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ON CERTAIN SUBCLASSES OF STARLIKE AND CONVEX FUNCTIONS ASSOCIATED WITH WRIGHT FUNCTION

NIRANJAN BASAVANTHAPPA GATTI[1], SAURABH PORWAL[2,*] AND NANJUNDAN MAGESH[3]

[1]Department of Mathematics,
Government Science College
Chitradurga-577501 Karnataka, India.
e-mail : nbg_71@rediffmail.com
[2,*]Department of Mathematics
Ram Sahai Government Degree College,
Bairi-Shivrajpur, Kanpur-209205, (U.P.), India
e-mail : saurabhjcb@rediffmail.com
ORCID Address: https://orcid.org/0000-0003-0847-3550.
[3]Post-Graduate and Research Department of Mathematics
Govt Arts College (Men), Krishnagiri - 635 001, Tamilnadu, India.
e-mail : nmagi_2000@yahoo.co.in
ORCID Address: http://orcid.org/0000-0002-0764-8390.

Abstract. In the present investigation to obtain certain sufficient conditions for normalized Wright function to be in certain classes of starlike and convex functions defined in unit disc $\Delta := \{z \in \mathbb{C} : |z| < 1\}$. Further, we obtain some inclusion relations and integral operator associated with Wright function.
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1. Introduction

Let $\mathscr{A}$ denote the class of all analytic functions in the open unit disc

$$\Delta := \{z \ : \ z \in \mathbb{C} \text{ and } |z| < 1\}$$

with the normalization $f(0) = 0$ and $f'(0) = 1$. As usual, we denote by $\mathscr{S}$ the subclass of $\mathscr{A}$ consisting of functions which are univalent in Δ. The functions of the class $\mathscr{S}$ can be represented by the power series expansion about the origin in the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Further, $\mathscr{T}$ denotes the subclass of $\mathscr{S}$ consisting the functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} |a_n| \ z^n. \tag{1.2}$$

A function $f \in \mathscr{A}$ is said to be a starlike functions of order α $(0 \leq \alpha < 1)$, if it satisfy the condition

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha, \qquad z \in \Delta.$$

A function $f \in \mathscr{A}$ is said to be a convex functions of order α $(0 \leq \alpha < 1)$, if it satisfy the condition

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \qquad z \in \Delta.$$

The class of starlike functions of order α and convex functions of order α are denoted by $\mathscr{S}^*(\alpha)$ and $\mathscr{K}(\alpha)$ respectively. These classes are extensively studied by Robertson [20] and Silverman [21]

$*$ Corresponding author: Saurabh Porwal.

A function $f \in \mathscr{A}$ be of the form (1.1) is said to be in the class $\mathscr{P}_\lambda(\alpha)$, if it satisfies the following condition

$$\Re\left(\frac{zf'(z)+\lambda z^2 f''(z)}{(1-\lambda)f(z)+\lambda zf'(z)}\right) > \alpha, \qquad z \in \Delta,$$

where $0 \leq \lambda < 1$ and $0 \leq \alpha < 1$.

The sufficient condition for $f \in \mathscr{A}$ be in the class $\mathscr{P}_\lambda(\alpha)$ as follows:

Lemma 1.1. *A function $f \in \mathscr{A}$ belongs to the class $\mathscr{P}_\lambda(\alpha)$, if*

$$\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)|a_n| \leq 1-\alpha. \tag{1.3}$$

Also, we denote by

$$\mathscr{P}^*_\lambda(\alpha) = \mathscr{P}_\lambda(\alpha) \cap \mathscr{T}.$$

Further, the necessary and sufficient condition for $f \in \mathscr{T}$ be in the class $\mathscr{P}^*_\lambda(\alpha)$ as follows:

Remark 1.1. Let $f \in \mathscr{A}$ be of the form (1.2), then $f \in \mathscr{P}^*_\lambda(\alpha)$, if and only if

$$\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)|a_n| \leq 1-\alpha. \tag{1.4}$$

A function $f \in \mathscr{A}$ be of the form (1.1)is said to be in the class $\mathscr{K}_\lambda(\alpha)$, if it satisfy the following condition

$$\Re\left(\frac{\lambda z^3 f'''(z)+(1+2\lambda)z^2 f''(z)+zf'(z)}{zf'(z)+\lambda z^2 f'(z)}\right) > \alpha, \qquad z \in \Delta,$$

where $0 \leq \lambda < 1$ and $0 \leq \alpha < 1$.

