Advanced Studies in Contemporary Mathematics
33 (2023), No. 3, pp. 203 - 212

www.jangjeon.or.kr
http://dx.doi.org/10.17777/ascm2023.33.3.203

SOME IDENTITIES OF THE GENERALIZED CHANGHEE-GENOCCHI POLYNOMIALS AND NUMBERS

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ABSTRACT. Recently, Kim-Jeong-Rim introduced Changhee-Genocchi polynomials and investigated explicit identities involving Gamma and Beta functions. In addition, Kwon-Kim-Park studied degenerate Changhee-Genocchi polynomials as degenerate version of those polynomials. The aim of this paper is to introduce generalized Changhee-Genocchi polynomials as general version of Changhee and Genocchi polynomials and to obtain new properties and identities using the Stirling numbers of the first kind, the Stirling numbers of the second kind, the Stirling polynomials of the second kind, Changhee polynomials, Changhee-Genocchi polynomials and Euler-Genocchi polynomials.

1. INTRODUCTION

Previously, many researchers have studied various special polynomials and numbers. In addition, they have obtained many impotrant identities by creating generating functions that represent two polynomials functions simultaniously and explored their properties. In particular, Belbachir and Hadj-Brahim introduced the Euler-Genocchi polynomials and derived some identitied involving these polynomial [2]. Goubi studied the generalized Euler-Genocchi polynomials in [7]. Recently, Kim-Jeong-Rim introduced the Changhee-Genocchi polynomials and investigated some explicit identities utilizing Gamma and Beta functions [6]. T. Kim, D.S.Kim and others have studied different versions of Changhee polynomials and relevant objects - Bernoulli, Whitney, Dowling polynomials, harmonic and hyperharmonic numbers, λ-linear functionals [1, 3, 4, 5, 8-19] and obtained many interesting results.

In this paper, we introduce the generalized Changhee-Genocchi polynomials and study their main properties. In particular, we derive new relations involving these polynomials and the Stirling numbers of the first kind, the Stirling numbers of the second kind, the Stirling polynomials of the second kind, Changhee polynomials, Changhee-Genocchi polynomials and Euler-Genocchi polynomials.

The outline of this paper is as follows. In section 1, we recall the definition of some polynomials and numbers: the Stirling numbers of the first and second kinds, the Stirling polynomials of the second kind, the Euler polynomials, the Bernoilli polynomials of the second kind, the Genocchi polynomials, the Euler-Genocchi polynomials, the Changhee polynomials and the Changhee-Genocchi polynomials. In section 2, we introduce the generalized Changhee-Genocchi polynomials using generating function related to the Changhee and Genocchi polynomials and represent the main results of this research.

In Theorem 1 and Theorem 6, we obtained the relation of the generalized Changhee-Genocchi polynomials $I_n^{(r)}(x)$ with the Stirling numbers of the first kind, the Changhee polynomials and the Changhee-Genocchi polynomials.

In Theorem 2, we observe certain correlations of the generalized Changhee-Genocchi polynomials $I_n^{(r)}(x)$ with the Stirling numbers of the second kind and the Euler polynomials.

Key words and phrases. Changhee-Genocchi polynomials; Generalized Changhee-Genocchi numbers and polynomials.

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In Theorems 4 and 5, we got the relationship of the generalized Changhee-Genocchi polynomials $I_n^{(r)}(x)$ and the Euler-Genocchi polynomials.

In Theorems 7, 8 and 9, we obtained some new properties and identities involving $I_n^{(r)}(x)$.

In Theorem 10, we observe the relationship between the Euler-Genocchi numbers, $I_n^{(r)}(-x)$ and the Stirling polynomials of second kind.

In Theorem 11, we represent the relation of the generalized Changhee-Genocchi polynomials $I_n^{(r)}(x)$, the Bernoulli polynomials of the second kind and the Changhee polynomials.

