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VL RECIPROCAL STATUS INDEX AND COINDEX OF CONNECTED GRAPHS

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Abstract

The reciprocal status of a vertex u in a connected graph G, is defined as the sum of reciprocal of the distances between u and all other vertices of a graph G. Relation between VL reciprocal status index and VL reciprocal status co-indices are established. Also these indices are computed for VL reciprocal status transmission regular graphs. Further the VL reciprocal status index and coindex of some standard graphs are obtained.

Keywords: Status index, Degree, Distance, Connected graph, Topological indices.
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1 Introduction and Preliminaries

A topological index is a graph invariant which is mathematically derived from the graph structure. The topological indices are used for quantitative structure-activity relationship ($QSAR$) and quantitative structure-property relationship ($QSPR$) [12] [13]. The oldest topological index is Wiener index, which was presented by the chemist Harold Wiener in 1947 [14], afterward a number of distance based topological indices were defined as hyper Wiener index, Harary index [6] [9] and so on.

Let G be a connected graph of order n and size m. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of G. The edge joining the vertices u and v is denoted by uv. The degree of a vertex u is the number of edges incident to it and is denoted by $d(u)$. The distance between the vertices u and v is denoted by $d(u,v)$ is the length of the shortest path joining u and v in G. The maximum distance between any pair of vertices in G is called the diameter of G and is denoted by $diam(G)$. For standard terminology and notion in graph theory, we follow the textbook of Harary [5].
Following definitions are used in upcoming section.

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Harary explained the *status* [5] of a vertex u, is defined as the sum of its distances from every other vertex of G and is denoted by $\sigma(u)$. That is,

$$\sigma(u) = \sum_{uv \in E(G)} d(u,v).$$

Recently, Ramane and Yalnaik [10] introduced the first and second status connectivity indices of a graph G to study the property of benzenoid hydrocarbons. The first and second status connectivity index is defined as

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] \ \ and \ \ S_2(G) = \sum_{uv \in E(G)} [\sigma(u) \cdot \sigma(v)].$$

The *reciprocal status* [11] of a vertex u is defined as the sum of reciprocal of its distances from every other vertex of G and is denoted by $rs(u)$. That is,

$$rs(u) = \sum_{v \in V(G), u \neq v} \frac{1}{d(u,v)}.$$

The Harary index $H(G)$ of a connected graph G is defined as

$$H(G) = \frac{1}{2} \sum_{\{uv\} \subseteq V(G), u \neq v} \frac{1}{d(u,v)} \tag{1}$$

The Equation (1) can be written as

$$H(G) = \frac{1}{2} \sum_{u \in V(G)} rs(u).$$

The first and second Zagreb indices of a graph G are defined as

$$Z_1(G) = \sum_{uv \in E(G)} [d(u) + d(v)] \quad and \quad Z_2(G) = \sum_{uv \in E(G)} [d(u) \cdot d(v)].$$

The first and second Zagreb co-indices of a graph G are defined as

$$\overline{Z_1}(G) = \sum_{uv \notin E(G)} [d(u) + d(v)] \quad and \quad \overline{Z_2}(G) = \sum_{uv \notin E(G)} [d(u) \cdot d(v)].$$

H. S. Ramane and S. Y. Talwar [11] introduced the first and second reciprocal status connectivity indices of a connected graph, defined as,

$$RS_1(G) = \sum_{uv \in E(G)} [rs(u) + rs(v)] \ \ and \ \ RS_2(G) = \sum_{uv \in E(G)} [rs(u) \cdot rs(v)].$$

T. Deepika, [4] introduced the VL index of a graph G is defined as,

$$VL(G) = \frac{1}{2} \sum_{uv \in E(G)} [d_e + d_f + 4]$$

where $d_e = d(u) + d(v) - 2$ and $d_f = (d(u) \cdot d(v)) - 2$.
The VL index shows a good correlation with the physical properties of octane isomers and polychlorinated biphenyl (PCB).

