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EXTENDED PSEUDO PROJECTIVE CURVATURE TENSOR ON $(LCS)_n$-MANIFOLD

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ABSTRACT. This paper is a survey of certain properties of extended pseudo projective curvature tensor on $(LCS)_n$-manifold. Here first we have studied that extended pseudo projective semi-symmetric and extended pseudo projective pseudo-symmetric $(LCS)_n$-manifold. Moreover, we also describe an $(LCS)_n$-manifold admitting $P^e(\xi, U) \cdot R = 0$ and $P^e(\xi, X) \cdot P^e = 0$.

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KEYWORDS AND PHRASES. $(LCS)_n$-manifold, Pseudo projective curvature tensor, Extended pseudo projective curvature tensor, η-Einstein manifold.

1. INTRODUCTION

The notion of Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds) with several examples were first introduced and studied by Shaikh [7], which generalizes the concept of LP-Sasakian manifolds given by Matsumoto [3] and by Mihai and Rosca [4]. Later, Shaikh et al. [10] proved the existence of ϕ-recurrent $(LCS)_n$-manifolds. Recently the same author [11] studied invariant submanifolds of $(LCS)_n$-manifolds. Moreover, Shaikh and Baishya [8, 9] have studied the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology. The notion of $(LCS)_n$-manifolds have been intensively studied by several geometers such as Hui and Atceken [2], Prakasha [5], Venkatesha and Naveen Kumar [12] and many others.

On the other hand, the author Bhagwath Prasad [6] developed a type of curvature tensor in an almost contact metric manifold called pseudo projective curvature tensor and is defined as follows:

$$(1)\qquad \begin{aligned} P(X,Y)Z &= aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] \\ &\quad -\frac{r}{n}[\frac{a}{n-1} + b][g(Y,Z)X - g(X,Z)Y], \end{aligned}$$

where a and b are constants such that a, $b \neq 0$ and r is scalar curvature of the manifold.

As a generalization of pseudo projective curvature tensor, the structure of extended pseudo projective curvature tensor P^e is defined as follows:

$$(2)\qquad \begin{aligned} P^e(X,Y)Z &= P(X,Y)Z - \eta(X)P(\xi,Y)Z \\ &\quad -\eta(Y)P(X,\xi)Z - \eta(Z)P(X,Y)\xi. \end{aligned}$$

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The paper is organized as follows. In section 2, we give some basic notations and preliminary results of $(LCS)_n$-manifold admitting extended pseudo projective curvature tensor used throughout the paper. In Sections 3 and 4, we perform extended pseudo projective semi-symmetric and extended pseudo projective pseudo-symmetric $(LCS)_n$-manifold and in both cases the manifold is reduces to η-Einstein provided a and b are not linearly dependent. Finally, Sections 5 and 6 are devoted to the study of $(LCS)_n$-manifold admitting $P^e(\xi, U) \cdot R = 0$ and $P^e(\xi, X) \cdot P^e = 0$. We have shown that the manifold always turns into η-Einstein with a and b are not linearly dependent to each other.

2. Preliminaries

An n-dimensional Lorentzian manifold is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits an smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to R$ is a non-degenerate inner product of signature $(-,+,\cdots,+)$, where T_pM denotes the tangent vector space of M at p and R is the real number space.

An n-dimensional Lorentzian manifold admit a unit timelike concircular vector field ξ and a non-zero 1-form η such that

$$g(X,\xi) = \eta(X), \tag{3}$$

$$(\nabla_X \eta)(Y) = \alpha[g(X,Y) + \eta(X)\eta(Y)], \quad (\alpha \neq 0), \tag{4}$$

for all vector fields X and Y, where ∇ refers the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{5}$$

where ρ being a certain scalar function given by $\rho = -(\xi\alpha)$. If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{6}$$

then by virtue of (4) and (5), we have

$$\phi X = X + \eta(X)\xi, \tag{7}$$

from which it follows that ϕ is a symmetric $(1,1)$ tensor. Thus the Lorentzian manifold M together with this unit timelike concircular vector field ξ, its associated 1-form η and a $(1,1)$ tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold)[7]. Especially, if we take $\alpha = 1$, then we obtain the Lorentzian para-Sasakian structure of Matsumoto [3].