The Stirling numbers of the first kind are defined by

$$\frac{1}{k!}(log(1+t))^k = \sum_{n=k}^{\infty} S_1(n,k)\frac{t^n}{n!}, \quad (\text{see}[1,6,12]).$$

and

$$(x)_n = \sum_{k=0}^{n} S_1(n,k)x^k, \quad (\text{see}[1,6,12]).$$

The Stirling numbers of the second kind are defined by

$$\frac{1}{k!}(e^t-1)^k = \sum_{n=k}^{\infty} S_2(n,k)\frac{t^n}{n!}, \quad (\text{see}[1,6,12,13]).$$

and

$$x^n = \sum_{k=0}^{n} S_2(n,k)(x)_k, \quad (\text{see}[1,6,12,13]).$$

The Stirling polynomials of the second kind are defined by

$$\frac{1}{k!}(e^t-1)^k e^{xt} = \sum_{n=k}^{\infty} S_2(n,k \mid x)\frac{t^n}{n!}, \quad (\text{see}[13]).$$

It is well known that the Euler polynomials are defined by

$$\frac{2}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!}, \quad (\text{see}[2]).$$

When $x=0$, $E_n = E_n(0)$ are called the Euler numbers.

The Bernoulli polynomials of the second kind are defined by generating function to be

$$\frac{t}{log(1+t)}(1+t)^x = \sum_{n=0}^{\infty} b_n(x)\frac{t^n}{n!}, \quad (\text{see}[10]).$$

The Bernoulli polynomials of the second kind of order r are defined by the generating function as

$$\left(\frac{t}{(log(1+t))}\right)^r (1+t)^x = \sum_{n=0}^{\infty} b_n^{(r)}\frac{t^n}{n!}, \quad (\text{see}[1-13]).$$

The Genocchi polynomials are given by

$$\frac{2t}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} G_n(x)\frac{t^n}{n!}, \quad (\text{see}[3,8,10]). \tag{1}$$

When x=0, $G_n = G_n(0)$ are called the Genocchi numbers.

The Euler-Genocchi polynomials are defined by the generating function to be

$$(2) \qquad \frac{2t^r}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}A_n^{(r)}(x)\frac{t^n}{n!}, \quad (\text{see}[3]).$$

When $x=0$, $A_n^{(r)}=A_n^{(r)}(0)$ are called the Euler-Genocchi numbers.
The Changhee polynomials are defined by

$$\frac{2(1+t)^x}{2+t}=\sum_{n=0}^{\infty}Ch_n(x)\frac{t^n}{n!}, \quad (\text{see}[2,4,5,6]).$$

The Changhee-Genocchi polynomials are defined by

$$(3) \qquad \frac{2log(1+t)}{2+t}(1+t)^x=\sum_{n=0}^{\infty}CG_n(x)\frac{t^n}{n!}, \quad (\text{see}[1,4,6]).$$

By replacing t by e^t-1 in (3), we get

$$(4) \qquad \begin{aligned}\frac{2t}{e^t+1}e^{xt}&=\sum_{k=0}^{\infty}CG_k(x)\frac{1}{k!}(e^t-1)^k\\&=\sum_{k=0}^{\infty}CG_k(x)\sum_{n=k}^{\infty}S_2(n,k)\frac{t^n}{n!}\\&=\sum_{n=0}^{\infty}\sum_{k=0}^{n}CG_k(x)S_2(n,k)\frac{t^n}{n!}.\end{aligned}$$

Thus, we obtain

$$G_n(x)=\sum_{k=0}^{n}CG_k(x)S_2(n,k).$$

2. Generalized Changhee-Genocchi numbers and polynomials

For $r\in\mathbb{Z}$ with $r\geq 0$, we consider the generalized Changhee-Genocchi polynomials which are given by

$$(5) \qquad \frac{2(log(1+t))^r}{2+t}(1+t)^x=\sum_{n=0}^{\infty}I_n^{(r)}(x)\frac{t^n}{n!}.$$

Note that $I_0^{(r)}(x)=\cdots=I_{r-1}^{(r)}(x)=0$.
When $x=0$, $I_n^{(r)}=I_n^{(r)}(0)$ are called the generalized Changhee-Genocchi numbers.
Putting particular values for r in (5), we get special types of the generalized Changhee-Genocchi polynomials:
when $r=0$, $I_n^{(0)}(x)=Ch_n(x)$ and when $r=1$, $I_n^{(1)}(x)=CG_n(x)$.

