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A VISUAL PROOF : IF $e < A < B$, THEN $A^B e^B < B^B e^A$

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ABSTRACT. In this paper, we give a visual proof of $A^B e^B < e^A B^B$ when $e < A < B$ without word. Moreover, we obtained $A^B > B^A$ when $e < A < B$ which is the same result as Gallant's but not different method.

1. INTRODUCTION

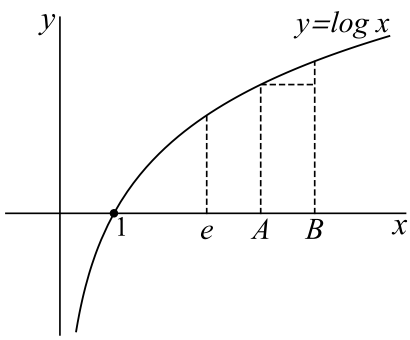
Inequalities play a fundamental role in all branches of mathematics, and various proof methods for various inequalities have been suggested by many researchers. For example, the Cauchy–Schwarz inequality is one of most widely used and most important inequalities in mathematics. Gregorio proved the Cauchy-Schward inequality by using analytic geometry in [3], and Levi also gave a water-based proof of that inequality (see [4]). In [1], authors proved that inequality by using a trapezoid partitioned into three right triangles.

In [5], Sun showed the Jensen's inequality without words, and Viloria-Cortez gave generalization of the Jensen's inequality by using definition of convex functions on n–coordinates in [6].

Gallant provided $A^B > B^A$ when $e < A < B$ in [2].

In this paper, we give a proof of $A^B e^B < e^A B^B$ when $e < A < B$ by using logarithm function without word.

2. MAIN RESULT



Note that

$$\int_A^B \log x dx \geq (B - A) \log A. \tag{1}$$

By (1), we see that

$$(B - A) \log A \leq B \log B - B - A \log A + A,$$

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and thus

$$B - A \le B \log\left(\frac{B}{A}\right). \tag{2}$$

By (2), we get

$$\left(\frac{A}{B}\right)^{B} e^{B} < e^{A},$$

and so we have

$$A^{B}e^{B} < e^{A}B^{B}.$$

Remark. We introduce the following well-known inequality to be compared with our inequality.

$$\begin{aligned}(B - A)\log B \ge \int_A^B \log x dx &= [x \log x - x]_A^B \\ &= B \log B - B - A \log A + A.\end{aligned} \tag{3}$$

By (3), we have

$$A \log A + B - A \ge A \log B,$$

and thus

$$A \log A + (B - A)\log A \ge A \log A + B - A \ge A \log B.$$

Hence, we obtain the followings :

$$B \log A \ge A \log B,$$

and

$$A^{B} \ge B^{A}$$

which is the same that Gallant's result.

3. Conclusion

Inequality occupies a very important place in all fields of mathematics. In addition to pure mathematics, inequalities play an increasingly important role in applied mathematics that finds the optimal condition in a given situation.

In this paper, we found the inequality

$$A^{B}e^{B} < e^{A}B^{B} \text{ when } e < A < B$$

by using logarithm function without word.

In addition, by the similar method to above proof, we obtained $A^{B} > B^{A}$ when $e < A < B$ which is the same result as Gallant's thing but not different proof method.

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