^{2n+1} $(n \geq 1)$ is said to be ϕ-pseudo projectively flat if $\phi^2(\tilde{P}(\phi X, \phi Y)\phi Z) = 0$, holds true for all vector fields X, Y, Z.*

In view of (2) in the above condition gives

$$\phi^2\{aR(\phi X, \phi Y)\phi Z)\} + b\phi^2\{S(\phi Y, \phi Z)\phi X - S(\phi X, \phi Z)\phi Y\} - \frac{r}{(2n+1)}\left(\frac{a}{2n} + b\right)\phi^2\{g(\phi Y, \phi Z)\phi X - g(\phi X, \phi Z)\phi Y\} = 0. \tag{20}$$

By considering (1) in (20), we get

$$a\phi^2\{f_1\{g(\phi Y,\phi Z)\phi X - g(\phi X,\phi Z)\phi Y\}+ f_2\{-g(\phi X,Z)\phi^2 Y - g(\phi Y,Z)\phi^2 X - 2g(\phi X,Y)\phi^2 Z\}+ b\{\phi^2\{S(\phi Y,\phi Z)\phi\ X\} - S(\phi X,\phi Z)\phi Y\}- \frac{r}{(2n+1)}\left(\frac{a}{2n}+b\right)\phi^2\{g(\phi Y,\phi Z)\phi X - g(\phi X,\phi Z)\phi Y\} = 0. \tag{21}$$

By virtue of (3), (4), (5), (6) and (11), the equation (21) turns into

$$\begin{aligned} & a\{f_1\{-g(Y,Z)\phi X - \eta(X)\eta(Z)\phi X + g(X,Z)\phi Y - \eta(X)\eta(Z)\phi Y\} \\ & + f_2\{-g(X,\phi Z)\phi^2 Y + g(Y,\phi Z)\phi^2 X - 2g(X,\phi Y)\phi^2 Z\}\} \\ & - b\{S(Y,Z)\phi X - 2n(f_1-f_3)\eta(Y)\eta(Z)\phi X - S(X,Z)\phi Y \\ & + 2n(f_1-f_3)\eta(X)\eta(Z)\phi Y\} - \frac{r}{(2n+1)}\left(\frac{a}{2n}+b\right)\{-g(Y,Z)\phi X \\ & + \eta(Y)\eta(Z)\phi X + g(X,Z)\phi Y - \eta(X)\eta(Z)\phi Y\} = 0. \end{aligned} \tag{22}$$

Taking inner product of (22) with respect to W, let $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold which on plugging $Y = Z = e_i$ and summing over i, $i = 1, 2, 3, \cdots, 2n+1$, we obtain

$$\begin{aligned} S(X,\phi W) = & \frac{1}{b}\{-af_1(2n-1) - 3af_2 - b(r - 2n(f_1 - f_3)) \\ & + \frac{r}{(2n+1)}(\frac{a}{2n}+b)(2n-1)\}g(X,\phi W). \end{aligned} \tag{23}$$

Now Replacing X by ϕX in (23), we have

$$S(X,W) = Ag(X,W) + B\eta(X)\eta(W), \tag{24}$$

$$\begin{aligned} \text{where, } A = & \left(\frac{(2n-1)a}{b} - 2n\right)f_1 + \frac{3a}{b}f_2 + 2nf_3 \\ & + r\left(1 - \frac{(a+2nb)(2n-1)}{2nb(2n+1)}\right), \\ B = & ((1-a+b)2n + a)f_1 - 3af_2 - 2n(1+b)f_3 \\ & + r\left(\frac{(a+2nb)(2n-1)}{2nb(2n+1)} - 1\right). \end{aligned}$$

Hence we can able to state the following result:

Theorem 3.1. *If a $(2n+1)$-dimensional generalized Sasakian space form M^{2n+1} is ϕ-pseudo projectively flat, then the manifold reduces to η-Einstein.*

Next by using (24) in (19), we get

$$L_v g(X,W) + 2[A+\lambda]g(X,W) + 2B\eta(X)\eta(W) = 0. \tag{25}$$

Putting $X = W = e_i$ in (25), yields

$$divV + 2(2n+1)[A+\lambda] + 2B = 0. \tag{26}$$

If V is solenoidal, the $divV = 0$. Then the above equation reduces to

$$\lambda = -\left[A + \frac{B}{2n+1}\right]. \tag{27}$$

Thus it leads to the following corollary:

Corollary 3.2. *The Ricci soliton (g, V, λ) in a ϕ-pseudo projectively flat generalized Sasakian space form is shrinking.*