In a $(LCS)_n$-manifold, the following relations hold:

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \tag{8}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{9}$$

$$R(X,Y)Z = (\alpha^2 - \rho)[g(Y,Z)X - g(X,Z)Y], \tag{10}$$

$$(\nabla_X \phi)(Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X], \tag{11}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{12}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{13}$$

$$Q\xi = (n-1)(\alpha^2 - \rho)\xi, \tag{14}$$

for any vector fields X, Y and Z, where R and S denotes respectively the curvature tensor and the Ricci tensor of the manifold.

Also in an $(LCS)_n$ manifold, the extended pseudo projective curvature tensor satisfies the following:

$$P^e(X,Y)\xi = 3[\frac{a}{n-1} + b][(n-1)(\alpha^2 - \rho) - \frac{r}{n}][\eta(Y)X - \eta(X)Y], \tag{15}$$

$$\begin{aligned} P^e(\xi,Y)Z &= S(Y,Z)\xi + 2[a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b)]g(Y,Z)\xi \\ &\quad -[a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b) + b(n-1)(\alpha^2 - \rho)] \\ &\quad [3\eta(Z)Y + \eta(Y)\eta(Z)\xi], \end{aligned} \tag{16}$$

$$\begin{aligned} P^e(X,\xi)\xi &= [a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b) + b(n-1)(\alpha^2 - \rho)] \\ &\quad [-3X - 3\eta(X)\xi] \end{aligned} \tag{17}$$

$$P^e(\xi,\xi)W = 0. \tag{18}$$

By virtue of (2), let $\{e_i\}$ be an orthonormal basis of the tangent space at each point of the manifold and using (1), (8) and (12), we get

$$\sum_{i=1}^{n} g(P^e(e_i,Y)Z, e_i) = (a + bn)S(Y,Z) + lg(Y,Z) + m\eta(Y)\eta(Z), \tag{19}$$

$$\begin{aligned} \text{where } l &= [a(\alpha^2 - \rho) - r(\frac{a}{n-1} + b)], \\ m &= (3-2n)[a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b) \\ &\quad + b(n-1)(\alpha^2 - \rho)]. \end{aligned}$$

3. Extended pseudo projective semi-symmetric $(LCS)_n$-manifold

Let M be an extended pseudo projective semi-symmetric $(LCS)_n$-manifold i.e., $R(X,Y) \cdot P^e = 0$. From which it follows that

$$\begin{aligned} &R(X,Y)P^e(U,V)W - P^e(R(X,Y)U,V)W \\ &-P^e(U,R(X,Y)V)W - P^e(U,V)R(X,Y)W = 0. \end{aligned} \tag{20}$$

By considering $Y = \xi$ in (20) and by using (10), gives either $(\alpha^2 - \rho) = 0$ or

$$\begin{aligned}(21)\quad & \eta(P^e(U,V)W)X - g(X,P^e(U,V)W)\xi - \eta(U)P^e(X,V)W \\ & +g(X,U)P^e(\xi,V)W - \eta(V)P^e(U,X)W + g(X,V)P^e(U,\xi)W \\ & -\eta(W)P^e(U,V)X + g(X,W)P^e(U,V)\xi = 0.\end{aligned}$$

Since $(\alpha^2 - \rho) \neq 0$ and by plugging $V = \xi$ in (21), we get

$$\begin{aligned}(22)\, P^e(U,X)W &= -2bS(U,W)X - 2bS(X,W)\eta(U)\xi \\ & -2bS(U,X)\eta(W)\xi - [a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b)] \\ & [2g(U,W)X + 2g(X,W)\eta(U)\xi - 2g(X,U)\eta(W)\xi] \\ & +[a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b) + b(n-1)(\alpha^2 - \rho)] \\ & [3g(U,X)\eta(W)\xi + 2\eta(X)\eta(W)\eta(U)\xi \\ & +3g(X,W)\eta(U)\xi + 3g(X,W)U + \eta(W)\eta(U)X].\end{aligned}$$

On contracting above equation and by virtue of (19), we obtain

$$\begin{aligned} S(X,W) &= Ag(X,W) + B\eta(X)\eta(W), \\ \text{where, } A &= \left(\frac{3n-4}{a+bn}\right)\left[a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b)\right] \\ & + \left(\frac{3n-3}{a+bn}\right) b(n-1)(\alpha^2 - \rho), \\ B &= \left(\frac{2n+1}{a+bn}\right)\left[a(\alpha^2 - \rho) - \frac{r}{n}(\frac{a}{n-1} + b)\right] \\ & + \left(\frac{2n-3}{a+bn}\right) b(n-1)(\alpha^2 - \rho).\end{aligned}$$