The sufficient condition for $f \in \mathcal{A}$ be in the class $\mathscr{K}_\lambda(\alpha)$ as follows:

Lemma 1.2. *A function $f \in \mathscr{A}$ belongs to the class $\mathscr{K}_\lambda(\alpha)$, if*

$$\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)|a_n| \leq 1-\alpha. \tag{1.5}$$

Also, we denote by

$$\mathscr{K}^*_\lambda(\alpha) = \mathscr{K}_\lambda(\alpha) \cap \mathscr{T}.$$

Further, the necessary and sufficient condition for $f \in \mathscr{T}$ be in the class $\mathscr{K}^*_\lambda(\alpha)$ as follows:

Remark 1.2. Let $f \in \mathscr{A}$ be of the form (1.2), then $f \in \mathscr{K}^*_\lambda(\alpha)$, if and only if

$$\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)|a_n| \leq 1-\alpha. \tag{1.6}$$

It is observed that

$$\mathscr{P}^*_0(\alpha) := \mathscr{S}^*(\alpha) \qquad \text{and} \qquad \mathscr{P}^*_1(\alpha) := \mathscr{K}(\alpha) = \mathscr{K}^*_0(\alpha).$$

The classes $\mathscr{P}_\lambda(\alpha)$ and $\mathscr{K}_\lambda(\alpha)$ gives a generalized and unified study of starlike and convex functions of order α. The aforesaid classes were extensively studied by Altıntaş [1], Altıntaş et al. [2,3], Kamali and Akbulut [11] and Kamali and Kadıoğlu [12]. Further, the applications of generalized Bessel functions and Poisson distribution series on these classes were studied by Murugusundaramoorthy *et al.* [14] and Porwal and Kumar [18].

In 1995, Dixit and Pal [7] introduce the class $\mathscr{R}^\tau(A, B)$ consisting of functions $f(z)$ of the form (1.1) which satisfy the inequality

$$\left|\frac{f'(z)-1}{(A-B)\tau - B(f'(z)-1)}\right| < 1, \qquad \tau \in \mathbb{C}\setminus\{0\}, \quad -1 \leq B < A \leq 1, \quad z \in \Delta.$$

By specializing the parameters in the class $\mathscr{R}^\tau(A, B)$ we obtain various well-known classes of univalent functions studied earlier by several researchers.

(1) If we put $\tau = \cos\alpha e^{-i\alpha}$ then it reduces to the class $\mathscr{R}(\alpha, A, B)$ studied by Dashrath [5].

(2) If we put $\tau = 1, A = \delta$ and $B = -\delta$ then it reduces to the class $\mathscr{R}(\delta)$ studied by Caplinger and Causey [4] and Padmanabhan [16].
(3) If we put $\tau = 1$ then it reduces to the class $\mathscr{R}(A,\ B)$ studied by Goel and Mehrok [8].
(4) If we put $\tau = 1,\ A = (1-2\rho)\delta$ and $B = -\delta$, where $0 \leq \rho < 1,\ 0 < \delta \leq 1$, then it reduces to the class $\mathscr{R}(\rho,\ \delta)$ studied by Juneja and Mogra [10].

Further, the class $\mathscr{R}^{\tau}(A,\ B)$ was generalized and studied by Dixit and Chandra [6] and Pathak *et al.* [17].

Lemma 1.3. *[7] If $f \in \mathscr{R}^{\tau}(A,\ B)$ is of the form (1.1), then*

$$|a_n| \leq \frac{(A-B)\,|\tau|}{n}, \qquad n \in \mathbb{N}\setminus\{1\}. \tag{1.7}$$

The result is sharp.

In 1933, Wright [22] introduced a special function, which is named as Wright function (see also [9])and is defined as

$$\mathscr{W}_{\nu,\,\mu}(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!\,\Gamma(n\,\nu+\mu)}, \qquad \nu > -1, \quad \mu \in \mathbb{C}, \tag{1.8}$$

where $\Gamma(\cdot)$ stands for the usual Gamma function. The series (1.8) is absolutely convergent for all $z \in \mathbb{C}$, while for $\nu = -1$ this is absolutely convergent in Δ. Also, Wright [22] shown that (1.8) is an entire function for $\nu > -1$.

Let $\mathbb{W}_{\nu,\,\mu}(z)$ represent the normalized Wright functions defined by

$$\mathbb{W}_{\nu,\,\mu}(z) = \Gamma(\mu)\, z\mathscr{W}_{\nu,\,\mu}(z) := \sum_{n=0}^{\infty} \frac{\Gamma(\mu) z^{n+1}}{n!\,\Gamma(n\,\nu+\mu)}, \qquad \nu > -1, \quad \mu > 0, \tag{1.9}$$

for all $z \in \Delta$. Also, $\mathbb{W}_{\nu,\,\mu}(z)$ can be written as

$$\mathbb{W}_{\nu,\,\mu}(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\mu) z^{n}}{(n-1)!\,\Gamma((n-1)\,\nu+\mu)}, \qquad \nu > -1, \quad \mu > 0. \tag{1.10}$$

The analytical and geometrical properties of normalized Wright function were studied by [13,15,19].