From (5), we get

$$\begin{aligned}
(6)\qquad \sum_{n=0}^{\infty} I_n^{(r)}(x)\frac{t^n}{n!} &= \frac{2(log(1+t))^r}{2+t}(1+t)^x \\
&= \frac{2r!}{2+t}(1+t)^x\frac{1}{r!}(log(1+t))^r \\
&= r!\sum_{l=0}^{\infty} Ch_l(x)\frac{t^l}{l!}\sum_{m=r}^{\infty} S_1(m,r)\frac{t^m}{m!} \\
&= r!\sum_{n=r}\sum_{m=r}^{n}\binom{n}{m}S_1(m,r)Ch_{n-m}(x)\frac{t^n}{n!}.
\end{aligned}$$

Therefore, by comparing coefficients in both sides of (6), we obtain the following theorem.

Theorem 1. *For* $n \geq 0$

$$r!\sum_{m=r}^{n}\binom{n}{m}S_1(m,r)Ch_{n-m}(x) = \begin{cases} I_n^{(r)}(x), & (if\, n \geq r), \\ 0, & (if\, n < r). \end{cases}$$

Replacing t by $e^t - 1$ in (5), yields

$$\begin{aligned}
(7)\qquad \sum_{n=0}^{\infty}\sum_{k=0}^{n} I_k^{(r)}(x)S_2(n,k)\frac{t^n}{n!} &= t^r\sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} \\
&= \sum_{n=r}^{\infty} E_{n-r}(x)\frac{t^n}{(n-r)!} \\
&= \sum_{n=r}^{\infty} E_{n-r}(x)\frac{n!}{(n-r)!}\frac{t^n}{n!} \\
&= \sum_{n=r}^{\infty} E_{n-r}(x)\frac{r!n(n-1)\cdots(n-r+1)}{r!}\frac{t^n}{n!} \\
&= r!\sum_{n=r}^{\infty}\binom{n}{r}E_{n-r}(x)\frac{t^n}{n!}.
\end{aligned}$$

Therefore, by comparing coefficients in both sides of (7), we arrive at the following result.

Theorem 2. *For* $n \geq 0$*, we have*

$$\sum_{k=0}^{n} I_k^{(r)}(x)S_2(n,k) = \begin{cases} r!\binom{n}{r}E_{n-r}(x), & (if\, n \geq r), \\ 0, & (if\, n < r). \end{cases}$$

By replacing t by e^t-1 in (5), we get

$$\begin{aligned}\sum_{n=0}^{\infty}\sum_{k=0}^{n} I_k^{(r)}(x)S_2(n,k)\frac{t^n}{n!} &= t^{r-1}\sum_{n=0}^{\infty} G_n(x)\frac{t^n}{n!}\\ &= \sum_{n=r}^{\infty} G_{n-r+1}(x)\frac{n!}{(n-r+1)!}\frac{t^n}{n!}\\ &= \sum_{n=r}^{\infty} G_{n-r+1}(x)\frac{(r-1)!n!}{(r-1)!(n-r+1)!}\frac{t^n}{n!}\\ &= (r-1)!\sum_{n=r}^{\infty}\binom{n}{r-1}G_{n-r+1}(x)\frac{t^n}{n!}.\end{aligned} \tag{8}$$

Thus, by comparing coefficients in both sides of (8), we get the following theorem.

Theorem 3. *For $n \geq 0$, we have*

$$\sum_{k=0}^{n} I_k^{(r)}(x)S_2(n,k) = \begin{cases}(r-1)!\binom{n}{r-1}G_{n-r+1}(x), & (if\, n \geq r),\\ 0, & (if\, 0 \leq n < r).\end{cases}$$

By replacing t by $log(1+t)$ in (2) and (5), we get

$$\begin{aligned}\sum_{k=0}^{\infty} A_k^{(r)}(x)\sum_{n=k}^{\infty} S_1(n,k)\frac{t^n}{n!} &= \frac{2(log(1+t))^r}{2+t}(1+t)^x\\ &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A_k^{(r)}(x)S_1(n,k)\frac{t^n}{n!}\\ &= \sum_{n=0}^{\infty} I_n^{(r)}(x)\frac{t^n}{n!}.\end{aligned} \tag{9}$$

Therefore, by comparing coefficients in both sides of (9), we get the following result.