The VL index which can also be written as,

$$VL(G) = \frac{1}{2} \sum_{uv \in E(G)} [d(u) + d(v) + d(u) \cdot d(v)].$$

Presently, in [7], two more indices introduced and studied their graph theoretical properties under the name of the *VL reciprocal status index $VLRS(G)$* and *VL reciprocal status co-index* $\overline{\mathrm{VLRS}}(G)$ of a graph G that are defined by

$$VLRS(G) = \frac{1}{2} \sum_{uv \in E(G)} [rs(u) + rs(v) + rs(u) \cdot rs(v)] \tag{1}$$

and

$$\overline{\mathrm{VLRS}}(G) = \frac{1}{2} \sum_{uv \notin E(G)} [rs(u) + rs(v) + rs(u) \cdot rs(v)]. \tag{2}$$

For example,

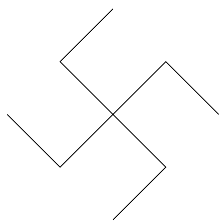

Figure 1:

For a graph given in FIGURE 1, $VLRS(G) = 119.75$ and $\overline{\mathrm{VLRS}}(G) = 44.75$.

2 VL reciprocal status index and co-indices

In this section, by considering the indices given in Eqs. (1) and (2), we state and prove some results on connected graphs.

Proposition 2.1. *Let G be a connected graph on n vertices. Then*

$$\overline{\mathrm{VLRS}}(G) = (n - 1 + H(G))H(G) - VLRS(G) - \frac{1}{4} \sum_{u \in V(G)} (rs(u))^2.$$

Proof. Consider,

$$\begin{aligned}
\overline{\text{VLRS}}(G) &= \frac{1}{2}\sum_{uv\notin E(G)}[rs(u)+rs(v)+rs(u)\cdot rs(v)] \\
&= \frac{1}{2}\left[\sum_{uv\notin E(G)} rs(u)+rs(v)+\sum_{uv\notin E(G)} rs(u)\cdot rs(v)\right] \\
&= \frac{1}{2}\left[\sum_{\{u,v\}\subseteq V(G)} rs(u)+rs(v)-\sum_{uv\notin E(G)} rs(u)+rs(v)+\sum_{\{u,v\}\subseteq V(G)} rs(u)\cdot rs(v)\right. \\
&\quad \left. -\sum_{uv\notin E(G)} rs(u)\cdot rs(v)\right] \\
&= \frac{1}{2}\left[(n-1)\sum_{u\in V(G)} rs(u)-RS_1(G)+\frac{1}{2}\left\{\left(\sum_{u\in V(G)} rs(u)^2\right)-\sum_{u\in V(G)}(rs(u))^2\right\}\right. \\
&\quad \left. -RS_2(G)\right] \\
&= \frac{1}{2}\left[2(n-1)H(G)-RS_1(G)+2(H(G))^2-\frac{1}{2}\sum_{u\in V(G)}(rs(u))^2\right. \\
&\quad \left. -RS_2(G)\right] \\
&= (n-1+H(G))H(G)-VLRS(G)-\frac{1}{4}\sum_{u\in V(G)}(rs(u))^2.
\end{aligned}$$

Hence,

$$\overline{\text{VLRS}}(G) = (n-1+H(G))H(G)-VLRS(G)-\frac{1}{4}\sum_{u\in V(G)}(rs(u))^2.$$

□

Corollary 2.2. *Let G be a connected graph with n vertices, m edges and $diam(G) \leq 2$. Then*

$$\begin{aligned}
\overline{\text{VLRS}}(G) &= (n-1+H(G))H(G)-\frac{m}{8}(n-1)(n+3)-\frac{1}{8}(n+1)Z_1(G)-\frac{1}{8}Z_2(G) \\
&\quad -\frac{1}{4}\sum_{u\in V(G)}(rs(u))^2.
\end{aligned}$$

Proof. For any graph G of $diam(G) \leq 2$, $rs(u) = \frac{1}{2}[(n-1)+d(u)]$ and $H(G) = m+\frac{1}{2}[\frac{n(n-1)}{2}-m] = \frac{1}{4}[2m+n(n-1)]$.