4. Quasi-Pseudo Projectively Flat Generalized Sasakian Space Forms

Let us consider a quasi-pseudo projectively flat generalized Sasakian space form, then we have

$$g(\tilde{P}(X,Y)Z,\phi W) = 0, \tag{28}$$

for every vector fields $X, Y, Z, W \in TM^{2n+1}$, where $g(\tilde{P}(X,Y)Z,\phi W) = \tilde{P}(X,Y,Z,\phi W)$.
By considering (2) in (28), we get

$$\begin{aligned}\tilde{P}(X,Y,Z,\phi W) =& aR(X,Y,Z,\phi W) + b\{S(Y,Z)g(X,\phi W) - \\ & S(X,Z)g(Y,\phi W)\} - \frac{r}{2n+1}\left(\frac{a}{2n}+b\right) \\ & \{g(Y,Z)g(X,\phi W) - g(X,Z)g(Y,\phi W)\}.\end{aligned} \tag{29}$$

Let $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold. Taking $Y = Z = e_i$ and summing over i, $i = 1, 2, 3, ..., 2n+1$, we obtain

$$S(X,\phi W) = \frac{r(a+2nb) - br(2n+1)}{(1-b)(2n+1)} g(X,\phi W). \tag{30}$$

Replacing X by ϕX in (30), gives

$$S(X,W) = Ag(X,W) + B\eta(X)\eta(W), \tag{31}$$

$$\text{where, } A = \frac{r(a-b)}{(1-b)(2n+1)},$$

$$B = 2n(f_1 - f_3) - \frac{r(a-b)}{(1-b)(2n+1)}.$$

This give on to the following result:

Theorem 4.1. *A $(2n+1)$-dimensional quasi-pseudo projectively flat generalized Sasakian space form M^{2n+1} with $b \neq 1$ is an η-Einstein manifold.*

By using (31) in (19), we get

$$L_v g(X,W) + 2[A+\lambda]g(X,W) + 2B\eta(X)\eta(W) = 0. \tag{32}$$

Contracting above equation, yields

$$divV + 2(2n+1)[A+\lambda] + 2B = 0. \tag{33}$$

If V is solenoidal i.e., $divV = 0$, then the above equation leads to

$$\lambda = -[\frac{2nr(a-b) + 2n(2n+1)(1-b)[f_1 - f_3]}{(1-b)(2n+1)^2}]. \tag{34}$$

Hence we obtain the following corollary:

Corollary 4.2. *The Ricci soliton (g, V, λ) in a quasi-pseudo projectively flat generalized Sasakian space form is shrinking, steady and expanding depends on the scalar curvature r is $> 0, = 0, <= 0$ respectively, provided $f_1 - f_3 = 0$.*

5. Pseudo Projective Pseudo-Symmetric Generalized Sasakian Space Form

Definition 5.1. *A $(2n+1)$-dimensional generalized Sasakian space form M^{2n+1} is said to be pseudo projective pseudo-symmetric if $R\cdot\tilde{P} = L_{\tilde{P}}\, Q(g,\tilde{P})$. i.e.,*

$$(R(\xi,Y)\tilde{P})(U,V)W = L_{\tilde{P}}[((\xi\wedge Y)\tilde{P})(U,V)W]. \tag{35}$$

Firstly, left hand side of (35) gives

$$\begin{aligned}(f_1-f_3)[\tilde{P}(U,V,W,Y)\xi - \eta(\tilde{P}(U,V)W)Y + \eta(U)\tilde{P}(Y,V)W \\ - g(Y,U)\tilde{P}(\xi,V)W - g(Y,V)\tilde{P}(U,\xi)W + \eta(V)\tilde{P}(U,Y)W \\ - g(Y,W)\tilde{P}(U,V)\xi + \eta(W)\tilde{P}(U,V)Y].\end{aligned} \tag{36}$$

Next, right hand side of (35) gives

$$\begin{aligned}L_{\tilde{P}}[\tilde{P}(U,V,W,Y)\xi - \eta(\tilde{P}(U,V)W)Y + \eta(U)\tilde{P}(Y,V)W \\ - g(Y,U)\tilde{P}(\xi,V)W - g(Y,V)\tilde{P}(U,\xi)W + \eta(V)\tilde{P}(U,Y)W \\ - g(Y,W)\tilde{P}(U,V)\xi + \eta(W)\tilde{P}(U,V)Y].\end{aligned} \tag{37}$$

Using (36) and (37) in (35) and taking inner product with ξ, we obtain

$$\begin{aligned}[L_{\tilde{P}} - (f_1-f_3)][\tilde{P}(U,V,W,Y) - \eta(\tilde{P}(U,V)W)\eta(Y) \\ + \eta(U)\eta(\tilde{P}(Y,V)W) + \eta(V)\eta(\tilde{P}(U,Y)W) + \eta(W)\eta(\tilde{P}(U,V)Y) \\ - g(Y,U)\eta(\tilde{P}(\xi,V)W) - g(Y,V)\eta(\tilde{P}(U,\xi)W) \\ - g(Y,W)\eta(\tilde{P}(U,V)\xi)] = 0.\end{aligned} \tag{38}$$

