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MHD Casson nanofluid with nonuniform heat source/sink and viscous dissipation

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Abstract

The current study is to carry out investigation of Magneto hydrodynamic Casson nanofluids with the combination of "($q^{'''}$) non-uniform heat source/sink" and "viscous dissipation". The equations probes couple of non-linear (ODE) with the help of correspondence transformation, 4^{th} order RK Shooting technique is executed for numerical results for velocity and temperature periphery. The outcomes parameters on velocity and temperature like "(γ)Casson fluid", "(M)magnetic", porosity, "Eckert number(EC)", "heat source/ sink", "Prandtl number', $f^{''}(\eta)$ and Nusselt number on the flow field has been investigated numerically represented graphically.

Keywords: Casson parameter, nanofluids, *viscous dissipation*, magneto hydrodynamic (MHD), "*Non-uniform heat source/sink".

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1 Introduction

Periphery non-Newtonian viscous fluids have impressed many researchers due to its large applications in manufacturing polymers, crystal growing, extrusion flexible sheets, jelly & honey industries etc. Crane[1] initiated boundary layer flow through stretching/shrinking surface. Later investigated originating problem has pulled great interest for many researchers like Cortell[2], Bhattacharyya et.al[3], Mukhopadhyay[4], Rashidi and Pour[5] and Pal[6] etc. "Magnetohydrodynamic (MHD)" periphery "flow due to an exponentially stretching sheet with_radiation effect has been examined by

Ishak[7]". "The characteristics of steady two-dimensional laminar boundary layer flow of a viscous and incompressible fluid past a moving wedge with suction or injection are theoretically investigated by Falkner and Skan[8]". "Heat and mass transfer in a two-dimensional radial flow of a viscous fluid through a saturated porous wedge-shaped region with confining walls is studied by Goyal and Kassoy[9]". Furthermore many researchers [10-22] investigated boundary layer with different parameters & non-Newtonian fluids along with variety of conditions by using numerical or analytical methods. Keeping the importance and several industrial applications in mind, the current study is carried out with the combination of viscous dissipation and heat source/sink for the MHD Casson nanofluid above permeable stretching sheet.

2 Formulation problem

Considering Casson fluid near stagnancy on heated stretching plane $y = 0$, "[3]ref the flow being limited to $y > 0$, where "y coordinate nor-mal surface. Equal and opposite forces are applied along the x-axis (measured along the surface) so that the surface is stretched keeping the origin fixed". The rheology modified equation of in compressible flow of Casson nanofluid defined by (Ref.[4]),

$$\tau_{ij} = \begin{cases} 2(\mu_B + \frac{P_Y}{\sqrt{2\pi}})e_{ij}, & \pi > \pi_c \\ 2(\mu_B + \frac{P_Y}{\sqrt{2\pi}})e_{ij}, & \pi < \pi_c \end{cases}$$

"$\pi = e_{ij}e_{ij}$, and e_{ij} $(i,j)^t h$ component of deformation rate, n is the product of deformation rate with itself, π_c critical value of product based on the non-Newtonian model, μ_B plastic dynamic viscosity of the non-Newtonian fluid, π product component of deformation rate with itself, π_c critical value and P_y is the yield stress of the fluid". The governed periphery equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v_{nf}(1 + \frac{1}{\gamma})(\frac{\partial^2 u}{\partial y^2}) - (\frac{\sigma B_0^2}{\rho_{nf}} + \frac{v_{nf}}{k_0})u \tag{2}$$

$$u\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{v}{\rho C_p}(1 + \frac{1}{\gamma})(\frac{\partial v}{\partial y})^2 + \frac{q^{''}}{\rho C_p} \tag{3}$$

v_{nf} Kinematic viscosity, ρ_f density base fluid, ρ_{nf} represents Casson fluid density, $\gamma = \mu_B \frac{\sqrt{2\pi_c}}{P_y}$ means non-Newtonian (Casson) parameter, σ electrical conductivity of the fluid, C_p represents specific heat, α_{nf} means thermal diffusivity of the fluid, T temperature and k_0 represents permeability of the porous medium. $q^{'''}$ from equation (3) given as

$$\mathrm{q}''' = \frac{\mathrm{Ku_w}(x)}{xv}\left[A^*(T_w - T_\infty)f' + (T - T_\infty)B^*\right] \tag{4}$$

