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Temporal intuitionistic fuzzy pairs

Krassimir Atanassov[1,2] and Vassia Atanassova[1]

[1] Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria,
[2] Intelligent Systems Laboratory
Prof. Asen Zlatarov University
1, "Prof. Yakimov" Blvd, Burgas 8010, Bulgaria
emails: *krat@bas.bg* and *vassia.atanassova@gmail.com*

Abstract: The concept of a Temporal Intuitionistic Fuzzy Pair (TIFP) is introduced as an extension of the concept of an intuitionistic fuzzy pair. Some geometrical interpretations of the TIFPs are given. The basic relations, operations and operators are defined over TIFPs.

Keywords: Intuitionistic fuzzy pair, Temporal intuitionistic fuzzy pair

AMS Classification: 03E72.

1. Introduction

The concept of an Intuitionistic Fuzzy Set (IFS) was introduced in 1983 [4] and from the beginning, the concept of Intuitionistic Fuzzy Pair (IFP) started being used. E. Szmidt, J. Kacprzyk and K. Atanassov, as well as many other colleagues working in the area of the intuitionistic fuzziness, used the concept of an IFP without a special definition in a lot of their publications, using different names: IFP, intuitionistic fuzzy couple, intuitionistic fuzzy value and others. In [10], formal definition of the IFP was given and its basic properties were discussed, offering the researchers in the area of the intuitionistic fuzziness to use only one concept. In the next years, the concept of an IFP was extended to IFP of n-th type, by analogy with the IFSs of n-th type, that incorrectly 25 years after their introduction were called Pythagorean fuzzy sets (for the case $n = 2$) and Orthopair fuzzy sets (for a fixed real number n).

In [5, 6, 7], the concept of a Temporal IFS (TIFS) was introduced and its properties were studied.

Here, by analogy with the ideas for IFPs and TIFSs, we give a formal definition of the concept of a Temporal IFP (TIFP) and definitions of a lot of operations, relations and operators, that can be defined over TIFPs.

2. Definition and geometrical interpretations of an TIFP

Let T be a fixed temporal scale and let everywhere $t \in T$.

The Temporal Intuitionistic Fuzzy Pair (TIFP) is an object with the form $\langle a(t), b(t)\rangle$, where $a, b : T \to [0, 1]$ are functions and for each t: $a(t), b(t) \in [0, 1]$ and $a(t) + b(t) \leq 1$. The TIFP is used as an evaluation of some object or process and which components ($a(t)$ and $b(t)$) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. in a fixed time-moment t. The geometrical interpretations of the TIFPs are the same as of the IFPs. Two of them are shown on Figs. 1 - 4. The first one (which is analogous to the standard fuzzy set interpretation) is shown on Fig. 1.

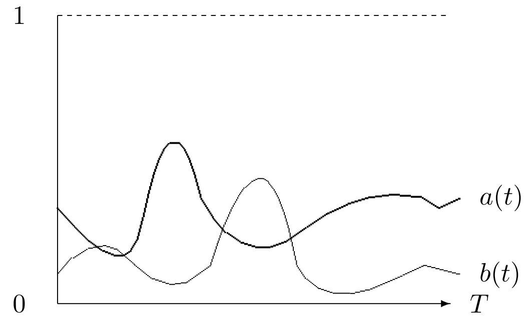


Fig. 1: First geometrical interpretation of a TIFP

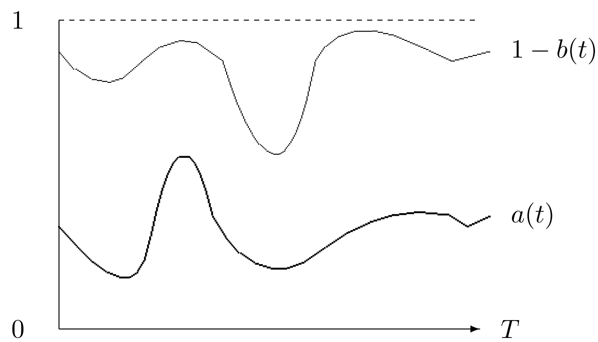
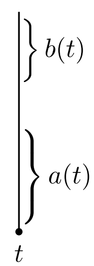