Next we define,

$$\mathscr{T}\mathbb{W}_{\nu,\,\mu}(z) = 2z - \mathbb{W}_{\nu,\,\mu}(z) = z - \sum_{n=2}^{\infty} \frac{\Gamma(\mu) z^{n}}{(n-1)!\,\Gamma((n-1)\,\nu+\mu)}, \qquad \nu > -1, \quad \mu > 0. \tag{1.11}$$

The convolution (or, Hadamard product) of two power series $f(z)$ of the form (1.1) and

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

is given by

$$(f * g)(z) = f(z) * g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \qquad z \in \mathbb{U}.$$

Now, we consider a linear operator $\Omega(\nu,\ \mu)\ :\ \mathscr{A} \to \mathscr{A}$ defined by

$$\Omega(\nu,\ \mu) f(z) = \mathbb{W}_{\nu,\,\mu}(z) * f(z) := z + \sum_{n=2}^{\infty} \frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)} \frac{a_n z^n}{(n-1)!}. \tag{1.12}$$

The applications of special functions are widely used in Geometric Function Theory. Note worthy contributions in this directions may be found in [9,13,15,19]. Motivating with the above mentioned work, we obtain sufficient conditions for normalized Wright functions belonging to the classes $\mathscr{P}_{\lambda}(\alpha)$ and $\mathscr{K}_{\lambda}(\alpha)$ and connections of these subclasses with $\mathscr{R}^{\tau}(A,\ B)$. Finally, we discuss an integral operator associated with normalized Wright function.

The following lemma is an easy consequences from the definition of normalized Wright function.

Lemma 1.4. *For all $\nu \geq 0$ and $\mu > 0$, we have*

(a) $\sum_{n=0}^{\infty} \frac{\Gamma(\mu)}{(n+1)!\,\Gamma((n+1)\nu+\mu)} = \mathbb{W}_{\nu,\mu}(1) - 1.$

(b) $\sum_{n=0}^{\infty} \frac{\Gamma(\mu)}{n!\,\Gamma((n+1)\nu+\mu)} = \mathbb{W}'_{\nu,\mu}(1) - \mathbb{W}_{\nu,\mu}(1).$

(c) $\sum_{n=0}^{\infty} \frac{\Gamma(\mu)}{(n-1)!\,\Gamma((n+1)\nu+\mu)} = \mathbb{W}''_{\nu,\mu}(1) - 2\mathbb{W}'_{\nu,\mu}(1) + 2\mathbb{W}_{\nu,\mu}(1).$

(d) $\sum_{n=0}^{\infty} \frac{\Gamma(\mu)}{(n-2)!\,\Gamma((n+1)\nu+\mu)} = \mathbb{W}'''_{\nu,\mu}(1) - 3\mathbb{W}''_{\nu,\mu}(1) + 6\mathbb{W}'_{\nu,\mu}(1) - 6\mathbb{W}_{\nu,\mu}(1).$

2. Main Results

Theorem 2.1. *Let $\nu, \mu > 0$. If for some $\lambda\ (0 \leq \lambda < 1)$, $\alpha\ (0 \leq \alpha < 1)$ and the inequality*

$$\lambda\mathbb{W}''_{\nu,\mu}(1) + (1-\alpha\lambda)\mathbb{W}'_{\nu,\mu}(1) - \alpha(1-\lambda)\mathbb{W}_{\nu,\mu}(1) \leq 2(1-\alpha) \tag{2.1}$$

is satisfied, then $\mathbb{W}_{\nu,\mu}(z) \in \mathscr{P}_\lambda(\alpha)$.

Proof. Since

$$\mathbb{W}_{\nu,\mu}(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)} \frac{z^n}{(n-1)!}.$$

To prove that $\mathbb{W}_{\nu,\mu}(z) \in \mathscr{P}_\lambda(\alpha)$, and by virtue of Lemma 1.1, it suffices to show that