Where A^* and B^* represents heat source/sink respectively. Note that their arises two cases

1. $A^* > 0$, $B^* > 0$ Correlate internal heat production and

2. $A^* < 0$, $B^* < 0$ Correlate internal heat inclusion. Induced magnetic field is negligibly very small should be assumed. Boundary conditions

$$\begin{aligned} u = u_x(x) = bx, \ \ v = 0 \\ T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 \ \ at \ \ y = 0 \\ u \to 0, T \to T_\infty \ \ as \ \ y \to \infty \end{aligned} \tag{5}$$

Where $u_w = bx$, $b > 0$ means stretching sheet velocity and A is constant. We launch suitable similarity variables

$$\psi = x\sqrt{b\nu_f}f(\eta), \ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \ \eta = \sqrt{\frac{b}{\nu_f}}y \tag{6}$$

Where ψ represents stream function defined by

$$u = \frac{\partial\psi}{\partial y} \ \ \text{and} \ \ v = -\frac{\partial\psi}{\partial x}$$

Substituting (6) in mathematical statement (2) and (3), results are obtained,

$$(1 + \frac{1}{\gamma})f^{'''} + \phi_1(ff^{''} - f^{'2} - \frac{M}{\phi_2}f^{'}) - kf^{'} = 0 \tag{7}$$

$$\theta^{''} + (\frac{k_f}{k_{nf}}\phi_3)(\text{Pr}f\theta^{'} + \text{Ec}(1 + \frac{1}{\gamma})f^{''} + (A^*f' + B^*\theta)) = 0 \tag{8}$$

And the conditions in (5) becomes

$$\begin{aligned} f = 0, \ \ f' = 1, \ \ \theta'' = 1 \ \ at \ \ \eta \to 0 \\ f' = 0, \ \ \theta \to 0 \ \ \ as \ \ \eta \to \infty \end{aligned} \tag{9}$$

Here $M = \frac{\sigma B_0^2}{b\rho_f}$ represents magnetic parameter", γ denotes Casson parameter, $k = \frac{\nu_f}{k_0 b}$ represents porosity parameter, ϕ denotes nanoparticle volume fraction $\left(\text{Where } \phi_1 = \frac{\nu_f}{\nu_{nf}}, \ \phi_2 = \frac{\rho_{nf}}{\rho_f} \ and \ \phi_3 = \frac{\alpha_f k_{nf}}{\alpha_{nf} k_f}\right)$, $\text{Pr} = \frac{\nu_f}{\alpha_f}$ is the Pr and tl number and $Ec = \frac{U_w^2}{C_p[T_w - T_\infty]}$ denotes (Eckert number).

C_f Skin friction coefficient & $N_(u_x)$ Nusselt number given by

$$C_f = \frac{\tau_w}{\rho_f u_w^2} \ \ and \ \ Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{10}$$

Where shear stress and heat flux from the surface are given by

$$\tau_w = \left(\mu_B + \frac{Py}{\sqrt{2\pi_c}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0} \ \ and \ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

Substituting the transformations in (6), (10) and (11), we obtain

$$\mathrm{Re}_x^{\frac{1}{2}} C_f = \left(1 + \frac{1}{\gamma}\right) f''(0), \ \ \mathrm{Re}_x^{-\frac{1}{2}} Nu_x = -\theta'(0) \tag{12}$$

Where $\mathrm{Re}_x = \frac{u_w^2}{v}$ Reynolds number.