Also we have [11]

$$RS_1(G) = (n-1)m+\frac{1}{2}Z_1(G)$$

and

$$RS_2(G) = \frac{1}{4}[(n-1)^2 m + (n-1)Z_1(G) + Z_2(G)].$$

Therefore by the Proposition 2.1.

$$\begin{aligned}
\overline{\text{VLRS}}(G) &= (n-1+H(G))H(G) - VLRS(G) - \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2 \\
&= (n-1+H(G))H(G) - \frac{1}{2}\sum_{uv\in E(G)}[rs(u)+rs(v)+rs(u)\cdot rs(v)] \\
&- \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2 \\
&= (n-1+H(G))H(G) - \frac{1}{2}\left[\sum_{uv\in E(G)}(rs(u)+rs(v)) + \sum_{uv\in E(G)}(rs(u)\cdot rs(v))\right] \\
&- \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2 \\
&= (n-1+H(G))H(G) - \frac{1}{2}[RS_1(G)+RS_2(G)] - \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2 \\
&= (n-1+H(G))H(G) - \frac{1}{2}\Big[(n-1)m + \frac{1}{2}Z_1(G) + \frac{1}{4}((n-1)^2 m \\
&+ (n-1)Z_1(G) + Z_2(G))\Big] - \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2 \\
&= (n-1+H(G))H(G) - \frac{m}{8}(n-1)(n+3) - \frac{1}{8}(n+1)Z_1(G) - \frac{1}{8}Z_2(G) \\
&- \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2.
\end{aligned}$$

Hence,

$$\begin{aligned}
\overline{\text{VLRS}}(G) &= (n-1+H(G))H(G) - \frac{m}{8}(n-1)(n+3) - \frac{1}{8}(n+1)Z_1(G) - \frac{1}{8}Z_2(G) \\
&- \frac{1}{4}\sum_{u\in V(G)}(rs(u))^2.
\end{aligned}$$

□

Proposition 2.3. *Let G be a connected graph with n vertices, m edges and $diam(G) \le 2$. Then*

$$\overline{\text{VLRS}}(G) = \frac{1}{16}n(n-1)^2(n+3) - \frac{1}{8}m(n-1)(n+3) + \frac{1}{8}(n+1)\overline{Z_1}(G) + \frac{1}{8}\overline{Z_2}(G).$$

Proof. For any graph G of $diam(G) \leq 2$, $rs(u) = \frac{1}{2}[(n-1) + d(u)]$. Therefore