$$\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!} \leq 1-\alpha.$$

Now,

$$\begin{aligned}
&\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad= \sum_{n=2}^{\infty}\{\lambda(n-1)(n-2) + (1+2\lambda-\alpha\lambda)(n-1) + (1-\alpha)\}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad= \sum_{n=2}^{\infty}\lambda(n-1)(n-2)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\qquad+ \sum_{n=2}^{\infty}(1+2\lambda-\alpha\lambda)(n-1)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\qquad+ \sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad= \sum_{n=2}^{\infty}\lambda\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-3)!} + \sum_{n=2}^{\infty}(1+2\lambda-\alpha\lambda)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-2)!}\\
&\qquad+ \sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad= \lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n-1)!} + (1+2\lambda-\alpha\lambda)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!}\\
&\qquad+ (1-\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\\
&\quad= \lambda\{\mathbb{W}''_{\nu,\mu}(1) - 2\mathbb{W}'_{\nu,\mu}(1) + 2\mathbb{W}_{\nu,\mu}(1)\} + (1+2\lambda-\alpha\lambda)\{\mathbb{W}'_{\nu,\mu}(1) - \mathbb{W}_{\nu,\mu}(1)\}\\
&\qquad+ (1-\alpha)\{\mathbb{W}_{\nu,\mu}(1) - 1\}\\
&\quad= \lambda\mathbb{W}''_{\nu,\mu}(1) + (1-\alpha\lambda)\mathbb{W}'_{\nu,\mu}(1) - \alpha(1-\lambda)\mathbb{W}_{\nu,\mu}(1) - (1-\alpha)
\end{aligned}$$

$$\leq 1-\alpha$$

by the given hypothesis. This completes the proof of Theorem 2.1. ∎

Theorem 2.2. *Let* $\nu,\ \mu>0$. *If for some* $\lambda\ (0\leq\lambda<1)$, $\alpha\ (0\leq\alpha<1)$ *and the inequality*

$$\lambda\mathbb{W}'''_{\nu,\mu}(1)+(1+2\lambda-\alpha\lambda)\mathbb{W}''_{\nu,\mu}(1)+(1-\alpha)\mathbb{W}_{\nu,\mu}(1)\leq 2(1-\alpha) \tag{2.2}$$

is satisfied, then $\mathbb{W}_{\nu,\mu}(z)\in\mathscr{K}_{\lambda}(\alpha)$.

Proof. Since

$$\mathbb{W}_{\nu,\mu}(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{z^n}{(n-1)!}.$$

To prove that $\mathbb{W}_{\nu,\mu}(z)\in\mathscr{K}_{\lambda}(\alpha)$, and by virtue of Lemma 1.2, it suffices to show that

$$\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\leq 1-\alpha.$$

Now,

$$\begin{aligned}
&\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad=\sum_{n=2}^{\infty}\{\lambda(n-1)(n-2)(n-3)+(1+5\lambda-\alpha\lambda)(n-1)(n-2)\\
&\qquad+(3+4\lambda-2\alpha\lambda-\alpha)(n-1)+(1-\alpha)\}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad=\sum_{n=2}^{\infty}\lambda(n-1)(n-2)(n-3)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\qquad+\sum_{n=2}^{\infty}(1+5\lambda-\alpha\lambda)(n-1)(n-2)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\qquad+\sum_{n=2}^{\infty}(3+4\lambda-2\alpha\lambda-\alpha)(n-1)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\qquad+\sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad=\sum_{n=2}^{\infty}\lambda\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-4)!}\\
&\qquad+\sum_{n=2}^{\infty}(1+5\lambda-\alpha\lambda)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-3)!}\\
&\qquad+\sum_{n=2}^{\infty}(3+4\lambda-2\alpha\lambda-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-2)!}\\
&\qquad+\sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad=\lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n-2)!}\\
&\qquad+(1+5\lambda-\alpha\lambda)\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n-1)!}
\end{aligned}$$

$$+(3+4\lambda-2\alpha\lambda-\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!}$$
$$+(1-\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}$$
$$=\lambda\{\mathbb{W}'''_{\nu,\mu}(1)-3\mathbb{W}''_{\nu,\mu}(1)+6\mathbb{W}'_{\nu,\mu}(1)-6\mathbb{W}_{\nu,\mu}(1)\}$$
$$+(1+5\lambda-\alpha\lambda)\{\mathrm{W}''_{\nu,\mu}(1)-2\mathrm{W}'_{\nu,\mu}(1)+2\mathbb{W}_{\nu,\mu}(1)\}$$
$$+(3+4\lambda-2\alpha\lambda-\alpha)\{\mathbb{W}'_{\nu,\mu}(1)-\mathbb{W}_{\nu,\mu}(1)\}$$
$$+(1-\alpha)\{\mathbb{W}_{\nu,\mu}(1)-1\}$$
$$=\lambda\mathrm{W}'''_{\nu,\mu}(1)+(1+2\lambda-\alpha\lambda)\mathbb{W}''_{\nu,\mu}(1)+(1-\alpha)\mathbb{W}'_{\nu,\mu}(1)-(1-\alpha)$$
$$\leq 1-\alpha$$

