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ON DEGENERATE BELL POLYNOMIALS ASSOCIATED WITH SPECIAL NUMBERS AND POLYNOMIALS

TAEKYUN KIM, DAE SAN KIM, UITAE PYO, AND JONGKYUM KWON

ABSTRACT. The aim of this paper is to study the Bell polynomials and three kinds of their degenerate versions, namely the degenerate Bell polynomials, the type 2 degenerate Bell polynomials and the modified degenerate Bell polynomials. We derive some identities on the Bell polynomials and the three kinds of their degenerate versions associated with some special numbers and polynomials, including the Stirling numers of the second kind, the degenerate Stirling numbers of the second kind, the derangement numbers and the degenerate Fubini polynomials.

1. INTRODUCTION

We have witnessed that various degenerate versions of many special numbers and polynomials have been studied not only with their number-theoretic or combinatorial interests but also with their applications to others, like probability, differential equations and symmetry. This exploration for degenerate versions began with Carlitz's pioneering work on the degenerate Bernoulli and degenerate Euler polynomials in [2]. It is remarkable that in the course of this quest people used many different tools, which include generating functions, combinatorial methods, p-adic analysis, umbral calculus, operator theory, differential equations, special functions, probability theory and analytic number theory (see [3,5,6,7,9,10,13-17] and the references therein).

The aim of this paper is to study the Bell polynomials and three kinds of their degenerate versions, namely the degenerate Bell polynomials, the type 2 degenerate Bell polynomials and the modified degenerate Bell polynomials. Some identities on the Bell polynomials and the three kinds of their degenerate versions are derived in association with some special numbers and polynomials, including the Stirling numers of the second kind, the degenerate Stirling numbers of the second kind, the derangement numbers and the degenerate Fubini polynomials.

The outline of this paper is as follows. In Section 1, we recall degenerate exponentials, degenerate Stirling numbers of both kinds, derangement numbers and degenerate Fubini polynomials. Also, we remind the reader of degenerate Bell polynomials and type 2 degenerate Bell polynomials. The main results of this paper are stated in Section 2. In Theorem 1, we derive an identity involving the degenerate Bell, the Stirling numbers of the second kind and the derangement numbers. In Theorem 2, we get an identity on the degenerate Bell polynomials. In Theorem 3, we deduce an identity relating the type 2 degenerate Bell polynomials and the degenerate Fubini polynomials. In Theorem 4, we obtain an identity connecting the modified degenerate Bell polynomials and the Stirling numbers of the second kind. Finally, we get an identity on the ordinary Bell polynomials in Theorem 5.

For any $\lambda \in \mathbb{R}$, the degenerate exponentials are defined by

$$(1) \qquad e_{\lambda}^{x}(t) = \sum_{n=0}^{\infty} \frac{(x)_{n,\lambda}}{n!} t^{n}, \quad (\text{see } [5-10, 13-17]),$$

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where

$$(x)_{0,\lambda}=1,\quad (x)_{n,\lambda}=x(x-\lambda)\cdots(x-(n-1)\lambda),\quad (n\geq 1),\quad \text{(see [7])}.$$

When $x=1$, $e_\lambda(t)=e_\lambda^1(t)=\sum_{n=0}^{\infty}\frac{(1)_{n,\lambda}}{n!}t^n$.

It is known that the degenerate Stirling numbers of the first kind are defined by

$$(x)_n=\sum_{k=0}^{n}S_{1,\lambda}(n,k)(x)_{k,\lambda},\quad (n\geq 0),\quad \text{(see [7])}, \tag{2}$$

where $(x)_0=1,\ (x)_n=x(x-1)\cdots(x-n+1),\ (n\geq 1)$.
The degenerate Stirling numbers of the second kind are defined by

$$(x)_{n,\lambda}=\sum_{k=0}^{n}S_{2,\lambda}(n,k)(x)_k,\quad (n\geq 0),\quad \text{(see [7])}. \tag{3}$$

Let $\log_\lambda t$ be the compositional inverse function of $e_\lambda(t)$. It is called the degenerate logarithm and given by

$$\log_\lambda(1+t)=\sum_{n=1}^{\infty}\frac{\lambda^{n-1}(1)_{n,\frac{1}{\lambda}}}{n!}t^n,\quad \text{(see [7])}, \tag{4}$$

