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# STUDY OF ROTATIONAL BAND STRUCTURE OF EVEN-EVEN $^{132,134}$SM NUCLEI

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Abstract. The study of neutron-deficient nuclei in the A∼130 mass region has been an interesting subject in nuclear structure physics as this region is considered to be transitional region. In the present work, the positive parity rotational band structure of even-even $^{132,134}$Sm up to the high spin states has been studied in a microscopic frame work of calculations known as Projected Shell Model (PSM). Yrast spectra for $^{132,134}$Sm have been obtained from the PSM calculations and has been found to be in good agreement with the available experimental data. Besides this, the present calculations have also reported the occurrence of back bending at the same spins at which experimental data shows in both the nuclei.

Keywords and phrases. Projected Shell Model (PSM); yrast spectra; back bending; band diagram.

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## 1. Introduction

The geometric shape is one of the most important property of a nucleus and can usually be changed by broken-pair excitation, particle-hole excitation and rotational motion. Atomic nuclei exhibit different shapes which are generally depend on single-particle structure, the collective behaviour and the total angular momentum of the nuclei. It has been found that most deformed nuclei have axially symmetric shapes (prolate or oblate), which is due to the observation of rotational band structures and measurements of their properties [1]. Another interesting phenomenon that occurs in many microscopic systems is shape coexistence. Shape coexistence of different shapes in a single nucleus, and shape transitions as a function of nucleon number, have been observed in light, medium and heavy nuclei [2, 3]. The distinctive character of shape coexistence in nuclei reflects the interplay between single-particle and collective degrees of freedom. Various experimental and theoretical studies of shapes and their evolution and transitions, give crucial information on the origin of nuclear collectivity and shell structures in nuclei far from stability [4]. Nuclei in the mass region A≈130-140 with N approaching to 82 shell closure show a softness with respect to $\gamma$, and hence these transitional nuclei are susceptible to shape changes driven by the rotational alignment of quasiparticle pairs [5]. The proton Fermi surface for these nuclei lies near the bottom of $h_{11/2}$ subshell and, hence, the occupation of these orbitals stabilizes prolate shapes.On the other hand, the neutron Fermi surface lies near the top of the $h_{11/2}$ subshell, rotational alignments of $h_{11/2}$ neutrons alter the nuclear shape of these transitional nuclei towards oblate deformation ($\gamma$ = -60°)[6]. In this mass region, the study of properties of even-even Samarium (Sm) isotopes has been a center for large number of experimental studies in the past[7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. One of the most striking feature of these nuclei is the occurrence of various shape transitions. It would be interesting to know the factors responsible for the variation of low lying yrast states and also the mechanism behind this manner in which transitions from sphericity to deformation take place. Various theoretical attempts have been made in the past to understand the systematics of the low lying yrast states in Sm isotopic mass chain [7, 15, 16, 17, 18]. Otsuka and Sugita [17] analysed the energies and E2 properties of the $0_1^+$ , $2_1^+$ , and $4_1^+$ levels of the Sm isotopes in the s–d–g framework by using Variation after Projection (VAP) method. They observed that the deformed shape transition in these isotopes depends on the boson number. Lister[7] demonstrated the lighter Samarium isotopes(134-138) to be axially symmetric rotors. J.B. Gupta [18]investigated the shape transition of light Sm isotopes in the IBM model and reproduced the spectra of $^{132,134}$Sm. In order to explain the structure of these nuclei, an attempt has been made in the present work to find the structure of even-even $^{132,134}$Sm (Z = 62) nuclei in the PSM and the present PSM calculations have been found to explain the experimental observation quite successfully in $^{132,134}$Sm nuclei.

## 2. METHODOLOGY

### 2.1. **Projected Shell Model (PSM).**

Projected shell model is the natural extension of the shell model which basically begins with the deformed Nilsson single-particle states at a deformation $\epsilon_2$. It makes use of angular-momentum projection technique in order to project out energies from the deformed Nilsson basis hence, makes shell model type of calculations possible for deformed nuclei. The brief philosophy of the presently used PSM [19]. Pairing correlations are incorporated into the Nilsson states by Bardeen-Cooper-Schrieffer (BCS) calculations. Finally, a two-body shell model Hamiltonian is diagonalized in the projected basis thereby obtaining the energy levels for a given spin. One of the most important feature of this quantum mechanical model is that it describes the inner details of the high-spin spectroscopy data with simple physical interpretations.