Fig. 2: Modified first geometrical interpretation of a TIFP

Therefore we can map to each time-moment t a unit segment of the form:



For the needs of the discussion below, we define the notion of Temporal Intuitionistic Fuzzy Tautological Pair (TIFTP) by:

$$x(t) \text{ is a TIFTP if and only if } a(t) \geq b(t),$$

while p is a Temporal Tautological Pair (TTP) iff $a(t) = 1$ and $b(t) = 0$ for each time-moment $t \in T$.

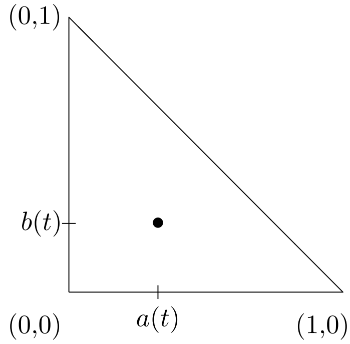


Fig. 3: Second (standard) geometrical interpretation of a TIFP

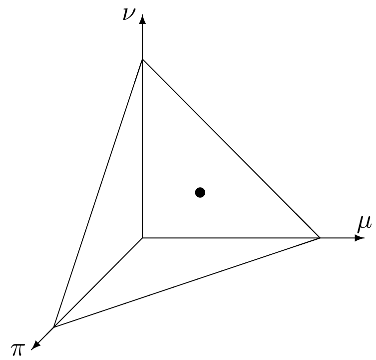


Fig. 4: Three dimensional geometrical interpretation of a TIFP

3. Relations over IFPs

Let us have two TIFPs $x(t) = \langle a(t), b(t)\rangle$ and $y(t) = \langle c(t), d(t)\rangle$. We define the following **standard** relations (i.e., similar to the relations over IFPs):

$$\begin{array}{lll}
x(t) <_{\Box} y(t) & \text{iff} & a(t) < c(t) \\
x(t) <_{\Diamond} y(t) & \text{iff} & b(t) > d(t) \\
x(t) < y(t) & \text{iff} & a(t) < c(t) \ \& \ b(t) > d(t) \\
x(t) \leq_{\Box} y(t) & \text{iff} & a(t) \leq c(t) \\
x(t) \leq_{\Diamond} y(t) & \text{iff} & b(t) \geq d(t) \\
x(t) \leq y(t) & \text{iff} & a(t) \leq c(t) \ \& \ b(t) \geq d(t) \\
x(t) >_{\Box} y(t) & \text{iff} & a(t) > c(t) \\
x(t) >_{\Diamond} y(t) & \text{iff} & b(t) < d(t)
\end{array}$$

$$
\begin{array}{lll}
x(t) > y(t) & \text{iff} & a(t) > c(t) \ \& \ b(t) < d(t) \\
x(t) \geq_{\Box} y(t) & \text{iff} & a(t) \geq c(t) \\
x(t) \geq_{\Diamond} y(t) & \text{iff} & b(t) \leq d(t) \\
x(t) \geq y(t) & \text{iff} & a(t) \geq c(t) \ \& \ b(t) \leq d(t) \\
x(t) =_{\Box} y(t) & \text{iff} & a(t) = c(t) \\
x(t) =_{\Diamond} y(t) & \text{iff} & b(t) = d(t) \\
x(t) = y(t) & \text{iff} & a(t) = c(t) \ \& \ b(t) = d(t)
\end{array}
$$