Theorem 4. *For $n \geq 0$, we have*

$$I_n^{(r)}(x) = \sum_{k=0}^{n} A_k^{(r)}(x)S_1(n,k).$$

By replacing t by e^t-1 in (5), we get

$$\begin{aligned}\sum_{n=0}^{\infty}\sum_{k=0}^{n} I_k^{(r)}(x)S_2(n,k)\frac{t^n}{n!} &= \frac{2t^r}{e^t+1}e^{tx}\\ &= \sum_{n=0}^{\infty} A_n^{(r)}(x)\frac{t^n}{n!}.\end{aligned} \tag{10}$$

Thus, we have the following theorem.

Theorem 5. *For $n \geq 0$, we have*

$$A_n^{(r)} = \sum_{k=0}^{n} I_k^{(r)}S_2(n,k).$$

By (5), we have

$$\sum_{n=r-1}^{\infty} I_n^{(r)}(x)\frac{t^n}{n!} = \frac{2log(1+t)}{2+t}(1+t)^x(r-1)!\frac{(log(1+t))^{r-1}}{(r-1)!} \tag{11}$$
$$= (r-1)!\sum_{l=0}^{\infty} CG_l(x)\frac{t^l}{l!}\sum_{m=r-1}^{\infty} S_1(m,r-1)\frac{t^m}{m!}$$
$$= (r-1)!\sum_{n=r-1}^{\infty}\sum_{m=r-1}^{n} S_1(m,r-1)CG_{n-m}(x)\frac{n!}{(n-m)!m!}\frac{t^n}{n!}$$
$$= (r-1)!\sum_{n=r}^{\infty}\sum_{m=r}^{n}\binom{n}{m} S_1(m,r-1)CG_{n-m}(x)\frac{t^n}{n!}.$$

Thus, by comparing coefficients in both sides of (11), we get the following theorem.

Theorem 6. *For $n \geq 0$, we have*

$$(r-1)!\sum_{m=r}^{n}\binom{n}{m}S_1(m,r-1)CG_{n-m}(x) = \begin{cases} I_n^{(r)}(x), & (if\, n \geq r), \\ 0, & (if\, n < r). \end{cases}$$

By (5), we have

$$(2+t)\sum_{n=0}^{\infty} I_n^{(r)}(x)\frac{t^n}{n!} = 2r!\sum_{m=r}^{\infty} S_1(m,r)\frac{t^m}{m!}\sum_{l=0}^{\infty}\binom{x}{l}t^l \tag{12}$$
$$= 2r!\sum_{n=0}^{\infty}\sum_{l=r}^{n}\binom{x}{l}S_1(m,r)\frac{n!}{(n-l)!}\frac{t^n}{n!}$$
$$= 2r!\sum_{n=0}^{\infty}\sum_{l=r}^{n}(n)_l\binom{x}{l}S_1(n-l,r)\frac{t^n}{n!}.$$

On the other hand, we see that

$$(2+t)\sum_{n=0}^{\infty} I_n^{(r)}(x)\frac{t^n}{n!} = \sum_{n=0}^{\infty} 2I_n^{(r)}(x)\frac{t^n}{n!} + \sum_{n=1}^{\infty} nI_{n-1}^{(r)}(x)\frac{t^n}{n!} \tag{13}$$
$$= \sum_{n=1}^{\infty}(2I_n^{(r)}(x) + nI_{n-1}^{(r)}(x))\frac{t^n}{n!}.$$

Therefore, by comparing coefficients in both sides of (12) and (13), we arrive at the following result.