Note that

$$\frac{1}{k!}\big(\log_\lambda(1+t)\big)^k=\sum_{n=k}^{\infty}S_{1,\lambda}(n,k)\frac{t^n}{n!},\quad (k\geq 0), \tag{5}$$

and

$$\frac{1}{k!}\big(e_\lambda(t)-1\big)^k=\sum_{n=k}^{\infty}S_{2,\lambda}(n,k)\frac{t^n}{n!},\quad \text{(see [7])}.$$

It is well known that the derangement numbers are defined by

$$d_n=n!\left(\sum_{k=0}^{n}\frac{(-1)^k}{k!}\right),\quad (n\geq 0),\quad \text{(see [6])}. \tag{6}$$

Here we recall that the derangement number d_n is the number of permutations of $\{1,2,\ldots,n\}$ with no fixed points. From (6), we see that the generating function of derangement numbers is given by

$$\sum_{n=0}^{\infty}d_n\frac{t^n}{n!}=\frac{1}{1-t}e^{-t},\quad \text{(see [6])}. \tag{7}$$

The degenerate Fubini polynomials are defined by

$$\frac{1}{1-x(e_\lambda(t)-1)}=\sum_{n=0}^{\infty}F_{n,\lambda}(x)\frac{t^n}{n!},\quad \text{(see [6])}. \tag{8}$$

Note that

$$F_{n,\lambda}(x)=\sum_{k=0}^{n}S_{2,\lambda}(n,k)k!x^k,\quad (n\geq 0),\quad \text{(see [6])}. \tag{9}$$

In [11], the degenerate Bell polynomials are defined by

$$e^{x(e_\lambda(t)-1)}=\sum_{n=0}^{\infty}\mathrm{Bel}_{n,\lambda}(x)\frac{t^n}{n!}. \tag{10}$$

Note that

$$\mathrm{Bel}_{n,\lambda}(x)=\sum_{k=0}^{n}S_{2,\lambda}(n,k)x^k,\quad (n\geq 0). \tag{11}$$

When $x=1$, $\mathrm{Bel}_{n,\lambda}=\mathrm{Bel}_{n,\lambda}(1)$ are called the degenerate Bell numbers.

The type 2 degenerate Bell polynomials are given by

$$e_\lambda\big(x(e_\lambda(t)-1)\big)=\sum_{n=0}^{\infty}\phi_{n,\lambda}(x)\frac{t^n}{n!},\quad (\text{see } [3,16,17]). \tag{12}$$

Note that

$$\phi_{n,\lambda}(x)=\sum_{k=0}^{n}(1)_{k,\lambda}S_{2,\lambda}(n,k)x^k.$$

2. Degenerate Bell polynomials associated with some special numbers and polynomials

Replacing t by $1-e_\lambda(t)$ in (6), we get

$$\begin{aligned} e_\lambda^{-1}(t)e^{e_\lambda(t)-1}&=\sum_{k=0}^{\infty}d_k\frac{1}{k!}\big(1-e_\lambda(t)\big)^k \\ &=\sum_{k=0}^{\infty}(-1)^k d_k\sum_{n=k}^{\infty}S_{2,\lambda}(n,k)\frac{t^n}{n!} \\ &=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}(-1)^k d_k S_{2,\lambda}(n,k)\right)\frac{t^n}{n!}. \end{aligned} \tag{13}$$

On the other hand, by (10), we get

$$\begin{aligned} e_\lambda^{-1}(t)e^{e_\lambda(t)-1}&=e_{-\lambda}(-t)e^{e_\lambda(t)-1} \\ &=\sum_{l=0}^{\infty}\frac{(1)_{l,-\lambda}(-1)^l}{l!}t^l\sum_{k=0}^{\infty}\mathrm{Bel}_{k,\lambda}\frac{t^k}{k!} \\ &=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k}\mathrm{Bel}_{k,\lambda}(1)_{n-k,-\lambda}(-1)^{n-k}\right)\frac{t^n}{n!}. \end{aligned} \tag{14}$$

Therefore, by (13) and (14), we obtain the following theorem.