$$\begin{aligned}
\overline{\text{VLRS}}(G) &= \frac{1}{2}\sum_{uv \notin E(G)} [rs(u) + rs(v) + rs(u) \cdot rs(v)] \\
&= \frac{1}{2}\sum_{uv \notin E(G)} \Big[\frac{1}{2}((n-1)+d(u)) + \frac{1}{2}((n-1)+d(v)) + \frac{1}{2}((n-1)+d(u)) \cdot \\
&\quad \frac{1}{2}((n-1)+d(v))\Big] \\
&= \frac{1}{2}\sum_{uv \notin E(G)} \Big[\frac{1}{2}((n-1)+d(u)) + \frac{1}{2}((n-1)+d(v))\Big] + \frac{1}{2}\sum_{uv \notin E(G)} \Big[\frac{1}{2}((n-1) \\
&\quad + d(u)) \cdot \frac{1}{2}((n-1)+d(v))\Big] \\
&= \frac{(n-1)}{2}\sum_{uv \notin E(G)} 1 + \frac{1}{4}\sum_{uv \notin E(G)} [d(u)+d(v)] + \frac{1}{8}\sum_{uv \notin E(G)} [(n-1)^2 + d(u)d(v) \\
&\quad + (n-1)(d(u)+d(v))] \\
&= \frac{(n-1)}{2}\left(\frac{n(n-1)}{2} - m\right) + \frac{1}{4}\overline{Z_1}(G) + \frac{1}{8}\Big[(n-1)^2 \sum_{uv \notin E(G)} 1 + \sum_{uv \notin E(G)} d(u)d(v) \\
&\quad + (n-1)\sum_{uv \notin E(G)} [d(u)+d(v)]\Big] \\
&= \frac{1}{4}n(n-1)^2 - \frac{m(n-1)}{2} + \frac{1}{4}\overline{Z_1}(G) + \frac{1}{8}\Big[(n-1)^2\left(\frac{n(n-1)}{2} - m\right) + \overline{Z_2}(G) \\
&\quad + (n-1)\overline{Z_1}(G)\Big] \\
&= \frac{1}{4}n(n-1)^2 - \frac{m(n-1)}{2} + \frac{1}{4}\overline{Z_1}(G) + \frac{1}{16}n(n-1)^3 - \frac{m(n-1)^2}{8} \\
&\quad + \frac{1}{8}(n-1)\overline{Z_1}(G) + \frac{1}{8}\overline{Z_2}(G) \\
&= \frac{1}{16}n(n-1)^2(n+3) - \frac{1}{8}m(n-1)(n+3) + \frac{1}{8}(n+1)\overline{Z_1}(G) + \frac{1}{8}\overline{Z_2}(G).
\end{aligned}$$

Hence,

$$\overline{\text{VLRS}}(G) = \frac{1}{16}n(n-1)^2(n+3) - \frac{1}{8}m(n-1)(n+3) + \frac{1}{8}(n+1)\overline{Z_1}(G) + \frac{1}{8}\overline{Z_2}(G).$$

□

Proposition 2.4. *Let G be a graph with n vertices and m edges. Let $\overline{G}$, the complement of G, be connected. Then*

$$VLRS(\overline{G}) \geq \frac{1}{4}n(n+1)(n-1)^2 - \frac{1}{2}m(n^2-1) - \frac{n}{4}\overline{Z_1}(G) + \frac{1}{8}\overline{Z_2}(G).$$

Proof. For any vertex u in $\overline{G}$ there are $n-1-d(u)$ vertices which are at distance 1 and the remaining vertices are at distance at least 2 from u. Therefore

$$rs_{\overline{G}} \leq (n-1-d(u)) + \frac{1}{2}d(u) = n-1-\frac{1}{2}d(u).$$

$$\begin{aligned}
VLRS(\overline{G}) &= \frac{1}{2}\sum_{uv\in E(\overline{G})}[rs_{\overline{G}}(u)+rs_{\overline{G}}(v)+rs_{\overline{G}}(u)\cdot rs_{\overline{G}}(v)] \\
&\geq \frac{1}{2}\sum_{uv\in E(\overline{G})}\left[n-1-\frac{1}{2}d(u)+n-1-\frac{1}{2}d(v)+\left(n-1-\frac{1}{2}d(u)\right)\cdot \right. \\
&\left.\left(n-1-\frac{1}{2}d(v)\right)\right] \\
&= \frac{1}{2}\sum_{uv\in E(\overline{G})}\left[2(n-1)-\frac{1}{2}(d(u)+d(v))+(n-1)^2-\frac{1}{2}(n-1)d(v)-\frac{(n-1)}{2}d(u)\right. \\
&\left.+\frac{1}{4}d(u)\cdot d(v)\right] \\
&= \frac{1}{2}\sum_{uv\notin E(\overline{G})}2(n-1)-\frac{1}{4}\sum_{uv\notin E(\overline{G})}(d(u)+d(v))+\frac{1}{2}\sum_{uv\notin E(\overline{G})}(n-1)^2-\frac{1}{4}(n-1) \\
&\sum_{uv\notin E(\overline{G})}(d(u)+d(v))+\frac{1}{8}\sum_{uv\notin E(\overline{G})}d(u)\cdot d(v) \\
&= (n-1)\left(\frac{n(n-1)}{2}-m\right)-\frac{1}{4}\overline{Z_1}(G)+\frac{1}{2}(n-1)^2\left(\frac{n(n-1)}{2}-m\right)-\frac{1}{4}(n-1) \\
&\overline{Z_1}(G)+\frac{1}{8}\overline{Z_2}(G) \\
&= \frac{1}{2}n(n-1)^2-m(n-1)-\frac{1}{4}\overline{Z_1}(G)+\frac{1}{4}n(n-1)^3-\frac{1}{2}m(n-1)^2-\frac{1}{4}(n-1) \\
&\overline{Z_1}(G)+\frac{1}{8}\overline{Z_2}(G) \\
&\geq \frac{1}{4}n(n+1)(n-1)^2-\frac{1}{2}m(n^2-1)-\frac{n}{4}\overline{Z_1}(G)+\frac{1}{8}\overline{Z_2}(G).
\end{aligned}$$