3 Numerical interpretation

Related ODE (7) & (8) are highly non-linear with the appropriate boundary conditions (9) are calculated mathematically by 4^{th} order RK shooting technique.

$$\frac{df_0}{d\eta} = f_1, \ \ \frac{df_1}{d\eta} = f_2, \ \ \left(1 + \frac{1}{\gamma}\right) \frac{df_2}{d\eta} = \phi_1 \left(f_0 f_2 - f_1^2 - \frac{M}{\phi_2} f_1\right) - k f_1 \tag{13}$$

$$\frac{d\theta_0}{d\eta} = \theta_1, \ \ \frac{d\theta_1}{d\eta} = -\frac{k_f}{k_{nf}} \varphi_3 \left[(\mathrm{Pr}\, \theta_1 f_0) - Ec \left(1 + \frac{1}{\gamma}\right) f_2^2 - (A^* f_1 + B^* \theta_0)\right] \tag{14}$$

Boundary conditions are,

$$f_0(0) = 0, \ f_1(0) = 1, \ \theta_0(0) = 1 \tag{15}$$

$$f_1(\infty) = 0, \ \ \theta_0(\infty) = 0 \tag{16}$$

Here "$f_0(\eta) = f(\eta)$ and $\theta_0(\eta) = \theta(\eta)$" ref[12]". This requires primary ideals $f_2(0)$ and $\theta_1(0)$ and hence suitable estimated values are chosen and afterwards integration is performed. $\Delta\eta = 0.001$ where $\Delta(\eta)$ is step size chosen with an error of tolerance $10^(- 6)$. To solve system of equations using Runge-Kutta fourth order shooting technique. In order to get the desired values one should need 3 more missing initial conditions. On the other hand standards of $f(\eta)$ & $\theta(\eta)$ are known when $\eta \to \infty$, these end settings are used to obtain an unknown initial conditions at $\eta = 0$ by using suitable shooting technique" ref[7].

4 Analysis of the Result

Casson nanofluid flow with viscous dissipation, heat source/sink & magneto hydrodynamics is explored in the current research. The numerical calculation has agreed to perform the $f^{'}(\eta)$(velocity profiles), $\theta(\eta)$, $f^{''}(\eta)$ and $\theta^{'}(\eta)$ for distinct parameters illustrates stream of individuality. The determination of this segment is to scrutinize the physical moment of different root as velocity and temperature profiles which are illustrated in Figs.[1-1].

Figs. [1-3] revels the results of parameter γ, M and k on $f^{'}(\eta)$. Fig. [1] explains fluid velocity falls for advanced ideals of Casson parameter γ due to the converse relation of γ differ stress shows increasing values of γ decrease the yield stress, that is, enhances in the Casson parameter reduces fluids flexibility. Fig.[2] shows the sound effects of m on fluid velocity, as m enhances fluid velocity drops. Because reason behind this is resistive type of force called a "Lorentz force". In Fig.[3] revels that as k (porous parameter) differs that the fluid velocity sharply in the vicinity of stretching pane. Fig. [4 - 12] revels that the effect of γ, M, k, Pr, Ec, ϕ_3, A^* and B^*" on $\theta^{'}(\eta)$ of Casson nanofluid over the horizontal stretching sheet. Fig.[4] depicts the outcomes of γ on $\theta^{'}(\eta)$ .Observed that nanofluid enhances with the increasing values of γ. It is evident from Fig.[5] $\theta^{'}(\eta)$ lifts for variety of M, magnifies, due to existence of magnetic field enhances in the flowing temperature at periphery. Fig.[6], influences k on the temperature profile, and porosity parameter increases as temperature of the fluid increases is noticed Fig. [7] For different values of nanoparticles ϕ improves as the fluid temperature also improves. Fig.[8] illustrates differing of Pr on temperature distribution of the fluid, as Pr enhances the temperature profile decreases.

From fig (9) demonstrates , viscous dissipation enhances as Ec raises the fluid temperature. Figs. [10 & 11] illustrates, As A^* & B^* raises there will enhances in temperature distribution at periphery. Commonly, positive values acts as heat power house and negative values acts as heat exhaustion of non-uniform heat source at periphery. Fig.[12] displays that for increase in nanoparticle's volume fraction ϕ, skin friction coefficient grow up where as local Nusselt number in figure[13 & 14] also shows the same effect for an increase in the Pr and Ec.

5 Concluding remarks

Current study explored velocity and heat transferred predictable two dimensional flows based on Casson nanofluid above a porous stretching shell with outcomes of irregular source/sink. Differing non bulk governing parameters results velocity distribution is discussed diagrammatically. The following are the effects observed.