Indeed, we have the following result:

Theorem 3.1. *An $(LCS)_n$-manifold is extended pseudo projective semi-symmetric manifold, then it reduces to η-Einstein manifold, provided a and b are not linearly dependent $(a + bn \neq 0)$.*

4. Extended pseudo projective pseudo-symmetric $(LCS)_n$-manifold

Let us consider the extended pseudo projective pseudo-symmetric $(LCS)_n$-manifold satisfying

$$(23)\quad (R(X,Y)\cdot P^e)(U,V)W = L_{P^e}[((X \wedge Y)\cdot P^e)(U,V)W],$$

holds on the set $U_{P^e} = \{x \in M : P^e \neq 0\}$ at x, where L_{P^e} is some function on U_{P^e} and P^e is the extended pseudo projective curvature tensor. Replacing $Y = \xi$ in (23), we get

$$\begin{aligned}(24)\quad (R(X,\xi)\cdot P^e)(U,V)W &= L_{P^e}[(X \wedge \xi)(P^e(U,V)W) \\ & -P^e((X \wedge \xi)U,V)W \\ & -P^e(U,(X \wedge \xi)V)W \\ & -P^e(U,V)(X \wedge \xi)W].\end{aligned}$$

Now the left hand side of (24) gives

$$(25)\qquad (\alpha^2-\rho)[P^e(U,V,W,\xi)X - P^e(U,V,W,X)\xi - \eta(U)P^e(X,V)W + g(X,U)P^e(\xi,V)W - \eta(V)P^e(U,X)W + g(X,V)P^e(U,\xi)W - \eta(W)P^e(U,V)X + g(X,W)P^e(U,V)\xi].$$

Similarly right hand side of (24) becomes

$$(26)\qquad L_{P^e}[P^e(U,V,W,\xi)X - P^e(U,V,W,X)\xi - \eta(U)P^e(X,V)W + g(X,U)P^e(\xi,V)W - \eta(V)P^e(U,X)W + g(X,V)P^e(U,\xi)W - \eta(W)P^e(U,V)X + g(X,W)P^e(U,V)\xi].$$

By considering the equations (25), (26) in (24) and then by using $V=\xi$, we have

$$(27)\qquad (L_{P^e} - (\alpha^2-\rho))[P^e(U,\xi,W,\xi)X - P^e(U,\xi,W,X)\xi - \eta(U)P^e(X,\xi)W + g(X,U)P^e(\xi,\xi)W - \eta(\xi)P^e(U,X)W + g(X,\xi)P^e(U,\xi)W - \eta(W)P^e(U,\xi)X + g(X,W)P^e(U,\xi)\xi] = 0,$$

from which it follows that either $L_{P^e} = (\alpha^2-\rho)$ or

$$(28)\qquad P^e(U,\xi,W,\xi)X - P^e(U,\xi,W,X)\xi - \eta(U)P^e(X,\xi)W + g(X,U)P^e(\xi,\xi)W - \eta(\xi)P^e(U,X)W + g(X,\xi)P^e(U,\xi)W - \eta(W)P^e(U,\xi)X + g(X,W)P^e(U,\xi)\xi = 0,$$

Taking into account of (15)-(18) in (28) and then by applying inner product with respect to T, we obtain

$$\begin{aligned}(29)\ P^e(U,X,W,T) = {} & -2bS(U,W)g(X,T) - 2bS(X,W)\eta(U)\eta(T) \\ & -2bS(U,X)\eta(W)\eta(T) \\ & -2[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)][g(U,W)g(X,T) \\ & +g(U,X)\eta(W)\eta(T) + g(X,W)\eta(U)\eta(T)] \\ & +[(a+b(n-1))(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)] \\ & [\eta(W)\eta(U)g(X,T) + 3g(U,X)\eta(W)\eta(T) \\ & +4\eta(W)\eta(U)\eta(X)\eta(T) \\ & +3g(X,W)\eta(U)\eta(T) + 3g(X,W)g(U,T)].\end{aligned}$$