and the following **specific** relations

$$
\begin{array}{lll}
x <^T_{\Box} y & \text{iff} & (\forall t \in T)(a(t) < c(t)), \\
x <^T_{\Diamond} y & \text{iff} & (\forall t \in T)(b(t) > d(t)), \\
x <^T y & \text{iff} & (\forall t \in T)(a(t) < c(t) \ \& \ b(t) > d(t)), \\
x \leq^T_{\Box} y & \text{iff} & (\forall t \in T)(a(t) \leq c(t)), \\
x \leq^T_{\Diamond} y & \text{iff} & (\forall t \in T)(b(t) \geq d(t)), \\
x \leq^T y & \text{iff} & (\forall t \in T)(a(t) \leq c(t) \ \& \ b(t) \geq d(t)), \\
x >^T_{\Box} y & \text{iff} & (\forall t \in T)(a(t) > c(t)), \\
x >^T_{\Diamond} y & \text{iff} & (\forall t \in T)(b(t) < d(t)), \\
x >^T y & \text{iff} & (\forall t \in T)(a(t) > c(t) \ \& \ b(t) < d(t)), \\
x \geq^T_{\Box} y & \text{iff} & (\forall t \in T)(a(t) \geq c(t)), \\
x \geq^T_{\Diamond} y & \text{iff} & (\forall t \in T)(b(t) \leq d(t)), \\
x \geq^T y & \text{iff} & (\forall t \in T)(a(t) \geq c(t) \ \& \ b(t) \leq d(t)), \\
x =^T_{\Box} y & \text{iff} & (\forall t \in T)(a(t) = c(t)) \\
x =^T_{\Diamond} y & \text{iff} & (\forall t \in T)(b(t) = d(t)) \\
x =^T y & \text{iff} & (\forall t \in T)(a(t) = c(t) \ \& \ b(t) = d(t))
\end{array}
$$

4. Opertions over IFPs

In some definitions below, we use functions sg and $\overline{\text{sg}}$ defined by

$$
\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},
$$

and

$$
\overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}.
$$

Let us have two IFPs $x = \langle a, b \rangle$ and $y = \langle c, d \rangle$.

First, we define analogues of operations "conjunction" and "disjunction":

$$x(t)\&_1 y(t) = x(t) \cap y(t) = \langle \min(a(t), c(t)), \max(b(t), d(t)) \rangle$$

$$x(t) \vee_1 y(t) = x(t) \cup y(t) = \langle \max(a(t), c(t))), \min(b(t), d(t)) \rangle$$

$$x(t)\&_2 y(t) = x(t) + y(t) = \langle a(t) + c(t) - a(t).c(t), b(t).d(t) \rangle$$

$$x(t) \vee_2 y(t) = x(t).y(t) = \langle a(t).c(t), b(t) + d(t) - b(t).d(t) \rangle.$$

Second, by analogy with [8], where 185 intuitionistic fuzzy implications[1] and 54 intuitionistic fuzzy negations are defined, here we define the first 15 operations "implication" and the operations "negation", generated by them, but for the case of TIFPs. In Table 1, the definitions of implications $x(t) \to y(t)$ are given.