Theorem 7. *For $n \geq 0$, we note that*

$$2r!\sum_{l=r}^{n}(n)_l\binom{x}{l}S_1(n-l,r) = \begin{cases} 2I_n^{(r)}(x) + nI_{n-1}^{(r)}(x), & (if\, n \geq r), \\ 0, & (if\, n < r). \end{cases}$$

By (5), we observe that

$$t(2+t)\sum_{n=0}^{\infty} I_n^{(r)}\frac{t^n}{n!} = 2(log(1+t))^{r+1}\frac{t}{log(1+t)}(1+t)^x.$$

On the other hand, we see that

$$t(2+t)\sum_{n=0}^{\infty}I_n^{(r)}=2\sum_{n=1}^{\infty}nI_n^{(r)}(x)\frac{t^{n+1}}{(n+1)!}+\sum_{n=1}^{\infty}n(n+1)I_{n-1}^{(r)}(x)\frac{t^{n+1}}{(n+1)!}$$
$$=\sum_{n=1}^{\infty}(2(n+1)I_n^{(r)}(x)+n(n+1)I_{n-1}^{(r)}(x))\frac{t^{n+1}}{(n+1)!}. \tag{14}$$

Note that

(15)

$$2(log(1+t))^{r+1}\frac{t}{log(1+t)}(1+t)^x=2(r+1)!\frac{(log(1+t))^{r+1}}{(r+1)!}\sum_{m=0}^{\infty}b_m(x)\frac{t^m}{m!}$$
$$=2(r+1)!\sum_{l=r+1}^{\infty}S_1(l,r+1)\frac{t^l}{l!}\sum_{m=0}^{\infty}b_m(x)\frac{t^m}{m!}$$
$$=2(r+1)!\sum_{n=r+1}^{\infty}\sum_{l=r+1}^{n}S_1(l,r+1)b_{n-l}(x)\frac{n!}{l!(n-l)!}\frac{t^n}{n!}$$
$$=2(r+1)!\sum_{n=r}^{\infty}\sum_{l=r+1}^{n+1}S_1(l,r+1)b_{n-l+1}(x)\frac{(n+1)!}{l!(n-l+1)!}\frac{t^{n+1}}{(n+1)!}$$
$$=2(r+1)!\sum_{n=r}^{\infty}\sum_{l=r+1}^{n+1}\binom{n+1}{l}S_1(l,r+1)b_{n-l+1}(x)\frac{t^{n+1}}{(n+1)!}.$$

Thus, by comparing corresponding coefficients in (14) and (15), we get the following theorem.

Theorem 8. *For $n\geq 0$, we have*

$$2(r+1)!\sum_{l=r+1}^{n}\binom{n+1}{l}S_1(l,r+1)b_{n-l+1}(x)=\begin{cases}2nI_{n-1}^{(r)}(x)+n(n-1)I_{n-2}^{(r)}(x), & (if\, n\geq r+1),\\ 0, & (if\, n<r+1).\end{cases}$$

From (5), we observe that

$$\sum_{n=0}^{\infty}I_n^{(r)}(x)\frac{t^n}{n!}=\frac{2(log(1+t))^r}{2+t}(1+t)^x \tag{16}$$
$$=\left(\sum_{l=1}^{\infty}\frac{(-1)^{l-1}t^l}{l!}\right)\sum_{m=0}^{\infty}ch_m(x)\frac{t^m}{m!}$$
$$=\left(\sum_{l_1=1}^{\infty}\frac{(-1)^{l_1-1}t^{l_1}}{l!}\right)\cdots\left(\sum_{l_r=1}^{\infty}\frac{(-1)^{l_r-1}t^{l_r}}{l_r!}\right)\sum_{m=0}^{\infty}ch_m(x)\frac{t^m}{m!}$$
$$=\sum_{k=r}^{\infty}\sum_{l_1+\cdots+l_r=k}\binom{k}{l_1\cdots l_r}(-1)^{k-r}\frac{t^k}{k!}\sum_{m=0}^{\infty}ch_m(x)\frac{t^m}{m!}$$
$$=\sum_{n=r}^{\infty}\sum_{k=r}^{n}\sum_{l_1+\cdots+l_r=k}\binom{k}{l_1\cdots l_r}(-1)^{k-r}ch_{n-k}(x)\frac{n!}{k!(n-k)!}\frac{t^n}{n!}$$
$$=\sum_{n=r}^{\infty}\sum_{k=r}^{n}\sum_{l_1+\cdots+l_r=k}\binom{k}{l_1\cdots l_r}\binom{n}{k}(-1)^{k-r}ch_{n-k}(x)\frac{t^n}{n!}.$$

Therefore, by comparing coefficients in both sides of (16), we obtain the following result.

