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Remark on an intuitionistic fuzzy operation "division"

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Abstract: Operation "division" over intuitionistic fuzzy sets is introduced and some of its basic properties are studied.

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1 Introduction

In a series of papers written together my colleague and friend Slovakian Prof. Beloslaw Riecan, we introduced a set of different operations "subtraction". They and others, introduced by me were included in my book [1]. During the last five years, we discussed the idea to prepare a new series of research, introducing a set of different operations "division". Unfortunately, we had not started this research. In 2018 I planned to discuss this topic in the time of his visit in Bulgaria in May, but in that moment I was ill and our meeting was only for an hour and we decided to continue it in the time of my visit in Slovakia in October. Unfortunately, Prof. Riecan passed away a month before this. In the present remark I introduce the first of the possible definitions of operation "division". It was written around 2010, when I worked on [1], but I decided that it will be better, if I can construct more operations "division", before publishing them. Now, receiving inquiries by some colleagues whether there exists such operation over Intuitionistic Fuzzy Sets (IFSs), I decided to publish the first of these operations, but mentioning that the research is in the beginning stage.

In the paper, we use the definitions of concept IFS and the operations over it from [1].

2 Main results

Let the IFSs

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

be given. Let for each $x \in E$:

$$\mu_B(x) > 0$$

and therefore

$$1 - \nu_B(x) > 0.$$

For the two IFSs we define operation "division" as follows:

$$A : B = \{\langle x, \min\left(1, \frac{\mu_A(x)}{\mu_B(x)}\right), \min\left(\max\left(0, 1 - \frac{\mu_A(x)}{\mu_B(x)}\right),\right.$$

$$\left.\max\left(0, \frac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}\right)\right)\rangle | x \in E\}.$$

Theorem 1 The operation "division" is defined correctly.
Proof. Let the IFSs A and B be defined. Then, we see that

$$1 \leq \min\left(1, \frac{\mu_A(x)}{\mu_B(x)}\right) \leq 1,$$

$$0 \leq \min\left(\max\left(0, 1 - \frac{\mu_A(x)}{\mu_B(x)}\right), \max\left(0, \frac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}\right)\right) \leq 1,$$

and

$$\min\left(1, \frac{\mu_A(x)}{\mu_B(x)}\right) + \min\left(\max\left(0, 1 - \frac{\mu_A(x)}{\mu_B(x)}\right), \max\left(0, \frac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}\right)\right)$$

$$\leq \min\left(1, \frac{\mu_A(x)}{\mu_B(x)}\right) + \max\left(0, 1 - \frac{\mu_A(x)}{\mu_B(x)}\right).$$

Let

$$X \equiv \min\left(1, \frac{\mu_A(x)}{\mu_B(x)}\right) + \max\left(0, 1 - \frac{\mu_A(x)}{\mu_B(x)}\right).$$

If $1 \leq \frac{\mu_A(x)}{\mu_B(x)}$. Then

$$X = 1 + 0 = 1.$$

If $1 \geq \frac{\mu_A(x)}{\mu_B(x)}$. Then

$$X = \frac{\mu_A(x)}{\mu_B(x)} + 1 - \frac{\mu_A(x)}{\mu_B(x)} = 1.$$ □

For completeness, let us denote for every $a, b \in [0, 1]$:

$$\frac{a}{b} = \begin{cases} \frac{a}{b}, & \text{if } b \neq 0 \\ 1, & \text{if } b = 0 \end{cases}.$$

Then Theorem 1 is valid for the cases when $\mu_B(x) = 0$ and/or $\nu_B(x) = 1$.
Let

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Following [1], we define for every two IFSs A and B:

$$A \subseteq B \text{ if and only if } (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x)).$$

Now, we see that if $B \subseteq A$, i.e., for each $x \in E$:

$$\mu_B(x) \leq \mu_A(x) \quad \text{and} \quad \nu_B(x) \geq \nu_A(x).$$

Then

$$A : B = \{\langle x, \min\left(1, \frac{\mu_A(x)}{\mu_B(x)}\right), \min\left(\max\left(0, 1 - \frac{\mu_A(x)}{\mu_B(x)}\right),\right.$$

$$\left.\max\left(0, \frac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}\right)\right)\rangle | x \in E\}.$$

$$= \{\langle x, 1, 0)\rangle | x \in E\} = E^*.$$

If $A \subseteq B$, then

$$A : B = \{\langle x, \frac{\mu_A(x)}{\mu_B(x)}, \min\left(1 - \frac{\mu_A(x)}{\mu_B(x)}, \frac{\nu_A(x) - \nu_B(x)}{1 - \nu_B(x)}\right)\rangle | x \in E\}.$$