The multi-qp subspace chosen for the present calculations of even-even Sm isotopes is spanned by the basis set

$$|0\rangle,\ a^\dagger_{\nu1}a^\dagger_{\nu2}|0\rangle,\ a^\dagger_{\pi1}a^\dagger_{\pi2}|0\rangle,\ a^\dagger_{\nu1}a^\dagger_{\nu2}a^\dagger_{\pi1}a^\dagger_{\pi2}|0\rangle$$

Where $\nu{'^s}(\pi{'^s})$ representing the neutron(proton) Nilsson quantum numbers which run over considered (low-lying) quasiparticle states. For even-even nuclei,2-qp states and 4-qp states are possible. 2-qp states are formed by pair of quasineutron or quasiproton and 4-qp states are formed by a pair of quasineutron and a pair of quasiproton.

The Hamiltonian used in this framework consists of a sum of schematic (Quadrupole-Quadrupole + Monopole pairing + Quadrupole pairing) forces which represent different kinds of characteristic correlations between active nucleons. The total Hamiltonian is of the form

$$(1)\quad \hat{H} = \hat{H}_0 - \frac{1}{2}\chi\sum_{\mu}\hat{Q}^\dagger_\mu\hat{Q}_\mu - G_M\hat{P}^\dagger\hat{P} - G_Q\sum_{\mu}P^\dagger_\mu P_\mu$$

where $\hat{H}_0$ represents the spherical single particle shell model Hamiltonian, involving spin orbit interactions, the second term includes the quadrupole-quadrupole interaction and third and fourth terms denotes the monopole and quadrupole pairing interactions respectively. $\chi$ denotes the strength of quadrupole-quadrupole two-body interaction and is adjusted with the quadrupole deformation parameter, $\epsilon_2$.

In the present PSM calculations, we have used three major shells (N = 3, 4, 5) for both protons and neutrons. The shell model space is truncated at a deformation $\epsilon_2 = 0.270$ and $\epsilon_4 = 0.042$ for $^{132}$Sm and $\epsilon_2 = 0.260$ and $\epsilon_4 = 0.010$ for $^{134}$Sm isotope.

The monopole pairing strength $G_M$ is given by

$$G_M = \left\{ G_1 \mp G_2 \frac{N-Z}{A} \right\} \frac{1}{A} (MeV), \tag{2}$$

where "+" is used for protons and "-" is used for neutrons. $G_1$ and $G_2$ are two adjustable parameters. For the present work, the value of $G_1$ and $G_2$ are 20.12 and 13.40 respectively.

## 3. Results and Discussion

The availability of various theoretical interpretations of the experimental data on $^{132,134}$Sm motivated us to study the various ground state nuclear structure properties such as yrast spectra, band structure and back-bending in moment of inertia of the above mentioned isotopes within a microscopic framework known as Projected Shell Model (PSM) The results of the above mentioned Model are then compared with the available experimental to bring out new Physics of the various processes that take place within the nucleus.The detailed discussion of the results is presented here-under:

3.1. **Yrast Spectra.** The states having the lowest energy for given value of angular momentum are referred to as yrast states. In fig.1, the PSM results on yrast energy states for even-even $^{132,134}$Sm isotopes are plotted against the spin values and are compared with the experimental data. In fig. 1(a), it is found that the experimental data for yrast energy levels for even-even $^{132}$Sm isotope is available only upto spin $I = 16\hbar$ while PSM calculation are able to predict the results to higher values of spin i.e., upto last calculated spin $30\hbar$.Similarly,in fig. 1(b), the experimental data for $^{134}$Sm is available upto $I = 12\hbar$ but the PSM calculations predicts the results upto spin $30\hbar$.It has been found that the PSM results for the above mentioned isotopes predicted the bandhead correctly and follow the same trend as been observed experimentally.

Band diagram is a collection of band energies and it is obtained by plotting various energy bands with respect to spin . It is an absolutely necessary tool that helps in finding the adequacy of the multi-quasiparticle states involved in the calculations. It plays an important role in for the interpretation of the yrast states,which are obtained as a result of diagonalisation of Hamiltonian within the chosen deformed basis states. The energy differences between the various bands results in crossing of bands at a particular value of spin where some crucial phenomena i.e., back-bending in moment of inertia, staggering and many more may take place, and therefore plays a key role in the interpretation of the numerical results. Band diagram is basically formed by many quasiparticle states of the configuration space but we considered only few low-lying energy states which contribute to the yrast srectra. In these diagrams, the projected energies are shown for 2-qp and 4-qp configurations because for the calculations of the even-even nuclei, as in the present case, the contribution is from a pair of quasineutron or a pair of quasiproton for 2-qp and 2-quasineutron and 2-quasiproton for 4-qp configurations. The qp configuration for each band are given in the band diagram.