- As velocity profile decreases eventually Casson parameter increases.
- Increment in skin friction $-f^{''}(0)$ enlarges ϕ (nanoparticle volume fraction) and [k] porosity parameter k.
- Positive values behave like heat generator & negative values behave like heat absorption in non –uniform heat source/sink at boundary region.
- Ec which cause temperature rise at boundary region

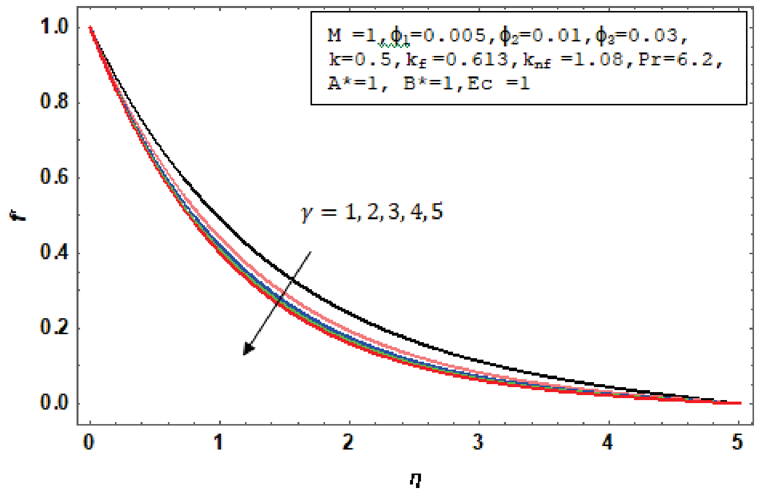

Figure.1. Plotting $f'(\eta)$ for variety of (Casson parameter) γ ,considering (M=1,Pr=6.2, Ec=1 ,k=0, $\varphi_1 = 0.005, \varphi_2 = 0.01, \varphi_3 = 0.03$)

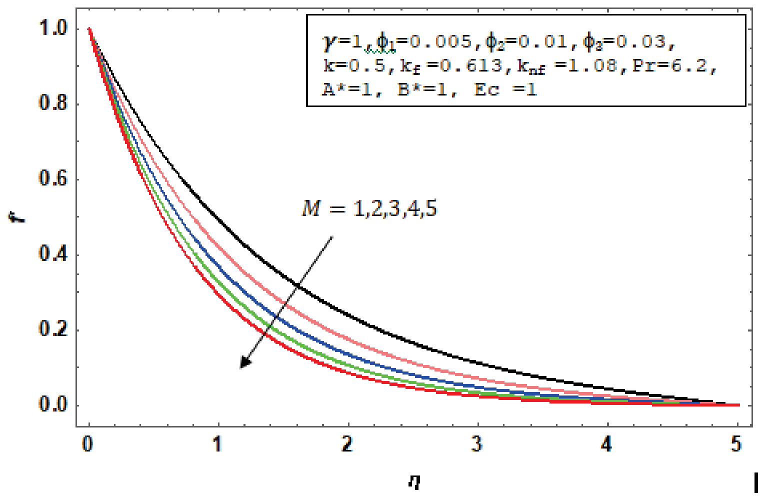

Figure 2. Plotting $f'(\eta)$ for effects of M(magnetic parameter)

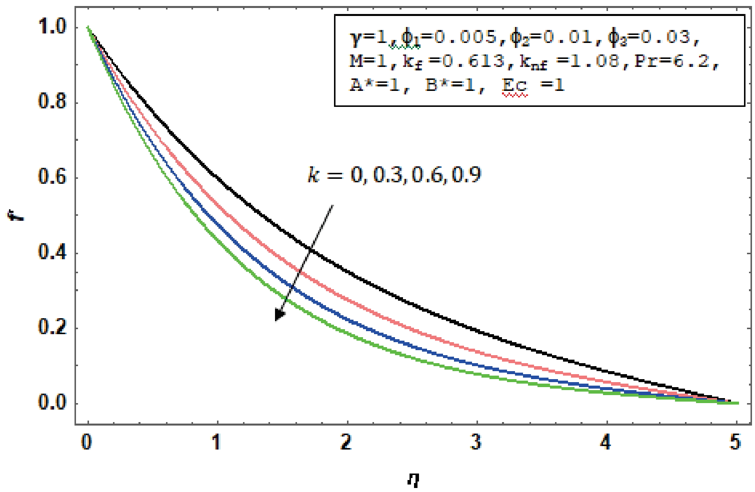

Figure 3.Representseffects of k(porosity parameter) on velocity profile$f'(\eta)$