Table 1

$\to_1$	$\langle \max(b(t), \min(a(t), c(t))), \min(a(t), d(t)) \rangle$
$\to_2$	$\langle \overline{\text{sg}}(a(t) - c(t)), d(t)\text{sg}(a(t) - c(t)) \rangle$
$\to_3$	$\langle 1 - (1 - c(t))\text{sg}(a(t) - c(t))), d(t)\text{sg}(a(t) - c(t)) \rangle$
$\to_4$	$\langle \max(b(t), c(t)), \min(a(t), d(t)) \rangle$
$\to_5$	$\langle \min(1, b(t) + c(t)), \max(0, a(t) + d(t) - 1) \rangle$
$\to_6$	$\langle b(t) + a(t)c(t), a(t)d(t) \rangle$
$\to_7$	$\langle \min(\max(b(t), c(t)), \max(a(t), b(t)), \max(c(t), d(t))),$ $\max(\min(a(t), d(t)), \min(a(t), b(t)), \min(c(t), d(t))) \rangle$
$\to_8$	$\langle 1 - (1 - \min(b(t), c(t)))\text{sg}(a(t) - c(t)),$ $\max(a(t), d(t))\text{sg}(a(t) - c(t))\text{sg}(d(t) - b(t)) \rangle$
$\to_9$	$\langle b(t) + a(t)^2 c(t), a(t)b(t) + a(t)^2 d(t) \rangle$
$\to_{10}$	$\langle c(t)\overline{\text{sg}}(1 - a(t)) + \text{sg}(1 - a(t))(\overline{\text{sg}}(1 - c(t)) + b(t)\text{sg}(1 - c(t))),$ $d(t)\overline{\text{sg}}(1 - a(t)) + a(t)\text{sg}(1 - a(t))\text{sg}(1 - c(t)) \rangle$
$\to_{11}$	$\langle 1 - (1 - c(t))\text{sg}(a(t) - c(t)), d(t)\text{sg}(a(t) - c(t))\text{sg}(d(t) - b(t)) \rangle$
$\to_{12}$	$\langle \max(b(t), c(t)), 1 - \max(b(t), c(t)) \rangle$
$\to_{13}$	$\langle b(t) + c(t) - b(t)c(t), a(t)d(t) \rangle$
$\to_{14}$	$\langle 1 - (1 - c(t))\text{sg}(a(t) - c(t)) - d(t)\overline{\text{sg}}(a(t) - c(t))\text{sg}(d(t) - b(t)),$ $d(t)\text{sg}(d(t) - b(t)) \rangle$
$\to_{15}$	$\langle 1 - (1 - \min(b(t), c(t)))\text{sg}(a(t) - c(t))\text{sg}(d(t) - b(t))$ $- \min(b(t), c(t))\text{sg}(a(t) - c(t))\text{sg}(d(t) - b(t)),$ $1 - (1 - \max(a(t), d(t)))\text{sg}(\overline{\text{sg}}(a(t) - c(t)) + \overline{\text{sg}}(d(t) - b(t)))$ $- \max(a(t), d(t))\overline{\text{sg}}(a(t) - c(t))\overline{\text{sg}}(d(t) - b(t)) \rangle$

In Table 2, the first 5 negations $\neg x$ are given.

The relations between the introduced implications and negations are shown in Table 3.

[1]In [11–14], L. Atanassova introduced 11implications ($\to_{139}$, ..., $\to_{149}$). P. Dworniczak generalized them in [18–20] (implications $\to_{150}$, ..., $\to_{152}$) and L. Atanassova modified Dworniczak's implications in [15, 16] (implications $\to_{154}, ..., \to_{165}$).

Table 2

$\neg_1$	$\langle x(t), b(t), a(t)\rangle$
$\neg_2$	$\langle x(t), \overline{\text{sg}}(a(t)), \text{sg}(a(t))\rangle$
$\neg_3$	$\langle x(t), b(t), a(t)b(t) + a(t)^2\rangle$
$\neg_4$	$\langle x(t), b(t), 1 - b(t)\rangle$
$\neg_5$	$\langle x(t), \overline{\text{sg}}(1 - b(t)), \text{sg}(1 - b(t))\rangle$

Table 3

$\neg_1$	$\rightarrow_1, \rightarrow_4, \rightarrow_5, \rightarrow_6, \rightarrow_7, \rightarrow_{10}, \rightarrow_{13}$
$\neg_2$	$\rightarrow_2, \rightarrow_3, \rightarrow_8, \rightarrow_{11}$
$\neg_3$	$\rightarrow_9$
$\neg_4$	$\rightarrow_{12}$
$\neg_5$	$\rightarrow_{14}, \rightarrow_{15}$

5. Operators over IFPs

There are three types of operators over IFPs. The first of them is of modal type.