Theorem 1. *For $n\geq 0$, we have*

$$\sum_{k=0}^{n}(-1)^{n-k}d_k S_{2,\lambda}(n,k)=\sum_{k=0}^{n}\binom{n}{k}\mathrm{Bel}_{k,\lambda}(1)_{n-k,-\lambda}(-1)^k.$$

From (10), we note that

$$e^{x(e_\lambda(-t)-1)}=\sum_{n=0}^{\infty}\mathrm{Bel}_{n,\lambda}(x)(-1)^n\frac{t^n}{n!}. \tag{15}$$

Thus, by (15), we get

$$\sum_{n=0}^{\infty}\mathrm{Bel}_{n+1,\lambda}(x)(-1)^{n+1}\frac{t^n}{n!}=\frac{d}{dt}e^{x(e_\lambda(-t)-1)}=\frac{-e^{x(e_\lambda(t)-1)}x}{e_{-\lambda}(t)(1-\lambda t)}. \tag{16}$$

$$\begin{aligned} xe^{x(e_\lambda(-t)-1)}&=\sum_{n=0}^{\infty}\left\{\sum_{k=0}^{n}\binom{n}{k}(-1)^k\mathrm{Bel}_{k+1,\lambda}(x)(1)_{n-k,-\lambda}\right\}\frac{t^n}{n!}(1-\lambda t) \\ &=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k}(-1)^k(1)_{n-k,-\lambda}\mathrm{Bel}_{k+1,\lambda}(x)\right)\frac{t^n}{n!} \\ &\quad-\lambda\sum_{n=1}^{\infty}\left(n\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k\mathrm{Bel}_{k+1,\lambda}(x)(1)_{n-k-1,-\lambda}\right)\frac{t^n}{n!}. \end{aligned} \tag{17}$$

By (15) and (17), we get the following theorem.

Theorem 2. *For $n \in \mathbb{Z}$ with $n \geq 0$, we have*

$$(-1)^n x\mathrm{Bel}_{n,\lambda}(x) = \sum_{k=0}^{n} \binom{n}{k} (-1)^k (1)_{n-k,-\lambda} \mathrm{Bel}_{k+1,\lambda}(x) - n\lambda \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \mathrm{Bel}_{k+1,\lambda}(x)(1)_{n-k-1,-\lambda}. \tag{18}$$

Letting $\lambda \to 0$ in (18), we get

$$(-1)^n x\mathrm{Bel}_n(x) = \sum_{k=0}^{n} \binom{n}{k} (-1)^k \mathrm{Bel}_{k+1}(x),$$

where $\mathrm{Bel}_n(x)$ are the ordinary Bell polynomials given by

$$e^{x(e^t-1)} = \sum_{n=0}^{\infty} \mathrm{Bel}_n(x) \frac{t^n}{n!}, \quad (\text{see } [1-10, 13-18]).$$

On the one hand, from (12) we have

$$\frac{d}{dt} e_\lambda\big(x(e_\lambda(t)-1)\big) = \sum_{n=0}^{\infty} \phi_{n+1,\lambda}(x) \frac{t^n}{n!}. \tag{19}$$

On the other hand, we also have

$$\frac{d}{dt} e_\lambda\big(x(e_\lambda(t)-1)\big) = \frac{x e_\lambda^{1-\lambda}(t)}{1+\lambda x(e_\lambda(t)-1)} e_\lambda\big(x(e_\lambda(t)-1)\big). \tag{20}$$

Equivalently, we have

$$e_\lambda^{\lambda-1}(t) \frac{d}{dt} e_\lambda\big(x(e_\lambda(t)-1)\big) = \frac{x}{1+\lambda x(e_\lambda(t)-1)} e_\lambda\big(x(e_\lambda(t)-1)\big). \tag{21}$$

Thus, by (8), (19) and (21), we get

$$\begin{aligned} &\sum_{n=0}^{\infty} \sum_{k=0}^{n} \binom{n}{k} (\lambda-1)_{n-k,\lambda} \phi_{k+1,\lambda}(x) \frac{t^n}{n!} \\ &= x \sum_{l=0}^{\infty} F_{l,\lambda}(-\lambda x) \frac{t^l}{l!} \sum_{k=0}^{\infty} \phi_{k,\lambda}(x) \frac{t^k}{k!} \\ &= x \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \binom{n}{k} \phi_{k,\lambda}(x) F_{n-k,\lambda}(-\lambda x) \right) \frac{t^n}{n!}. \end{aligned} \tag{22}$$

Therefore, by comparing the coefficients on both sides of (22), we obtain the following theorem.