We see directly that for each IFS A:

$$A : A = \{\langle x, \min\left(1, \frac{\mu_A(x)}{\mu_A(x)}\right),$$

$$\min\left(\max\left(0, 1 - \frac{\mu_A(x)}{\mu_A(x)}\right), \max\left(0, \frac{\nu_A(x) - \nu_A(x)}{1 - \nu_A(x)}\right)\right)\rangle | x \in E\}$$

$$= \{\langle x, 1, 0)\rangle | x \in E\} = E^*;$$

$$A : E^* = \{\langle x, \min\left(1, \frac{\mu_A(x)}{1}\right),$$

$$\min\left(\max\left(0, 1 - \frac{\mu_A(x)}{1}\right), \max\left(0, \frac{\nu_A(x) - 0}{1}\right)\right)\rangle | x \in E\}$$

$$= \{\langle x, \min(1, \mu_A(x)), \min(\max(0, 1 - \mu_A(x)), \max(0, \nu_A(x)))\rangle | x \in E\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} = A;$$

$$E^* : A = \{\langle x, \min\left(1, \frac{1}{\mu_A(x)}\right),$$

$$\min\left(\max\left(0, 1 - \frac{1}{\mu_A(x)}\right), \max\left(0, \frac{0 - \nu_A(x)}{1}\right)\right)\rangle | x \in E\}$$

$$= \{\langle x, 1, \min(0, 0))\rangle | x \in E\} = E^*;$$

$$O^* : A = \{\langle x, \min\left(1, \frac{0}{\mu_A(x)}\right),$$

$$\min\left(\max\left(0, 1 - \frac{0}{\mu_A(x)}\right), \max\left(0, \frac{1-\nu_A(x)}{1-\nu_A(x)}\right)\right)\rangle | x \in E\}$$

$$= \{\langle x, 0, \min(\max(0,1), \max(0,1))\rangle | x \in E\} = O^*;$$

$$A : O^* = \{\langle x, \min\left(1, \frac{\mu_A(x)}{0}\right), \min\left(\max\left(0, 1 - \frac{\mu_A(x)}{0}\right),\right.$$

$$\left.\max\left(0, \frac{\nu_A(x)-1}{1-1}\right)\right)\rangle | x \in E\}$$

$$= \{\langle x, \min(1,1), \min(\max(0, 1-1), \max(0,0))\rangle | x \in E\} = E^*.$$

Theorem 2. For every two IFSs A and B:

$$(A.B) : B = A,$$

where for each $x \in E$: $\mu_B(x) > 0$.

Proof. Using the definition of operation "multiplication" from [1], that is

$$A.B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle | x \in E\},$$

we obtain:

$$(A.B) : B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle | x \in E\} : B$$

$$= \{\langle x, \min\left(1, \frac{\mu_A(x)\mu_B(x)}{\mu_B(x)}\right), \min\left(\max\left(0, 1 - \frac{\mu_A(x)\mu_B(x)}{\mu_B(x)}\right),\right.$$

$$\left.\max\left(0, \frac{\nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) - \nu_B(x)}{1-\nu_B(x)}\right)\right)\rangle | x \in E\}$$

$$= \{\langle x, \min(1, \mu_A(x)), \min(\max(0, 1-\mu_A(x)),$$

$$\max\left(0, \frac{\nu_A(x) - \nu_A(x)\nu_B(x)}{1-\nu_B(x)}\right)\rangle | x \in E\}$$

$$= \{\langle x, \mu_A(x), \min(1-\mu_A(x), \max(0, \nu_A(x)))\rangle | x \in E\}$$

$$= \{\langle x, \mu_A(x), \min(1-\mu_A(x), \nu_A(x))\rangle | x \in E\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} = A. \qquad \square$$

We can check directly, that for every three IFSs A, B and C, if $B, C, \subseteq A$, then,

$$(A : B) : C = (A : C) : B.$$

Indeed, let $B, C, \subseteq A$. Then

$$(A : B) : C = E^* : C = E^* = E^* : B = (A : C) : B.$$

On the other hand, when $B \subset A \subseteq C$ or $C \subset A \subseteq B$, or $A \subseteq B \cap C$, there will exist cases for which equality $(A : B) : C = (A : C) : B$ will not be true.

In [2], the concept of a groupid $\langle X, *, e_* \rangle$ is defined as a groupoid $\langle X, * \rangle$ such that for each $x \in X$: $x * x = e_*$ and $x * e_* = x$. Therefore, from validity of equalities

$$A : A = E^*$$

and

$$A : E^* = A$$

it follows the validity of

Theorem 3. If $\mathcal{U}(E)$ is the set of all IFSs over universe E, then $\langle \mathcal{U}(E), :, E^* \rangle$ is a groupid.

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