Theorem 9. *For $n \geq 0$, we have*

$$\sum_{k=r}^{n} \sum_{l_1+\cdots+l_r=k} \binom{k}{l_1 \cdots l_r} \binom{n}{k} = \begin{cases} I_n^{(r)}(x), & (if\, n \geq r), \\ 0, & (if\, n < r). \end{cases}$$

Replacing x by $-x$ and t by $e^t - 1$ in (5), gives

$$\begin{aligned} \frac{2t^r}{(e^t+1)} &= e^{xt} \sum_{k=0}^{\infty} I_k^{(r)}(-x) \frac{(e^t-1)^k}{k!} \\ &= \sum_{k=0}^{\infty} I_k^{(r)}(-x) \sum_{k=n}^{\infty} S_2(n,k \mid x) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{n} I_k^{(r)}(-x) S_2(n,k \mid x) \frac{t^n}{n!}. \end{aligned} \tag{17}$$

By (1) and (17), we get

$$\sum_{n=0}^{\infty} A_n^{(r)} \frac{t^n}{n!} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} I_k^{(r)}(-x) S_2(n,k \mid x) \frac{t^n}{n!}. \tag{18}$$

Thus, by comparing coefficients in both sides of (18), we obtain the following theorem.

Theorem 10. *For $n \geq 0$, we have*

$$A_n^{(r)} = \sum_{k=0}^{n} I_k^{(r)}(-x) S_2(n,k \mid x) \frac{t^n}{n!}.$$

By (5), we have

$$\frac{t^r}{(log(1+t))^r} \sum_{l=0}^{\infty} I_l^{(r)}(x) \frac{t^l}{l!} = \frac{2t^r}{2+t} (1+t)^x. \tag{19}$$

By (19), we observe that

$$\begin{aligned} \left(\frac{t}{(log(1+t))} \right)^r \sum_{l=0}^{\infty} I_l^{(r)}(x) \frac{t^l}{l!} &= \sum_{m=0}^{\infty} b_m^{(r)} \frac{t^m}{m!} \sum_{l=0}^{\infty} I_l^{(r)}(x) \frac{t^l}{l!} \\ &= \sum_{n=0}^{\infty} \sum_{m=0}^{n} b_m^{(r)} I_{n-m}^{(r)}(x) \frac{n!}{m!(n-m)!} \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \sum_{m=0}^{n} \binom{n}{m} b_m(x)^{(r)} I_{n-m}^{(r)}(x) \frac{t^n}{n!}. \end{aligned} \tag{20}$$

On the other hand, we see that

$$\begin{aligned}\frac{2t^r}{2+t}(1+t)^x &= t^r\sum_{n=0}^{\infty}Ch_n(x)\frac{t^n}{n!}\\ &= t^r\sum_{n=0}^{\infty}Ch_n(x)\frac{t^n}{n!}\\ &= \sum_{n=r}^{\infty}Ch_{n-r}(x)\frac{n!}{(n-r)!}\frac{t^n}{n!}\\ &= \sum_{n=r}^{\infty}(n)_rCh_{n-r}(x)\frac{t^n}{n!}.\end{aligned} \tag{21}$$

Thus, by comparing corresponding coefficients in (20) and (21), we arrive at the following theorem.

Theorem 11. *For $n \geq 0$,we have*

$$\sum_{m=0}^{n}\binom{n}{m}b_m(x)^{(r)}I_{n-m}^{(r)}(x)\frac{t^n}{n!} = \begin{cases}(n)_rCh_{n-r}(x), & (if\, n \geq r)\,,\\ 0, & (if\, 0 \leq n < r)\,.\end{cases}$$

3. Conclusion

Recently, many researchers studied various generalized versions of the special polynomials and numbers by using their generating functions and obtained some interesting results. Motivated by these investigations, in this paper we introduced new type of polynomials - the generalized Changhee-Genocchi polynomials and studied their properties. We derived interesting relations involving these polynomials and the Stirling numbers of the first kind, the Stirling numbers of the second kind, the Stirling polynomials of the second kind, Changhee polynomials, Changhee-Genocchi polynomials and Euler-Genocchi polynomials. Obtained outcomes can have good perspectives for further study of generalized versions of different types of polynomials and their applications in physics, science and engineering as well as in mathematics.