by the given hypothesis. This completes the proof of Theorem 2.2. ∎

3. Inclusion Relations

Theorem 3.1. *Let $\nu,\ \mu>0$. If for some $\lambda\ (0\leq\lambda<1)$, $\alpha\ (0\leq\alpha<1)$, $f\in\mathscr{R}^{\tau}(A,\ B)$, where $\tau\in\mathbb{C}\backslash\{0\}$ and the inequality*

$$(A-B)|\tau|\{\lambda\mathbb{W}''_{\nu,\mu}(1)+(1-\alpha\lambda)\mathbb{W}'_{\nu,\mu}(1)+\alpha(\lambda-1)\mathbb{W}_{\nu,\mu}(1)-(1-\alpha)\}\leq 1-\alpha \tag{3.1}$$

is satisfied, then $\Omega(\nu,\ \mu)f\in\mathscr{K}_{\lambda}(\alpha)$.

Proof. Let f be of the form (1.1) belongs to the class $\mathscr{R}^{\tau}(A,\ B)$. To show that $\Omega(\nu,\ \mu)f\in\mathscr{K}_{\lambda}(\alpha)$ by virtue of Lemma 1.2, it suffices to show that

$$\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}|a_n|\leq 1-\alpha.$$

Since $f\in\mathscr{R}^{\tau}(A,\ B)$, then by Lemma 1.3, we have

$$|a_n|\leq\frac{(A-B)|\tau|}{n}.$$

Now,

$$\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}|a_n|$$
$$\leq(A-B)|\tau|\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}$$
$$=(A-B)|\tau|\sum_{n=2}^{\infty}\{\lambda(n-1)(n-2)+(1+2\lambda-\alpha\lambda)(n-1)+(1-\alpha)\}$$
$$\times\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}$$
$$=(A-B)|\tau|\left\{\sum_{n=2}^{\infty}\lambda(n-1)(n-2)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\right.$$
$$+\sum_{n=2}^{\infty}(1+2\lambda-\alpha\lambda)(n-1)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}$$
$$\left.+\sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\right\}$$
$$=(A-B)|\tau|\left\{\sum_{n=2}^{\infty}\lambda\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-3)!}+\sum_{n=2}^{\infty}(1+2\lambda-\alpha\lambda)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-2)!}\right.$$

$$+\sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\Bigg\}$$

$$= (A-B)|\tau|\left\{\lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n-1)!}+(1+2\lambda-\alpha\lambda)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!}\right.$$

$$\left.+(1-\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\right\}$$

$$= (A-B)|\tau|\left[\lambda\{\mathbb{W}''_{\nu,\,\mu}(1)-2\mathbb{W}'_{\nu,\,\mu}(1)+2\mathbb{W}_{\nu,\,\mu}(1)\}+(1+2\lambda-\alpha\lambda)\{\mathbb{W}'_{\nu,\,\mu}(1)-\mathbb{W}_{\nu,\,\mu}(1)\}\right.$$

$$\left.+(1-\alpha)\{\mathbb{W}_{\nu,\,\mu}(1)-1\}\right]$$

$$= (A-B)|\tau|\{\lambda\mathbb{W}''_{\nu,\,\mu}(1)+(1-\alpha\lambda)\mathbb{W}'_{\nu,\,\mu}(1)+\alpha(\lambda-1)\mathbb{W}_{\nu,\,\mu}(1)-(1-\alpha)\}$$

$$\leq 1-\alpha$$

by the given hypothesis. This completes the proof of Theorem 3.1. ∎

Theorem 3.2. *Let $\nu,\ \mu>0$. If for some λ $(0\leq\lambda<1)$, α $(0\leq\alpha<1)$, $f\in\mathscr{B}^{\tau}(A,\ B)$, where $\tau\in\mathbb{C}\backslash\{0\}$ and the inequality*

$$(A-B)|\tau|\left\{\lambda\mathbb{W}'_{\nu,\,\mu}(1)+[1-\lambda(1+\alpha)]\mathbb{W}_{\nu,\,\mu}(1)\right.$$
$$\left.-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\,\mu-\nu}(1)-1]-(1-\alpha)\right\}\leq 1-\alpha \tag{3.2}$$

is satisfied, then $\Omega(\nu,\ \mu)f\in\mathscr{P}_{\lambda}(\alpha)$.