On contracting above equation by considering (19), we have

$$\begin{aligned} S(X,W) &= Cg(X,W) + D\eta(X)\eta(W), \\ \text{where, } C &= \frac{[a(3n-4)+3b(n-1)^2](\alpha^2-\rho) + \frac{(3-2n)r}{n}(\frac{a}{n-1}+b)}{(a+bn)}, \\ D &= \frac{(5n-6)[(a+b(n-1))(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)]}{(a+bn)}.\end{aligned}$$

Hence we have the following result:

Theorem 4.1. *An extended pseudo projective pseudo-symmetric $(LCS)_n$-manifold with a and b are not linearly dependent $(a+bn \neq 0)$ is always reduces to η-Einstein manifold.*

5. $(LCS)_n$ MANIFOLD SATISFYING $P^e(\xi, U) \cdot R = 0$

In this section, we consider $(LCS)_n$-manifold satisfying $P^e(\xi, U) \cdot R = 0$, from which it can be easily obtain that

$$P^e(\xi, U)R(X,Y)Z - R(P^e(\xi, U)X, Y)Z \\ -R(X, P^e(\xi, U)Y)Z - R(X,Y)P^e(\xi, U)Z = 0. \tag{30}$$

Using (16) in (30), we get

$$2bS(U, R(X,Y)Z)\xi - 2bS(U,X)R(\xi,Y)Z - 2bS(U,Y)R(X,\xi)Z \\ -2bS(U,Z)R(X,Y)\xi + 2[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)][g(U, R(X,Y)Z)\xi \\ -g(U,X)R(\xi,Y)Z - g(U,Y)R(X,\xi)Z - g(U,Z)R(X,Y)\xi] \\ +[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b) + b(n-1)(\alpha^2-\rho)][-3\eta(R(X,Y)Z)U \\ -\eta(U)\eta(R(X,Y)Z)\xi + 3\eta(X)R(U,Y)Z + R(\xi,Y)Z\eta(U)\eta(X) \\ +3\eta(Y)R(X,U)Z + \eta(U)\eta(Y)R(X,\xi)Z + 3\eta(Z)R(X,Y)U \\ +\eta(U)\eta(Z)R(X,Y)\xi = 0. \tag{31}$$

Taking inner product of (31) with ξ, gives

$$-2bS(U, R(X,Y)Z) - 2bS(U,X)\eta(R(\xi,Y)Z) - 2bS(U,Y)\eta(R(X,\xi)Z) \\ +2[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)][-g(U,R(X,Y)Z) \\ -g(U,X)\eta(R(\xi,Y)Z) - g(U,Y)\eta(R(X,\xi)Z)] \\ +[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b) + b(n-1)(\alpha^2-\rho)][-3\eta(R(X,Y)Z)\eta(U) \\ +\eta(U)\eta(R(X,Y)Z) + 3\eta(X)\eta(R(U,Y)Z) + \eta(R(\xi,Y)Z)\eta(U)\eta(X) \\ +3\eta(Y)\eta(R(X,U)Z) + 3\eta(Z)\eta(R(X,Y)U) \\ +\eta(U)\eta(Y)\eta(R(X,\xi)Z)] = 0. \tag{32}$$

Putting $Y = Z = e_i$ in (32), $\{e_i\}_{i=1}^n$ be an orthonormal basis of the tangent space, we have

$$S(U,X) = Eg(U,X) + F\eta(U)\eta(X),$$

$$\text{where, } E = \frac{(\alpha^2-\rho)[(2n-1)a + 3b(n-1)] - \frac{r}{n}(\frac{a}{n-1}+b)(2n-1)}{2\left[1 - \frac{r}{n(n-1)(\alpha^2-\rho)}\right]a + 2\left[1 - \frac{r}{n(\alpha^2-\rho)}\right]b},$$

$$F = \frac{(\alpha^2-\rho)(a + 4b(n-1)) - \frac{4r}{n}(\frac{a}{n-1}+b)}{2\left[1 - \frac{r}{n(n-1)(\alpha^2-\rho)}\right]a + 2\left[1 - \frac{r}{n(\alpha^2-\rho)}\right]b}.$$

In view of above discussion, we can state the following:

Theorem 5.1. *An $(LCS)_n$-manifold satisfying $P^e(\xi, U) \cdot R = 0$ is always turns into η-Einstein manifold provided a and b are not linearly dependent* $\left(2\left[1 - \frac{r}{n(n-1)(\alpha^2-\rho)}\right]a + 2\left[1 - \frac{r}{n(\alpha^2-\rho)}\right]b \neq 0\right)$.