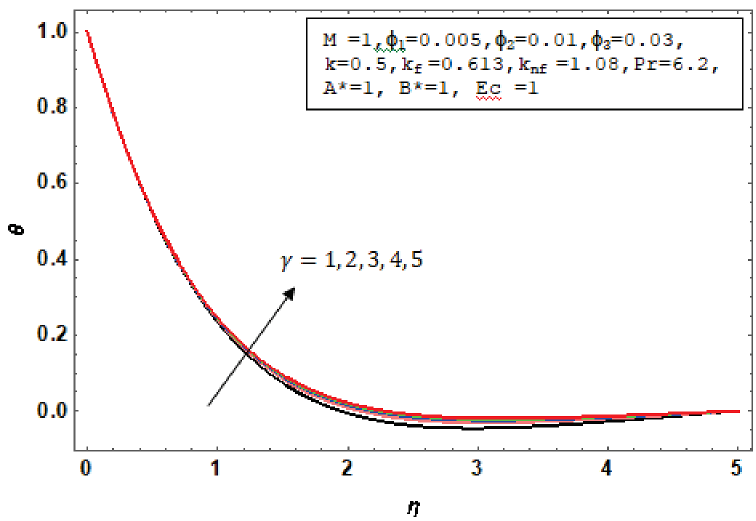

Figure.4. Mapping γ(Casson parameter) on temperature profile$\theta(\eta)$

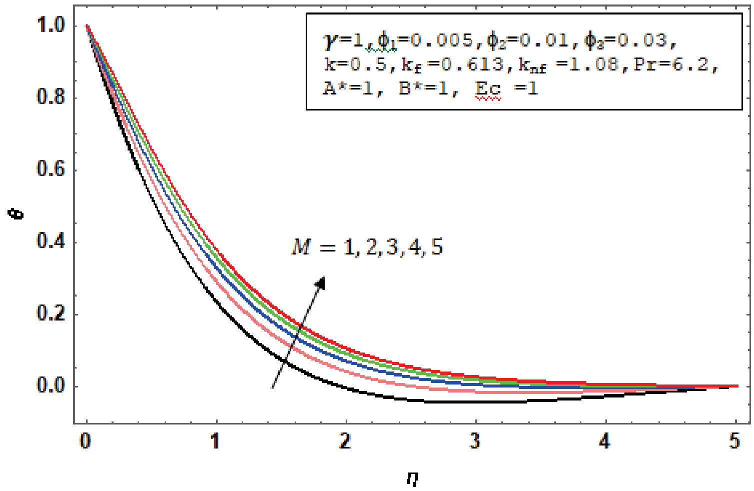

Figure.5. Indicating temperature profile $\theta(\eta)$ for heterogeneous M (magnetic parameter)

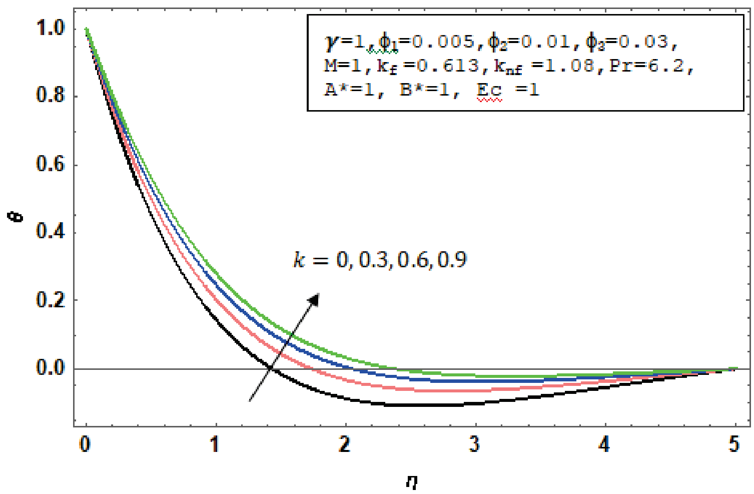

Figure.6. Mapping temperature profile $\theta(\eta)$ a variety of Porosity parameter k