Proof. Let f be of the form (1.1) belongs to the class $\mathscr{R}^{\tau}(A,\ B)$. To show that $\Omega(\nu,\ \mu)f\in\mathscr{P}_{\lambda}(\alpha)$ by virtue of (1.3) from Lemma 1.1, it suffices to show that

$$\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}|a_n|\leq 1-\alpha.$$

Since $f\in\mathscr{R}^{\tau}(A,\ B)$, then by Lemma 1.3, we have

$$|a_n|\leq\frac{(A-B)|\tau|}{n}.$$

Now,

$$\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}|a_n|$$

$$\leq (A-B)|\tau|\sum_{n=2}^{\infty}[n[1+\lambda(n-1)]-n\lambda\alpha-\alpha(1-\lambda)]\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}$$

$$= (A-B)|\tau|\left\{\sum_{n=2}^{\infty}\lambda(n-1)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\right.$$

$$\left.+\sum_{n=2}^{\infty}(1-\lambda\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}-\sum_{n=2}^{\infty}\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\right\}$$

$$= (A-B)|\tau|\left\{\lambda\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-2)!}+(1-\lambda\alpha)\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\right.$$

$$\left.-\alpha(1-\lambda)\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\right\}$$

$$= (A-B)|\tau|\left\{\lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!}+(1-\lambda\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\right.$$

$$
\begin{aligned}
&-\alpha(1-\lambda)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+2)!}\Bigg\}\\
&=(A-B)|\tau|\left\{\lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!}+(1-\lambda\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\right.\\
&\quad\left.-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}\left[\sum_{n=0}^{\infty}\frac{\Gamma(\mu-\lambda)}{\Gamma((n+1)\nu+\mu-\nu)}\frac{1}{(n+1)!}-\frac{\Gamma(\mu-\nu)}{\Gamma(\mu)}\right]\right\}\\
&=(A-B)|\tau|\left\{\lambda[\mathbb{W}'_{\nu,\mu},\ \mu(1)-\mathbb{W}_{\nu,\mu}(1)]+(1-\lambda\alpha)[\mathbb{W}_{\nu,\mu}(1)-1]\right.\\
&\quad\left.-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}\left[[\mathbb{W}'_{\nu,\mu-\nu}(1)-1]-\frac{\Gamma(\mu-\nu)}{\Gamma(\mu)}\right]\right\}\\
&=(A-B)|\tau|\left\{\lambda[\mathbb{W}'_{\nu,\mu}(1)-\mathbb{W}_{\nu,\mu}(1)]+(1-\lambda\alpha)[\mathbb{W}_{\nu,\mu}(1)-1]\right.\\
&\quad\left.-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1]+\alpha(1-\lambda)\right\}\\
&=(A-B)|\tau|\left\{\lambda\mathbb{W}'_{\nu,\mu}(1)-\lambda\mathbb{W}_{\nu,\mu}(1)+(1-\lambda\alpha)\mathbb{W}_{\nu,\mu}(1)-1+\lambda\alpha\right.\\
&\quad\left.-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1]+\alpha-\alpha\lambda\right\}\\
&=(A-B)|\tau|\left\{\lambda\mathbb{W}'_{\nu,\mu}(1)+[1-\lambda(1+\alpha)]\mathbb{W}_{\nu,\mu}(1)\right.\\
&\quad\left.-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1]-(1-\alpha)\right\}\\
&\le 1-\alpha
\end{aligned}
$$

by the given hypothesis. This completes the proof of Theorem 3.2 ∎

4. An Integral Operator

In this section, we introduce a new integral operator associated with normalized Wright function and obtain sufficient conditions for this operator belonging to the classes $\mathscr{P}^*_\lambda(\alpha)$ and $\mathscr{K}^*_\lambda(\alpha)$.

First we define an integral operators $\mathscr{J}^{\nu}_{\mu}(z)$ and $\mathscr{T}\mathscr{J}^{\nu}_{\mu}(z)$ as follow:

$$
\mathscr{J}^{\nu}_{\mu}(z)=\int_0^z\frac{\mathbb{W}_{\nu,\mu}(t)}{t}dt \qquad\text{and}\qquad \mathscr{T}\mathscr{J}^{\nu}_{\mu}(z)=\int_0^z\frac{\mathscr{T}\mathbb{W}_{\nu,\mu}(t)}{t}dt \tag{4.1}
$$

Theorem 4.1. *Let $\nu,\ \mu>0$. If for some $\lambda\,(0\le\lambda<1)$ and $\alpha\,(0\le\alpha<1)$, then $\mathscr{T}\mathscr{J}^{\nu}_{\mu}(z)$ is in the class $\mathscr{K}^*_\lambda(\alpha)$ if and only if*

$$
\lambda\mathbb{W}''_{\nu,\mu}(1)+(1-\alpha\lambda)\mathbb{W}'_{\nu,\mu}(1)-\alpha(1-\lambda)\mathbb{W}_{\nu,\mu}(1)\le 2(1-\alpha). \tag{4.2}
$$