In Fig.2(a), for $^{132}$Sm one observes that the yrast spectra upto spin $10^+$ coincides with the g-band arising from 0-qp intrinsic state. At spin $10^+$, the g-band is crossed by 2-qp neutron band having configuration $2\nu h_{11/2}[-7/2, -7/2], K = 0$, which contributes to yrast upto spin $12^+$. At spin $12^+$,the above mentioned 2-qp neutron band is crossed by 2-qp proton band having configuration $2\pi d_{5/2}[-3/2, 5/2], K = 1$, and this band contributes to yrast band upto spin $20^+$. This band is then crossed by 4-qp band having configuration $2\nu h_{11/2}[-7/2, 5/2] + 2\pi d_{5/2}[-3/2, 5/2], K = 0$ at spin $20^+$ and afterwards it contributes to the yrast spectra upto the last calculated spin $30^+$.

In Fig.2(b), for $^{134}$Sm, it is evident from the band diagram that the yrast spectra upto spin $12^+$ coincides with the g-band and at spin $12^+$ the g-band is crossed by 2-qp proton band having configuration $2\pi d_{5/2}[-3/2, 5/2], K = 1$ and contributes to yrast band upto spin $18^+$. At spin $18^+$, the above mentioned 2-qp proton band is joined by another 2-qp proton band having configuration $2\pi d_{5/2}[1/2, 5/2], K = -2$ and they collectively contributed to yrast band upto spin $24^+$. At spin $24^+$,these two bands are joined by another one 4-qp band $2\nu h_{11/2}[-7/2, 9/2]+2\pi d_{5/2}[-3/2, -5/2]$,$K = 1$ and these bands then together contribute upto $26^+$.At spin $26^+$, two 4-qp bands namely $2\nu h_{11/2}[-7/2, 9/2]+2\pi d_{5/2}[-3/2, -5/2]$, $K = 1$ and $2\nu h_{11/2}[-7/2, 1/2] + 2\pi d_{5/2}[-3/2, -5/2], K = -3$ together contribute yrast upto spin $28^+$. After that spin only one 4-qp band $2\nu h_{11/2}[-7/2, 9/2]+2\pi d_{5/2}[-3/2, -5/2]$,

$K = 1$ contributed to yrast band upto last calculated spin $30^{+}$.

3.2. **Back-Bending in moment of inertia.** Back-bending takes place if two bands cross each other with large angle or if they rotate with very different angular velocities.Thus, with the band mixing, the yrast line jumps from a given angular momentum with small deformation to a higher angular momentum with large deformation,resulting in a drastic increase in the moment of inertia at this angular momentum.It can be represented as a plot between twice the kinetic moment of inertia and square of rotational frequency ($\hbar^2\omega^2$), calculated using the formulae [20]

$$2\Im^{(1)} = \frac{(2I-1)}{\omega}(\hbar^2 MeV^{-1}) \tag{3}$$

$$\hbar\omega = \frac{E_\gamma}{\sqrt{(I+1)(I+2)-K^2}-\sqrt{(I-1)I-K^2}} \tag{4}$$

Fig. 3 represents the back-bending plots for even-even Sm isotopes where PSM and experimental results are plotted for the sake of comparison. It can be seen from, Fig. 3(a) that, in $^{132}$Sm, the experimental results exhibit back-bending at spin $14^{+}$, and the theoretical PSM results also reproduce the back-bending at the same spin i.e. at $14^{+}$. Moreover, the calculations are also able to predict another back-bending at spin $22^{+}$ , but the corresponding experimental data is not available to make the comparison at higher spins. For $^{134}$Sm, the experimental data is not available but we have predicted the back-bending theoretically at spin $12^{+}$.