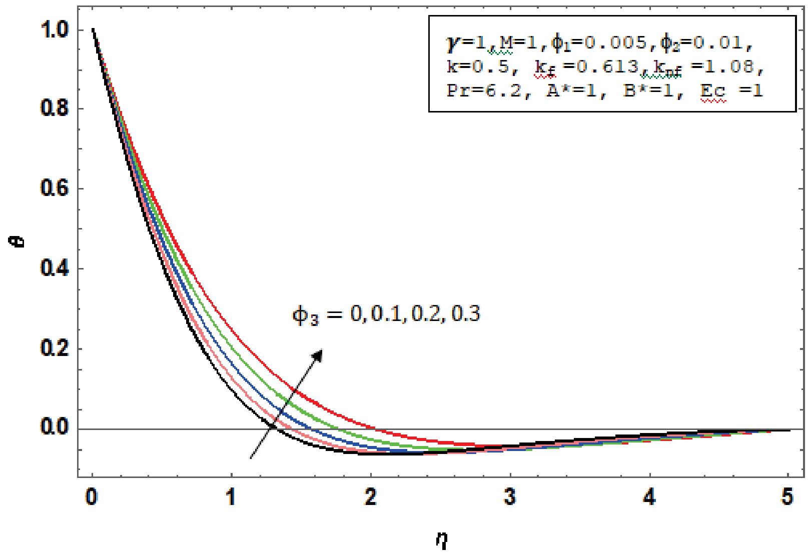

Figure.7. Plotting $\theta(\eta)$ for heterogeneous values of ϕ_3 "(nanoparticle parameter)"