Proof. From the representation (4.1), we have

$$
\mathscr{T}\mathscr{J}^{\nu}_{\mu}(z)=z-\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{z^n}{n!}\ .
$$

To prove that $\mathscr{T}\mathscr{J}^{\nu}_{\mu}(z)\in\mathscr{K}_\lambda(\alpha)$ from Remark 1.2, it is enough to prove that

$$
\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\le 1-\alpha.
$$

Now,

$$
\begin{aligned}
&\sum_{n=2}^{\infty}n[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\\
&=\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{n=2}^{\infty}\{\lambda(n-1)(n-2)+(1+2\lambda-\alpha\lambda)(n-1)+(1-\alpha)\}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&= \sum_{n=2}^{\infty}\lambda(n-1)(n-2)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad+\sum_{n=2}^{\infty}(1+2\lambda-\alpha\lambda)(n-1)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\quad+\sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&= \sum_{n=2}^{\infty}\lambda\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-3)!}+\sum_{n=2}^{\infty}(1+2\lambda-\alpha\lambda)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-2)!}\\
&\quad+\sum_{n=2}^{\infty}(1-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&= \lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n-1)!}+(1+2\lambda-\alpha\lambda)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!}\\
&\quad+(1-\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\\
&= \lambda\mathbb{W}''_{\nu,\mu}(1)+(1-\alpha\lambda)\mathbb{W}'_{\nu,\mu}(1)-\alpha(1-\lambda)\mathbb{W}_{\nu,\mu}(1)-(1-\alpha)\\
&\le 1-\alpha
\end{aligned}
$$

by given hypothesis. Thus the proof of Theorem 4.1 is established. I ∎

Theorem 4.2. *Let $\nu,\ \mu>0$. If for some $\lambda\ (0\le\lambda<1)$ and $\alpha\ (0\le\alpha<1)$, then $\mathscr{T}\mathscr{J}_{\mu}^{\nu}(z)$ is in the class $\mathscr{P}_{\lambda}^{*}(\alpha)$ if and only if*

$$
\lambda\mathbb{W}'_{\nu,\mu}(1)+[1-\lambda(1+\alpha)]\mathbb{W}_{\nu,\mu}(1)-\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1]\le 2(1-\alpha). \tag{4.3}
$$

Proof. From the representation (4.1), we have

$$
\mathscr{T}\mathscr{J}_{\mu}^{\nu}(z)=z-\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{z^n}{n!}\ .
$$

To prove that $\mathscr{T}\mathscr{J}_{\mu}^{\nu}(z)\in\mathscr{P}_{\lambda}^{*}(\alpha)$ from Remark 1.1, it is sufficient to prove that

$$
\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\le 1-\alpha.
$$

Now,

$$
\begin{aligned}
&\sum_{n=2}^{\infty}[1+\lambda(n-1)](n-\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\\
&\quad= \sum_{n=2}^{\infty}\{n[1+\lambda(n-1)]-n\lambda\alpha-\alpha(1-\lambda)\}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\\
&\quad= \sum_{n=2}^{\infty}\lambda(n-1)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}+\sum_{n=2}^{\infty}(1-\lambda\alpha)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}\\
&\qquad-\sum_{n=2}^{\infty}\alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\\
&\quad= \lambda\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-2)!}+(1-\lambda\alpha)\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{(n-1)!}
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \alpha(1-\lambda)\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n-1)\nu+\mu)}\frac{1}{n!}\\
&= \lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!} + (1-\lambda\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\\
&\quad - \alpha(1-\lambda)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+2)!}\\
&= \lambda\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{n!} + (1-\lambda\alpha)\sum_{n=0}^{\infty}\frac{\Gamma(\mu)}{\Gamma((n+1)\nu+\mu)}\frac{1}{(n+1)!}\\
&\quad - \alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}\left\{\sum_{n=0}^{\infty}\frac{\Gamma(\mu-\lambda)}{\Gamma((n+1)\nu+\mu-\nu)}\frac{1}{(n+1)!} - \frac{\Gamma(\mu-\nu)}{\Gamma(\mu)}\right\}\\
&= \lambda[\mathbb{W}'_{\nu,\mu}(1) - \mathbb{W}_{\nu,\mu}(1)] + (1-\lambda\alpha)[\mathbb{W}_{\nu,\mu}(1)-1]\\
&\quad - \alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}\left\{[\mathbb{W}_{\nu,\mu-\nu}(1)-1] - \frac{\Gamma(\mu-\nu)}{\Gamma(\mu)}\right\}\\
&= \lambda[\mathbb{W}'_{\nu,\mu}(1) - \mathbb{W}_{\nu,\mu}(1)] + (1-\lambda\alpha)[\mathbb{W}_{\nu,\mu}(1)-1]\\
&\quad - \alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1] + \alpha(1-\lambda)\\
&= \lambda\mathbb{W}'_{\nu,\mu}(1) - \lambda\mathbb{W}_{\nu,\mu}(1) + (1-\lambda\alpha)\mathbb{W}_{\nu,\mu}(1) - 1 + \lambda\alpha\\
&\quad - \alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1] + \alpha - \alpha\lambda\\
&= \lambda\mathbb{W}'_{\nu,\mu}(1) + [1-\lambda(1+\alpha)]\mathbb{W}_{\nu,\mu}(1) - \alpha(1-\lambda)\frac{\Gamma(\mu)}{\Gamma(\mu-\nu)}[\mathbb{W}_{\nu,\mu-\nu}(1)-1] - (1-\alpha)\\
&\le 1-\alpha
\end{aligned}
$$