Theorem 3. *For $n \geq 0$, we have*

$$\sum_{k=0}^{n} \binom{n}{k} (\lambda-1)_{n-k,\lambda} \phi_{k+1,\lambda}(x) = x \sum_{k=0}^{n} \binom{n}{k} \phi_{k,\lambda}(x) F_{n-k,\lambda}(-\lambda x).$$

Remark. From (10), we note that

$$\mathrm{Bel}_{n,\lambda}(x) = e^{-x} \sum_{k=0}^{\infty} \frac{(k)_{n,\lambda}}{k!} x^k = e^{-x} \left(x \frac{d}{dx} \right)_{n,\lambda} e^x, \quad (n \geq 0). \tag{23}$$

Thus, we have

$$
\begin{aligned}
\mathrm{Bel}_{n+1,\lambda}(x) &= \sum_{k=0}^{n} S_{2,\lambda}(n+1,k)x^k \\
&= \sum_{k=0}^{n+1} \left\{ S_{2,\lambda}(n,k-1) + (k-n\lambda)S_{2,\lambda}(n,k) \right\} x^k \\
&= \sum_{k=1}^{n+1} S_{2,\lambda}(n,k-1)x^k + \sum_{k=0}^{n+1} S_{2,\lambda}(n,k)(k-n\lambda)x^k \\
&= x\sum_{k=0}^{n} S_{2,\lambda}(n,k)x^k + \left(x\frac{d}{dx} - n\lambda \right) \sum_{k=0}^{n} S_{2,\lambda}(n,k)x^k \\
&= x\mathrm{Bel}_{n,\lambda}(x) + \left(x\frac{d}{dx} - n\lambda \right) \mathrm{Bel}_{n,\lambda}(x), \quad \text{(see [14])}.
\end{aligned} \tag{24}
$$

By (24), we get

$$
\begin{aligned}
\mathrm{Bel}_{n+1,\lambda}(x) &= x\mathrm{Bel}_{n,\lambda}(x) + \left(x\frac{d}{dx} - n\lambda \right) \mathrm{Bel}_{n,\lambda}(x) \\
&= x\sum_{k=0}^{n} \binom{n}{k} (1-\lambda)_{n-k,\lambda} \mathrm{Bel}_{k,\lambda}(x), \quad \text{(see [14])}.
\end{aligned} \tag{25}
$$

3. Further Remark

In this section, we consider the modified degenerate Bell polynomials given by

$$
e_\lambda\left(x(e^t-1)\right) = \sum_{n=0}^{\infty} \phi^*_{n,\lambda}(x)\frac{t^n}{n!}. \tag{26}
$$

From (26), we have

$$
\begin{aligned}
\sum_{n=0}^{\infty} \phi^*_{n,\lambda}(x)\frac{t^n}{n!} = e_\lambda\left(x(e^t-1)\right) &= \sum_{k=0}^{\infty} \frac{(1)_{k,\lambda}}{k!} x^k (e^t-1)^k \\
&= \sum_{k=0}^{\infty} (1)_{k,\lambda} x^k \sum_{n=k}^{\infty} S_{2,\lambda}(n,k)\frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} (1)_{k,\lambda} x^k S_2(n,k) \right) \frac{t^n}{n!}.
\end{aligned} \tag{27}
$$

Thus, by (27), we get

$$
\phi^*_{n,\lambda} = \sum_{k=0}^{n} (1)_{k,\lambda} S_2(n,k) x^k, \quad (n \geq 0). \tag{28}
$$

From (26), we note that

$$
\begin{aligned}
\sum_{n=0}^{\infty} \phi^*_{n+1,\lambda}(x)\frac{t^n}{n!} &= \frac{d}{dt}\left(e_\lambda(x(e^t-1)) \right) \\
&= xe^t e_\lambda^{1-\lambda}\left(x(e^t-1) \right).
\end{aligned} \tag{29}
$$

Equivalently, we have

$$
e^t \sum_{n=0}^{\infty} \phi^*_{n+1,\lambda}(x)(-1)^n \frac{t^n}{n!} = xe_\lambda^{1-\lambda}\left(x(e^{-t}-1) \right). \tag{30}
$$