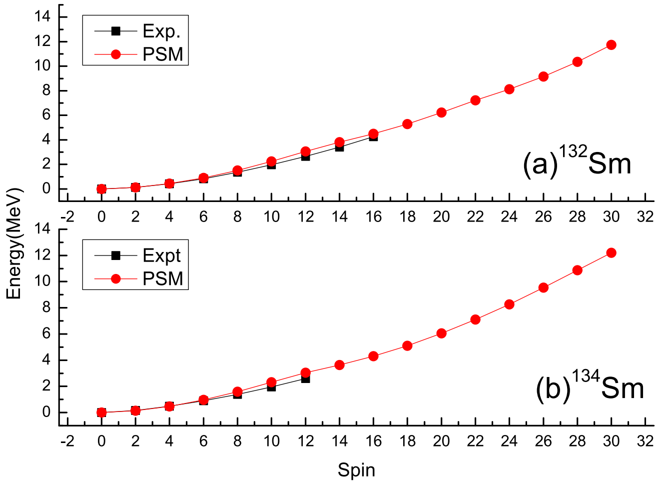
Thus, it has been found from the present calculations that back-bending in moment of inertia for $^{132,134}$Sm occurs at almost same spin at which the band crossing takes place and this supports the accuracy of the calculated PSM results. However, the comparison cannot be made at higher spins due to the unavailability of the experimental data.

## 4. SUMMARY

Various conclusions that can drawn from the results of calculations carried out for $^{132,134}$Sm isotopes are given below:

- From the comparison of the PSM approach with experimental data, one can say that yrast bands and other structure properties have been well reproduced by the applied theoretical quantum framework under consideration.
- Band structures of these nuclei have also been clearly depicted by the band diagrams. It is quite clear from the band diagrams that the yrast spectra is mainly contributed by 2-qp bands at lower spin and by 4-qp bands at higher spin for the whole range of calculated spins.
- The band heads for positive-parity states of $^{132,134}$Sm nuclei are well reproduced. Moreover, back bending in the moment of inertia has also been predicted by the PSM model applied for all nuclei.

## 5. Tables and Figures



(6)

Figure 1. Yrast spectra for (a) $^{132}$Sm (b) $^{134}$Sm.

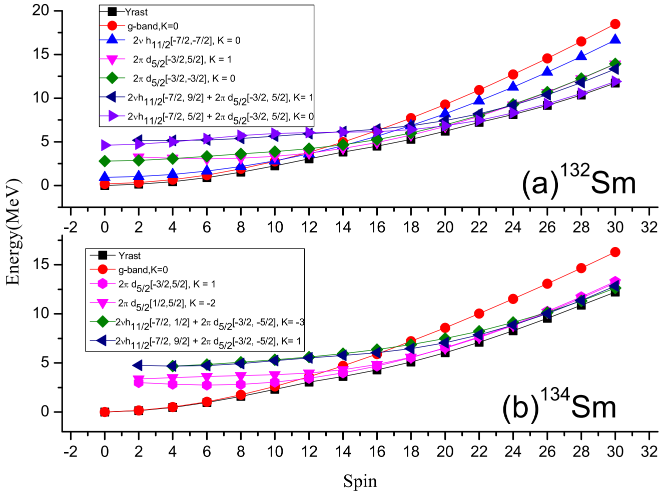


Figure 2. Band diagrams for (a) $^{132}$Sm (b) $^{134}$Sm

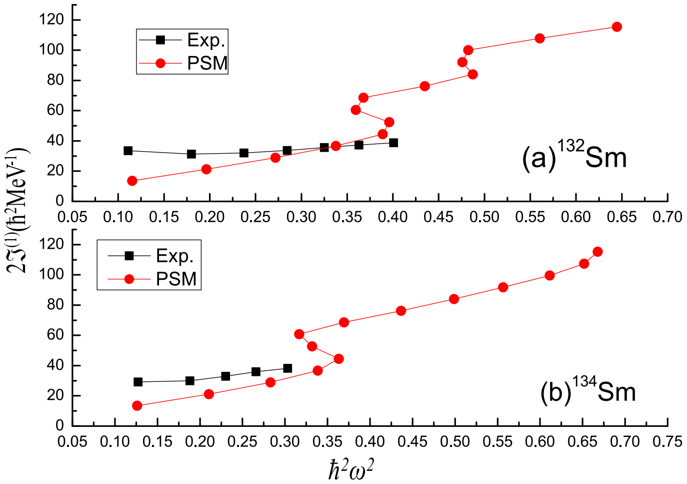


FIGURE 3. Moment of inertia is plotted against angular frequency squared ($\hbar^2\omega^2$)

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