by given hypothesis. Thus the proof of Theorem 4.2 is established. ∎

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References

[1] O. Altıntaş, On a subclass of certain starlike functions with negative coefficients, Math. Japon. **36** (1991), no. 3, 489–495.

[2] O. Altıntaş and S. Owa, Neighborhoods of certain analytic functions with negative coefficients, Internat. J. Math. Math. Sci. **19** (1996), no. 4, 797–800.

[3] O. Altintaş, Ő. Őzkan and H. M. Srivastava, Neighborhoods of a class of analytic functions with negative coefficients, Appl. Math. Lett. **13** (2000), no. 3, 63–67.

[4] T. R. Caplinger and W. M. Causey, A class of univalent functions, Proc. Amer. Math. Soc., **39**(2) (1973), 357-361.

[5] Dashrath, On some classes related to spiral-like univalent and multivalent functions, Ph.D. Thesis, Kanpur University, Kanpur, (U.P.), India, 1984.

[6] K. K. Dixit and V. Chandra, On the coefficients of some classes of analytic functions related to complex order, Bull. Cal. Math. Soc., **101** (4) (2009), 351-358.

[7] K. K. Dixit and S. K. Pal, On a class of univalent functions related to complex order, Indian J. Pure Appl. Math., **26** (1995), no. 9, 889-896.

[8] R.M. Goel and B.S. Mehrok, A subclass of univalent functions, J. Aust. Math. Soc., **35**(1983), 1-17.

[9] R. Gorenflo, Y. Luchko, F. Mainardi, Analytic properties and applications of Wright functions, Fract Calc Appl Anal., **2** (1999), no. 4, 383-414.

[10] O.P. Juneja and M.L. Mogra, A class of univalent functions, Bull. Sci. Math. 2^e Series, **103** (1979), 435-447.

[11] M. Kamali and S. Akbulut, On a subclass of certain convex functions with negative coefficients, Appl. Math. Comput. **145** (2003), no. 2-3, 341–350.

[12] M. Kamali and E. Kadıoğlu, On a new subclass of certain starlike functions with negative coefficients, Atti Sem. Mat. Fis. Univ. Modena **48** (2000), no. 1, 31–44.

[13] S. Maharana, J. K. Prajapat and D. Bansal, Geometric properties of Wright function, Math. Bohem., **143** (2018) no. 1, 99-111.

[14] G. Murugusundaramoorthy, K. Vijaya and M. Kasthuri, A note on subclasses of starlike and convex functions associated with Bessel functions, J. Nonlinear Funct. Anal., (2014), Art ID 11, 1-11.
[15] N. Mustafa, Geometric properties of normalized wright functions, Math Comput Appl., **21** (2016), no. 14, 1-10.
[16] K.S. Padmanabhan, On a certain class of functions whose derivatives have a positive real part in the unit disc, Ann. Polon. Math., **23**(1)(1970), 73-81.
[17] A.L. Pathak, K.K. Dixit, S. Porwal and R. Tripathi, On the coefficients of some classes of multivalent functions related to complex order, Malaya J. Mat., **7**(3)(2019), 388-392.
[18] S. Porwal and M. Kumar, A unified study on starlike and convex functions associated with Poisson distribution series, Afr. Mat. **27** (2016), no. 5-6, 1021–1027.
[19] J. K. Prajapat, Certain geometric properties of the Wright functions, Integral Transforms Spec Funct., **26** (2015), no. 3,203-212.
[20] M. S. Robertson, On the theory of univalent functions, Ann. Math., **37** (1936), no. 2, 374-408.
[21] H. Silverman, Univalent functions with negative coefficients, Proc. Amer. Math. Soc., **51** (1975), 109-116.
[22] E. M. Wright, On the coefficients of power series having exponential singularities. J. Lond. Math. Soc., **8** (1933), 71-79.