Let as above, $x(t) = \langle a(t), b(t)\rangle$ be an IFP and let $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$. Then the modal type of operators defined over x have the forms:

$$\begin{aligned}
\Box\, x(t) &= \langle a(t), 1 - a(t)\rangle, \\
\Diamond x(t) &= \langle 1 - b(t), b(t)\rangle, \\
D_\alpha(x(t)) &= \langle a(t) + \alpha(1 - a(t) - b(t)), b(t) + (1 - \alpha)(1 - a(t) - b(t))\rangle, \\
F_{\alpha,1-\alpha}(x(t)) &= \langle a(t) + \alpha(1 - a(t) - b(t)), b(t) + \beta(1 - a(t) - b(t))\rangle, \\
&\quad \text{where } \alpha + \beta \leq 1, \\
G_{\alpha,\beta}(x(t)) &= \langle \alpha a(t), \beta b(t)\rangle, \\
H_{\alpha,\beta}(x(t)) &= \langle \alpha a(t), b(t) + \beta(1 - a(t) - b(t))\rangle, \\
H^*_{\alpha,\beta}(x(t)) &= \langle \alpha a(t), b(t) + \beta(1 - \alpha a(t) - b(t))\rangle, \\
J_{\alpha,\beta}(x(t)) &= \langle a(t) + \alpha(1 - a(t) - b(t)), \beta b(t)\rangle, \\
J^*_{\alpha,\beta}(x(t)) &= \langle a(t) + \alpha(1 - a(t) - \beta b(t)), \beta b(t)\rangle, \\
X_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(x(t)) &= \langle \alpha a(t) + \beta(1 - a(t) - \gamma b(t)), \delta b(t) + \varepsilon(1 - \zeta a(t) - b(t))\rangle \\
&\quad \text{where } \alpha + \varepsilon - \alpha\zeta \leq 1, \beta + \delta - \beta\gamma \leq 1, \beta + \varepsilon \leq 1.
\end{aligned}$$

The second type of operators over TIFPs is from another (similar to modal) type. Let $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$.

$$\begin{aligned}
\boxplus x(t) &= \langle \tfrac{a(t)}{2}, \tfrac{b(t)+1}{2}\rangle \\
\boxtimes x(t) &= \langle \tfrac{a(t)+1}{2}, \tfrac{b(t)}{2}\rangle \\
\boxplus_\alpha x(t) &= \langle \alpha a(t), \alpha b(t) + 1 - \alpha\rangle \\
\boxtimes_\alpha x(t) &= \langle \alpha a(t) + 1 - \alpha, \alpha b(t)\rangle,
\end{aligned}$$

$$\begin{aligned}
\boxplus_{\alpha,\beta} x(t) = & \quad \langle \alpha a(t), \alpha.b(t)+\beta\rangle, \text{ where } \alpha+\beta\le 1,\\
\boxtimes_{\alpha,\beta} x(t) = & \quad \langle \alpha a(t)+\beta, \alpha b(t)\rangle, \text{ where } \alpha+\beta\le 1,\\
\boxplus_{\alpha,\beta,\gamma} x(t) = & \quad \langle \alpha a(t), \beta b(t)+\gamma\rangle, \text{ where } \max(\alpha,\beta)+\gamma\le 1,\\
\boxtimes_{\alpha,\beta,\gamma} x(t) = & \quad \langle \alpha a+\gamma, \beta b(t)\rangle, \text{ where } \max(\alpha,\beta)+\gamma\le 1,\\
\boxdot_{\alpha,\beta,\gamma,\delta} x(t) = & \quad \langle \alpha a(t)+\gamma, \beta b(t)+\delta\rangle, \text{ where } \max(\alpha,\beta)+\gamma+\delta\le 1,\\
E_{\alpha,\beta} x(t) = & \quad \langle \beta(\alpha a(t)+1-\alpha), \alpha(\beta b(t)+1-\beta)\rangle (\text{see [17]}),\\
\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} x(t) = & \quad \langle \alpha a(t)-\varepsilon b(t)+\gamma, \beta b(t)-\zeta a(t)+\delta\rangle,\\
& \quad \text{where } \max(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\le 1,\\
& \quad \min(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\ge 0 \text{ and } \beta+\varepsilon\le 1.
\end{aligned}$$