Thus, by (30), we get

$$\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k}\phi_{k+1,\lambda}^{*}(x)(-1)^{k}\right)\frac{t^{n}}{n!}=x\sum_{k=0}^{\infty}(1-\lambda)_{k,\lambda}x^{k}\frac{1}{k!}\left(e^{-t}-1\right)^{k}$$
$$=x\sum_{k=0}^{\infty}(1-\lambda)_{k,\lambda}x^{k}\sum_{n=k}^{\infty}S_{2}(n,k)(-1)^{n}\frac{t^{n}}{n!}$$
$$=\sum_{n=0}^{\infty}\left(x\sum_{k=0}^{n}(1-\lambda)_{k,\lambda}x^{k}S_{2}(n,k)(-1)^{n}\right)\frac{t^{n}}{n!}. \tag{31}$$

By comparing the coefficients on both sides of (31), we get the following theorem.

Theorem 4. *For $n \geq 0$, we have*

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}\phi_{k+1,\lambda}^{*}(x)=(-1)^{n}x\sum_{k=0}^{n}(1-\lambda)_{k,\lambda}S_{2}(n,k)x^{k}, \tag{32}$$

where n is a nonnegative integer.

By letting $\lambda \to 0$ in (32), we obtain

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}\mathrm{Bel}_{k+1}(x)=(-1)^{n}x\sum_{k=0}^{n}S_{2}(n,k)x^{k}=(-1)^{n}x\mathrm{Bel}_{n}(x). \tag{33}$$

We observe that

$$\sum_{k=0}^{n+1}\left(\binom{n+1}{k}-\binom{n}{k}\right)(-1)^{k}\mathrm{Bel}_{k}(x)=\sum_{k=1}^{n+1}\binom{n}{k-1}(-1)^{k}\mathrm{Bel}_{k}(x)$$
$$=\sum_{k=0}^{n}\binom{n}{k}(-1)^{k+1}\mathrm{Bel}_{k+1}(x). \tag{34}$$

By (33) and (34), we get

$$\sum_{k=0}^{n}\binom{n}{k}(-1)^{k}\mathrm{Bel}_{k}(x)=\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^{k}\mathrm{Bel}_{k}(x)+\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^{k+1}\mathrm{Bel}_{k+1}(x)$$
$$=\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^{k}\mathrm{Bel}_{k}(x)+(-1)^{n}x\mathrm{Bel}_{n-1}(x).$$

Continuing this process, we obtain the following theorem.

Theorem 5. *For $n \geq 0$, we have*

$$\sum_{k=0}^{n}\binom{n}{k}(-1)^{k}\mathrm{Bel}_{k}(x)=1+(-1)x\mathrm{Bel}_{0}(x)+(-1)^{2}x\mathrm{Bel}_{1}(x)+\cdots+(-1)^{n}x\mathrm{Bel}_{n-1}(x)$$
$$=1+x\sum_{k=0}^{n-1}(-1)^{k+1}\mathrm{Bel}_{k}(x).$$

4. Conclusion

The explorations for degenerate versions of some special numbers and polynomials are not just limited to polynomials but also extended to transcendental functions, like gamma functions (see [11,12]).

In this paper, we studied the Bell polynomials and three kinds of their degenerate versions, namely the degenerate Bell polynomials, the type 2 degenerate Bell polynomials and the modified degenerate Bell polynomials. We derived some identities on the Bell polynomials and the three kinds of their degenerate versions associated with the Stirling numers of the second kind, the degenerate Stirling numbers of the second kind, the derangement numbers and the degenerate Fubini polynomials.

As one of our future projects, we would like to continue to investigate various degenerate versions of some special numbers and polynomials, and to find their applications to physics, science and engineering as well.

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DEPARTMENT OF MATHEMATICS, KWANGWOON UNIVERSITY, SEOUL 139-701, REPUBLIC OF KOREA
Email address: `tkkim@kw.ac.kr`

DEPARTMENT OF MATHEMATICS, SOGANG UNIVERSITY, SEOUL 121-742, REPUBLIC OF KOREA
Email address: `dskim@sogang.ac.kr`

DEPARTMENT OF MATHEMATICS, KWANGWOON UNIVERSITY, SEOUL 139-701, REPUBLIC OF KOREA
Email address: `vydmlxov@naver.com`

DEPARTMENT OF MATHEMATICS EDUCATION, GYEONGSANG NATIONAL UNIVERSITY, JIN-JU, REPUBLIC OF KOREA
Email address: `mathkjk26@gnu.ac.kr`