Hence,

$$VLRS(\overline{G}) \geq \frac{1}{4}n(n+1)(n-1)^2-\frac{1}{2}m(n^2-1)-\frac{n}{4}\overline{Z_1}(G)+\frac{1}{8}\overline{Z_2}(G).$$

□

3 VL reciprocal status index and co-index of some transmission regular graphs

A bijection α on vertex set of G is called an *automorphism* of G if it preserves edge set of G. In other words α defines an automorphism if for each $u, v \in V(G)$, we have $e = uv \in E(G)$ if and only if $\alpha(e) = \alpha(u).\alpha(v) \in E(G)$. Let $Aut(G) = \{\alpha \mid \alpha : V(G) \to V(G)$ is a bijection which preserves the adjacency$\}$. It is known $Aut(G)$ forms an algebraic group under the composition of mappings. On the other hand a graph G is called *vertex-transitive* if for every two vertices u and v of G, there exists an automorphism α of G such that $\alpha(u) = \alpha(v)$. It is known that any vertex-transitive graph is vertex degree regular, transmission regular and self-centered. Indeed the graph depicted in Figure 2 is 14-transmission regular graph but not degree regular and therefore not vertex-transitive (see [1] [2]).

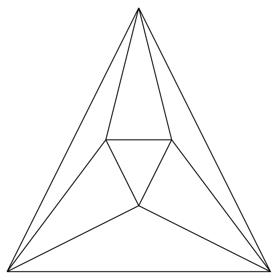

Figure 2: The VL reciprocal status transmission regular but not degree regular graph with the smallest order

The following is straightforward from the definition of VL reciprocal status connectivity indices.

Lemma 3.1. *For a connected k-VL reciprocal status transmission regular graph G with m edges,* $VLRS(G) = \frac{1}{2}[mk(k+2)]$.

Theorem 3.2 ([3]). *Let G be a connected graph on n vertices with the automorphism group $Aut(G)$ and the vertex set $V(G)$. Let $V_1, V_2, \ldots, V_t$ be all orbits of the action $Aut(G)$ on $V(G)$. Suppose also that for each $1 \le i \le t$, d_i and k_i are the vertex degree and the transmission of vertices in the orbit V_i, respectively. Then* $H(G) = \frac{1}{2}\sum_{i=1}^{t}|V_i|k_i$.
Specially if G is vertex-transitive (that is, t=1), then $H(G) = \frac{1}{2}nk$, where k is the transmission of each vertex of G.

