

GROUP OF ONE-DIMENSIONAL BOUNDED PURE PSEUDOREPRESENTATIONS OF A SIMPLE HERMITIAN SYMMETRIC LIE GROUP

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ABSTRACT. We continue the study of the group of one-dimensional bounded pure pseudorepresentations of a group. This group is computed for Hermitian symmetric simple connected simply connected Lie groups.

§ 1. INTRODUCTION

For the definitions, notation, and generalities concerning pseudocharacters, quasicharacters, pseudorepresentations, and quasirepresentations, and also for the definition of the Guichardet–Wigner pseudocharacter on a connected simply connected Hermitian symmetric simple Lie group, see [1]–[3]. The groups of one-dimensional bounded pure pseudorepresentations of a group were introduced in [4].

Obviously, the (tensor or, equivalently, ordinary pointwise) product of two one-dimensional pseudorepresentations of a group is a one-dimensional quasirepresentation. Indeed, if π and ρ are one-dimensional pseudorepresentations of G with $|\pi(g)| \leq C_\pi$ for all $g \in G$ and $|\pi(gh) - \pi(g)\pi(h)| \leq \varepsilon_\pi$ (ε_π is the defect of π) and if $|\rho(g)| \leq C_\rho$ for all $g \in G$ and $|\rho(gh) - \rho(g)\rho(h)| \leq \varepsilon_\rho$, then $C_\pi = 1$, since the restriction of π to every cyclic subgroup is an ordinary character of the subgroup, and a direct calculation shows that $\Delta \leq C_\pi \varepsilon_\rho + C_\rho \varepsilon_\pi + \varepsilon_\pi \varepsilon_\rho = \varepsilon_\rho + \varepsilon_\pi + \varepsilon_\pi \varepsilon_\rho$.

This means that the set of bounded one-dimensional pseudorepresentations of a given group G is a group with respect to the ordinary pointwise

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product, whose identity element is the identity character $\iota(g) = 1 \in \mathbb{C}$, $g \in G$, and the inverse one-dimensional pseudorepresentation to a given bounded one-dimensional pseudorepresentation π is the pseudorepresentation π^{-1} given by the formula $\pi^{-1}(g) = \pi(g^{-1})$, $g \in G$. Denote this group by $\text{BODP}(G)$.

Note that the additional condition that the restriction of a pseudorepresentation to every cyclic group is an ordinary representation is obvious both for $\pi\rho$ (since the product of ordinary characters is a character) and for π^{-1} (since the inverse of an ordinary character is a character).

§ 2. PRELIMINARIES

The group $\text{BODP}(G)$ of bounded one-dimensional pseudorepresentations of a group G contains the subgroup $\text{BODPP}(G)$ introduced in [4]. As follows from the results of [2] (Lemma 3.3.10), for a semisimple Hermitian symmetric Lie group G , every element of $\text{BODP}(G)$ belongs to the subgroup $\text{GBODP}(G)$ of $\text{BODPP}(G)$, and thus $\text{BODP}(G) = \text{BODPP}(G) = \text{GBODP}(G)$.

The group $\text{BODP}(G)$ obviously contains the group \widehat{G} of ordinary characters of G as a subgroup. The subgroup $\text{BODPP}(G)$ coincides with \widehat{G} if G is amenable, see Corollary 3.3.8 of [2]. The universal covering group $\widetilde{\text{SL}}(2, \mathbb{R})$ has nontrivial bounded one-dimensional pseudorepresentations [1–3] but has no nontrivial characters.

Let us describe the group $\text{BODP}(G)$ for simple connected simply connected Hermitian symmetric Lie groups.

§ 3. MAIN THEOREM

Theorem 1. *Let G be a simple connected simply connected Hermitian symmetric Lie group. Then the group $\text{BODP}(G)$ consists of the one-dimensional pseudorepresentations of G of the form $f_t: g \mapsto \exp it\chi(g)$, $g \in G$, where $t \in \mathbb{R}$ and χ stands for the Guichardet–Wigner pseudocharacter of G .*

Thus, the group $\text{BODP}(G)$ is isomorphic to \mathbb{R} .

Proof. The one-dimensional pseudorepresentations of a simple connected simply connected Hermitian symmetric Lie group are described in [2], where the explicit form of these pseudorepresentations is given in terms of the Guichardet–Wigner character on G . Since the product of these pseudorepresentations corresponds to the addition of the values of the parameter t , it follows that $\text{BODP}(G)$ is isomorphic to \mathbb{R} , which completes the proof.

The group G has no nontrivial ordinary characters, and thus the subgroup \tilde{G} of ordinary characters in G is the identity subgroup.

§ 4. CONCLUDING REMARKS

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