Which gives either $L_{\tilde{P}} = f_1 - f_3$ or

$$\begin{aligned}\tilde{P}(U,V,W,Y) - \eta(\tilde{P}(U,V)W)\eta(Y) + \eta(U)\eta(\tilde{P}(Y,V)W) \\ + \eta(V)\eta(\tilde{P}(U,Y)W) + \eta(W)\eta(\tilde{P}(U,V)Y) - g(Y,U)\eta(\tilde{P}(\xi,V)W) \\ - g(Y,V)\eta(\tilde{P}(U,\xi)W) - g(Y,W)\eta(\tilde{P}(U,V)\xi)] = 0.\end{aligned} \tag{39}$$

Plugging $U = Y = e_i$ in (39) and then using (17), we get

$$S(V,W) = Ag(X,W) + B\eta(V)\eta(W), \tag{40}$$

where, $A = 2n(f_1 - f_3)$

$$B = 2n(1-\frac{a}{b})(f_1-f_3) - \frac{1}{a}(2n+1) - \frac{r(a+4nb+b)}{a(2n+1)}.$$

Thus we can state the following:

Theorem 5.1. *A $(2n+1)$-dimensional pseudo projective pseudo-symmetric generalized Sasakian space form is an η-Einstein manifold.*

By virtue of (40) in (19), we get

$$L_v g(X,W) + 2[A+\lambda]g(X,W) + 2B\eta(X)\eta(W) = 0, \tag{41}$$

Again putting $X = W = e_i$ in (41), yields

$$divV + 2(2n+1)[A+\lambda] + 2B = 0. \tag{42}$$

If V is solenoidal, $divV = 0$. Then the above equation reduces to

(43)
$$\lambda = 2n\left[\frac{(2n+1)^2 + r(a+(4n+1)b)}{a(2n+1)^2}\right] - \left[1 + \frac{(b-a)}{b(2n+1)}\right] 2n(f_1 - f_3).$$

Hence we gain the following:

Corollary 5.2. *The Ricci soliton (g, V, λ) in a pseudo projective pseudo-symmetric generalized Sasakian space form is shrinking, steady and expanding depends on the scalar curvature r is $> 0, = 0, <= 0$ respectively, provided $f_1 - f_3 = 0$.*

6. Generalized Sasakian Space Form Satisfying $\tilde{P} \cdot \tilde{P} = 0$

Let us consider an generalized Sasakian space form satisfying $(\tilde{P}(\xi, X) \cdot \tilde{P})(Y, Z)U = 0$, from which it follows that

(44)
$$\tilde{P}(\xi, X)\tilde{P}(Y, Z)U - \tilde{P}(\tilde{P}(\xi, X)Y, Z)U - \tilde{P}(Y, \tilde{P}(\xi, X)Z)U - \tilde{P}(Y, Z)\tilde{P}(\xi, X)U = 0.$$

By taking an account of (1), (2), (11), (14)-(17) in (44) and then by considering $U = Z = \xi$, we get

(45)
$$\left[(a+2nb)(f_1 - f_3) - \frac{r}{2n+1}\left(\frac{a}{2n} + b\right)\right]^2 [\eta(Y)X - g(X, Y)\xi] = 0,$$

which implies that

(46)
$$(a+2nb)(f_1 - f_3) - \frac{r}{2n+1}\left(\frac{a}{2n} + b\right) = 0.$$

Now it can be easily obtained from the above expression that

(47)
$$r = 2n(2n+1)(f_1 - f_3).$$

By comparing (13) and (47) follows that

(48)
$$3f_2 + (2n-1)f_3 = 0.$$

Considering (48) in (11), we obtain

(49)
$$S(X, Y) = 2n(f_1 - f_3)g(X, Y).$$

Hence we can state the following:

Theorem 6.1. *A $(2n+1)$-dimensional generalized Sasakian space form admitting $\tilde{P} \cdot \tilde{P} = 0$ is an Einstein manifold with scalar curvature $r = 2n(2n+1)(f_1 - f_3)$.*

By using (49) in (19), we get

(50)
$$L_v g(X, Y) + 2[2n(f_1 - f_3) + \lambda]g(X, Y) = 0.$$

Plugging $X = Y = e_i$ in (50), yields

(51)
$$divV + 2(2n+1)[2n(f_1 - f_3) + \lambda] = 0.$$

If V is a solenoidal, $divV = 0$, then the above equation reduced to

(52)
$$\lambda = -2n(f_1 - f_3).$$

Hence we can conclude the following corollary:

Corollary 6.2. *The Ricci soliton (g, V, λ) in a generalized Sasakian space form admitting $\tilde{P} \cdot \tilde{P} = 0$ is shrinking.*

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