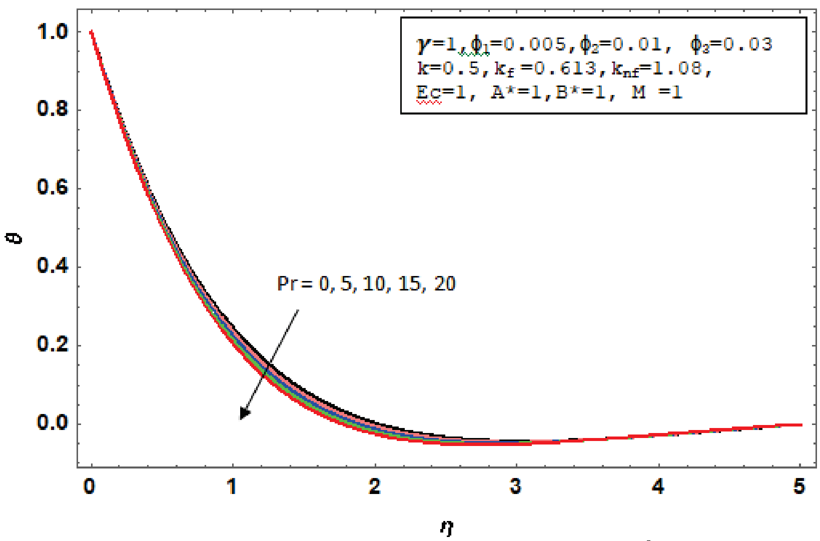

Figure.8. Effects of Pr (Prandtl number) for $\theta(\eta)$

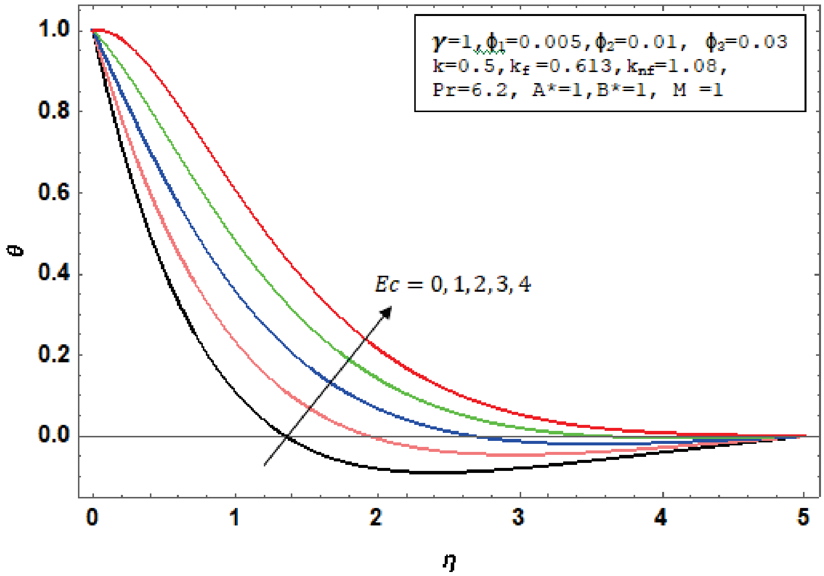

Figure.9. Plotting $\theta(\eta)$ (Temperature profile) for differing(Eckert number) Ec

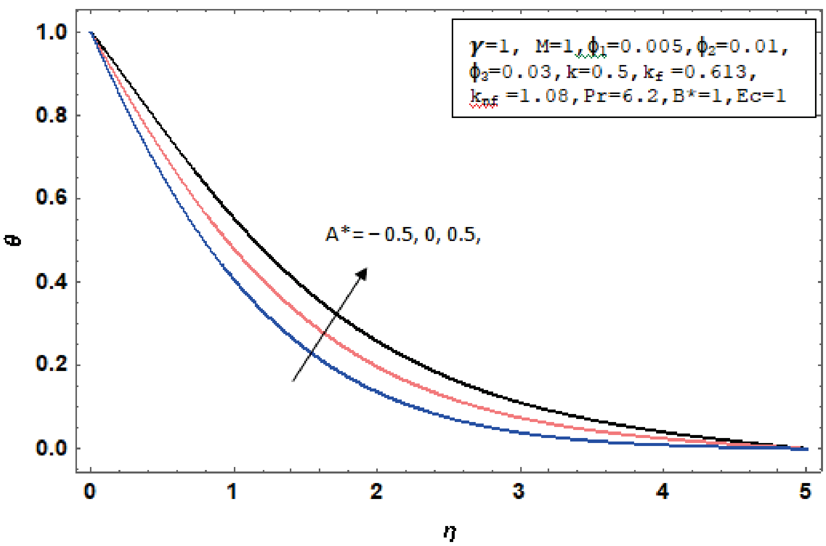

Figure.10.Specialeffects of Velocity profile $\theta(\eta)$ for different ideals of A^*