Acknowledgments

This paper is supported by Research Fund of Kwangwoon University in 2023.

References

[1] D.S. Kim, T. Kim, J. Seo, A Note on Changhee Polynomials and Numbers, Adv. Studies Theor. Phys., Vol. 7, no. 20, (2013), pp. 993 - 1003.

[2] H. Belbachir, S. Hadj-Brahim, Some explicit formulas for Euler-Genocchi polynomials, Integers, 19, (2019),♯A28, 14pp.

[3] H.I. Kwon, T. Kim, J.-W. Park, A note on degenerate Changhee-Genocchi polynomials and numbers, Global Journal of Pure and Applied Mathematics. Vol. 12, no. 5, (2016), pp. 4057–4064.

[4] J. Kwon, J.-W. Park, S.J. Yun, Some identities of the poly-Changhee and unipoly Changhee polynomials, Adv. Stud. Contemp. Math. (Kyungshang) Vol. 31, no. 4,(2021), pp. 471-484

[5] L.C. Jang , H. Kim, A note on the modified type 2 degenerate poly-Changhee-Genocchi numbers and polynomials, Adv. Stud. Contemp. Math. (Kyungshang) Vol. 31, no. 3, (2021), pp. 325-333.

[6] B.-M. Kim, J. Jeong, S.-H. Rim, Some explicit identities on Changhee-Genocchi polynomials and numbers. Adv. Difference Equ. 2016, Paper no. 202, 12 pp.

[7] M. Goubi, On a generalized family of Euler-Genocchi polynomials. Integers, 21(2021), paper no. A48, 13pp.

[8] T. Kim, D.S. Kim, Degenerate r-Whitney numbers and degenerate r-Dowling polynomials via boson operator, Adv. in Math. 140, (2022), no. 102394.

[9] T. Kim, D.S. Kim, Some identities involving degenerate Stirling numbers associated with several degenerate polynomials and numbers, Russ. J. Math. Phys. 30 (2023), no. 1, 62–75.

[10] T. Kim, D.S. Kim, Some identities on degenerate r-Stirling numbers via boson operators, Russ. J. Math. Phys. 29 (2022), no. 4, 508–517

[11] T. Kim, D.S. Kim, H.K. Kim, On generalized degenerate Euler–Genocchi polynomials, Applied Mathematics in Science and Engineering, 31, no. 1, (2022), 2159958.

[12] T. Kim, λ-analogue of Stirling numbers of the first kind, Adv. Stud. Contemp. Math. (Kyungshang) 27, no. 3, (2017), pp. 423-429.

[13] T. Kim, D.S. Kim, A note on nonlinear Changhee differential equations, Russ. J. Math. Phys. 23 (2016), no. 1, 88–92.

[14] T.; Kim, D.S. Combinatorial identities involving degenerate harmonic and hyperharmonic numbers. Adv. in Appl. Math. 148 (2023), Paper No. 102535

[15] T.; Kim, D. . A new approach to fully degenerate Bernoulli numbers and polynomials. Filomat 37 (2023), no. 7, 2269–2278.

[16] T. K.; Kim, D. S.; Some Identities Involving Degenerate Stirling Numbers Associated with Several Degenerate Polynomials and Numbers. Russ. J. Math. Phys. 30 (2023), no. 1, 62–75.

[17] T.; Kim, D. S. Degenerate Whitney numbers of first and second kind of Dowling lattices. Russ. J. Math. Phys. 29 (2022), no. 3, 358–377.

[18] Kim, T.; Kim, D. S. Degenerate r-Whitney numbers and degenerate r-Dowling polynomials via boson operators. Adv. in Appl. Math. 140 (2022), Paper No. 102394, 21 pp

[19] Kim, D. S.; Kim, T.; Kwon, J.; Lee, S.-H.; Park, S. On λ-linear functionals arising from p-adic integrals on $\mathbb{Z}_p$. Adv. Difference Equ. 2021, Paper No. 479, 12 pp.

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