6. $(LCS)_n$-MANIFOLD SATISFYING $P^e(\xi, X) \cdot P^e = 0$

Here an $(LCS)_n$-manifold admitting $P^e(\xi, X) \cdot P^e = 0$ turns into

$$(33) \quad P^e(\xi, X)P^e(U,V)W - P^e(P^e(\xi, X)U, V)W - P^e(U, P^e(\xi, X)V)W - P^e(U,V)P^e(\xi, X)W = 0,$$

which in view of (16), gives

$$(34) \quad 2bS(X, P^e(U,V)W)\xi - 2bS(X,U)P^e(\xi, V)W - 2bS(X,V)P^e(U,\xi)W - 2bS(X,W)P^e(U,V)\xi + 2[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)][g(X, P^e(U,V)W)\xi - g(X,U)P^e(\xi,V)W - g(X,V)P^e(U,\xi)W - g(X,W)P^e(U,V)\xi] - [a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b) + b(n-1)(\alpha^2-\rho)][3\eta(P^e(U,V)W)X + \eta(X)\eta(P^e(U,V)W)\xi - 3\eta(U)P^e(X,V)W - \eta(X)\eta(U)P^e(\xi,V)W - 3\eta(V)P^e(U,X)W - \eta(V)\eta(X)P^e(U,\xi)W - 3\eta(W)P^e(U,V)X - \eta(X)\eta(W)P^e(U,V)\xi] = 0.$$

On plugging $V = \xi$ in (34) and then taking an account of (15)-(18), we obtain that

$$(35) \quad P^e(U,X)W = \left[\frac{2b(2b-1)(n-1)(\alpha^2-\rho)}{3a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b) + b(n-1)(\alpha^2-\rho)}\right] S(U,W)\eta(X)\xi + \frac{2[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)](n-1)(2b-1)(\alpha^2-\rho)}{3[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b) + b(n-1)(\alpha^2-\rho)} (g(U,W)\eta(X)\xi - (2b-1)S(U,X)\eta(W)\xi) + [1 - \left(\frac{2b-1}{3}\right)(n-1)(\alpha^2-\rho) - 3[a(\alpha^2-\rho) - \frac{r}{n}\left(\frac{a}{n-1}+b\right) + b(n-1)(\alpha^2-\rho)]] (\eta(W)\eta(X)\eta(U)\xi + 2bS(X,W)U) - [a(\alpha^2-\rho) - \frac{r}{n}\left(\frac{a}{n-1}+b\right) + b(n-1)(\alpha^2-\rho)] [\eta(X)\eta(W)U - \eta(W)\eta(U)X] + 2[a(\alpha^2-\rho) - \frac{r}{n}\left(\frac{a}{n-1}+b\right)][g(X,W)U - g(U,W)X] - 2bS(U,W)X.$$

Contracting above equation over U, we conclude that

$$S(X,W) = Gg(X,W) + H\eta(X)\eta(W),$$

$$\text{where, } G = \frac{(2n-3)[a(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)]}{a+(2-n)b},$$

$$H = \frac{-1+(n+1)[(a+b(n-1))(\alpha^2-\rho) - \frac{r}{n}(\frac{a}{n-1}+b)]}{a+(2-n)b}.$$

Hence we can state the following theorem:

Theorem 6.1. *Let M be an $(LCS)_n$-manifold satisfying $P^e(\xi, X) \cdot P^e = 0$, then the manifold is reduces to η-Einstein provided a and b are not linearly dependent $(a + (2 - n)b \neq 0)$.*

7. Conclusion

In the present study, we have obtained that a Lorentzian concircular structure manifold briefly ($(LCS)_n$-manifold) satisfying $R(X,Y) \cdot P^e = 0$, $(R(X,Y) \cdot P^e)(U,V)W = L_{P^e}[((X \wedge Y) \cdot P^e)(U,V)W]$, $P^e(\xi, U) \cdot R = 0$ and $P^e(\xi, X) \cdot P^e = 0$ is always turns into η-Einstein manifold provided that a and b are not linearly dependent to each other.

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