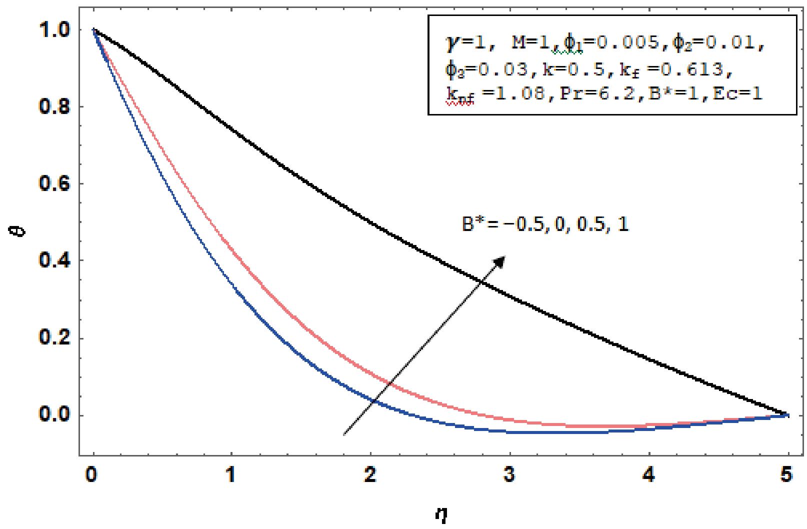

Figure.11.PlottingTemperature profile $\theta(\eta)$ for B^*

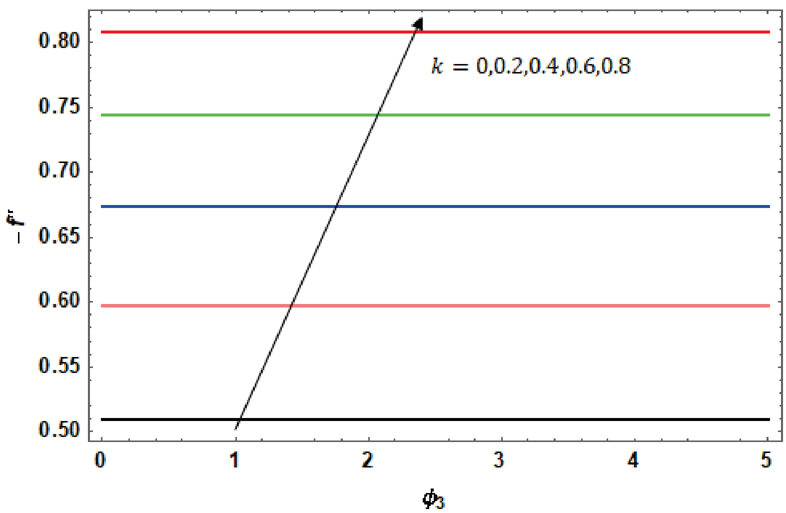

Figure.12. For Several values of k (Porosity parameter) plotting skin friction $-f''$ with nano particles φ_3

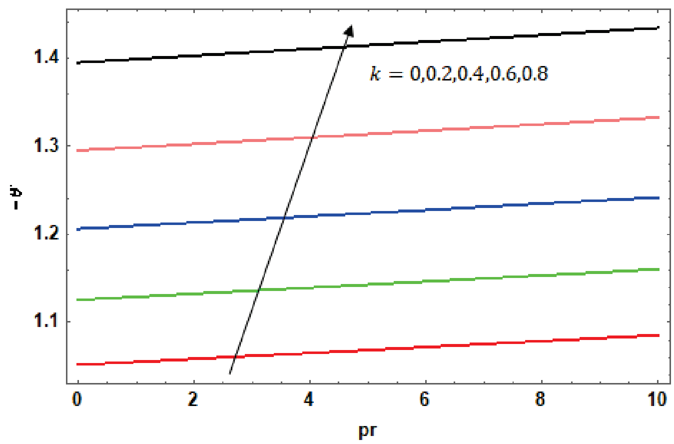

Figure.13. Plotting $-\theta'$with Prandtl number Pr for variety of k

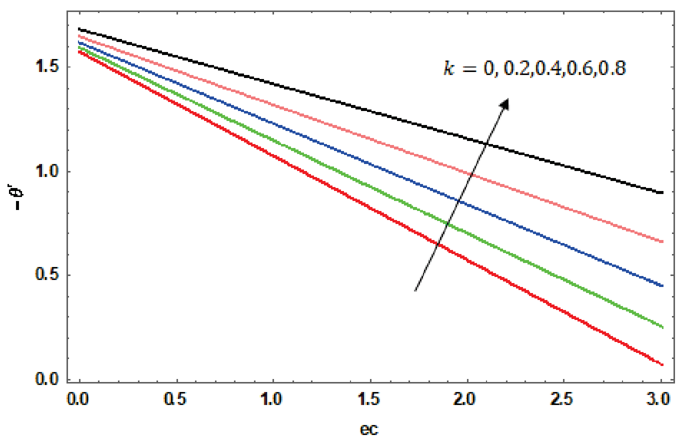

Figure.14. Effects of "porosity parameter k" plotting heat transfer coefficient $-\theta'(0)$ with " Eckert number Ec "

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