Theorem 3.3 ([11]). *Let G be a connected graph on n vertices with the automorphism group $Aut(G)$ and the vertex set $V(G)$. Let $V_1, V_2, \ldots, V_t$ be all orbits of the action $Aut(G)$ on $V(G)$. Suppose also that for each $1 \le i \le t$, d_i and k_i are the vertex degree and the transmission of vertices in the orbit V_i, respectively. Then*

$$RS_1(G) = \sum_{i=1}^{t}|V_i|d_ik_i, \quad and \quad \overline{RS_1}(G) = \sum_{i=1}^{t}\left(|V_i|k_i\left(1 - \frac{d_i}{n-1}\right)\right).$$

Specially if G is vertex-transitive (i.e. $t = 1$), then

$$RS_1(G) = ndk, \qquad RS_2(G) = \tfrac{1}{2}ndk^2,$$
$$\overline{RS_1}(G) = 2\tbinom{n}{2}k - ndk, \qquad \overline{RS_2}(G) = \left(\tbinom{n}{2} - \tfrac{nd}{2}\right)k^2,$$

where d and k are the degree and the transmission of each vertex of G, respectively.

Analogous to Theorem 3.2, Theorem 3.3 and as a consequence of Proposition 2.1, we have the following.

Theorem 3.4. *Let G be a connected graph on n vertices with the automorphism group $Aut(G)$ and the vertex set $V(G)$. Let $V_1, V_2, \ldots, V_t$ be all orbits of the action $Aut(G)$ on $V(G)$. Suppose also that for each $1 \leq i \leq t$, d_i and k_i are the vertex degree and the transmission of vertices in the orbit V_i, respectively. If G is vertex-transitive (that is, t=1), then*

$$VLRS(G) = \tfrac{ndk}{4}(k+2), \qquad \overline{\text{VLRS}}(G) = \tfrac{1}{2}\left[2\tbinom{n}{2}k - ndk + \left(\tbinom{n}{2} - \tfrac{nd}{2}\right)k^2\right].$$

Lemma 3.5. *Let G be a connected k-VL reciprocal status transmission regular graph with m edges. Then*

$$\overline{\text{VLRS}}(G) = \frac{1}{2}\left[2\binom{n}{2}k - 2mk + \binom{n}{2}k^2 - mk^2\right].$$

The *Kneser graph $KG_{p,k}$* is the graph whose vertices correspond to the k-element subsets of a set of p elements, and any two vertices are adjacent if and only if the two corresponding sets are disjoint. Clearly we must impose the restriction $p \geq 2k$. The Kneser graph $KG_{p,k}$ has $\binom{p}{k}$ vertices and it is actually regular of degree $\binom{p-k}{k}$. Therefore, by [8], the number of edges of $KG_{p,k}$ is $\frac{1}{2}\binom{p}{k}\binom{p-k}{k}$. Moreover the Kneser graph $KG_{n,1}$ is complete on n vertices, and $KG_{8,1}$ is known as 8-complete graph (see Figure 3).

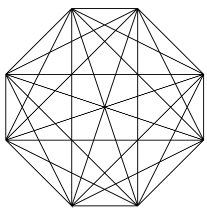

Figure 3: 8-complete graph

Lemma 3.6. *[11] The Kneser graph $KG_{p,k}$ is vertex-transitive and for each k-subset A,*

$$rs_{KG_{p,k}}(A) = \frac{2H(KG_{p,k})}{\binom{p}{k}}.$$

Following Proposition follows from Lemma 3.6 and 3.1.

Proposition 3.7. *For a Kneser graph $KG_{p,k}$ we have*

$$VLRS(KG_{p,k}) = \frac{1}{2}\left[2H(KG_{p,k})\binom{p-k}{k} + \binom{p-k}{k}\frac{2H(KG_{p,k})^2}{\binom{p}{k}}\right].$$

Proposition 3.8. *For a Kneser graph $KG_{p,k}$ we have*

$$\overline{\text{VLRS}}(KG_{p,k}) = \frac{1}{2}\left[2H(KG_{p,k})\left(\binom{p}{k} - \binom{p-k}{k} - 1\right) + \left(\frac{4(H(KG_{p,k}))^2}{\binom{p}{k}^2}\right)Q\right]$$

where Q is the number of edges which doesnot belong to $E(KG_{p,k})$.

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