The third type of operators is from level type. They are

$$P_{\alpha,\beta} x(t) = \langle \max(\alpha, a(t)), \min(\beta, b(t))\rangle,$$

$$Q_{\alpha,\beta} x(t) = \langle \min(\alpha, a(t)), \max(\beta, b(t))\rangle,$$

for $\alpha, \beta \in [0,1]$ and $\alpha+\beta \le 1$.

6. Quantifiers over temporal intuitionistic fuzzy predicates

Let x be a variable, obtaining values in some fixed set S and let $P(x)$ be an intuitionistic fuzzy predicate with a variable x (see [8]) if its truth-value is defined as

$$V(P(x)) = \langle a(x), b(x)\rangle.$$

The IF-interpretations of the (intuitionistic fuzzy) quantifiers *for all* ($\forall$) and *there exists* ($\exists$) are introduced in [6, 9, 21] by

$$V(\exists x P(x)) = \langle \sup_{y\in S} a(P(y)), \inf_{y\in S} b(P(y))\rangle,$$

$$V(\forall x P(x)) = \langle \inf_{y\in S} a(P(y)), \sup_{y\in S} b(P(y))\rangle.$$

If S is a finite set, then we can use the denotations

$$V(\exists x P(x)) = \langle \max_{y\in S} b(P(y)), \min_{y\in S} b(P(y))\rangle,$$

$$V(\forall x P(x)) = \langle \min_{y\in S} b(P(y)), \max_{y\in S} b(P(y))\rangle.$$

Now, we can extend the concept of an IFP to the form $P(x,t)$, where $x \in S$ and $t \in T$ and it will have a truth-value

$$V(P(x,t)) = \langle a(x,t), b(x,t)\rangle.$$

For it, the IF-interpretations of the temporal intuitionistic fuzzy quantifiers have the **standard** forms

$$V(\exists x P(x,t)) = \langle \sup_{y\in S} a(P(y,t)), \inf_{y\in S} b(P(y,t))\rangle,$$

$$V(\forall x P(x,t)) = \langle \inf_{y\in S} a(P(y,t)), \sup_{y\in S} b(P(y,t))\rangle.$$

If S is a finite set, then we can use the denotations

$$V(\exists x P(x,t)) = \langle \max_{y\in S} b(P(y,t)), \min_{y\in S} b(P(y,t))\rangle,$$

$$V(\forall x P(x,t)) = \langle \min_{y\in S} b(P(y,t)), \max_{y\in S} b(P(y,t))\rangle.$$

For difference of the standard (intuitionistic fuzzy) quantifiers, for the TIF-Predicates we can introduce also **specific** (intuitionistic fuzzy) quantifiers with the forms:

$$V(\exists_T t P(x,t)) = \langle \sup_{t\in T} a(P(x,t)), \inf_{t\in T} b(P(x,t))\rangle,$$

$$V(\forall_T t P(x,t)) = \langle \inf_{t\in T} a(P(x,t)), \sup_{t\in T} b(P(x,t))\rangle.$$

If T is a finite set, then we can use the denotations

$$V(\exists_T t P(x,t)) = \langle \max_{t\in T} b(P(x,t)), \min_{t\in T} b(P(x,t))\rangle,$$

$$V(\forall_T t P(x,t)) = \langle \min_{t\in T} b(P(y,t)), \max_{t\in T} b(P(x,t))\rangle.$$

7. An IFS generated by a TIFP

Let us have the temporal scale $T = \{t_1, t_2, ...\}$ and the TIFP $x(t) = \langle a(t), b(t)\rangle$. Then we can construct the set

$$T(x) = \{\langle t, a(t), b(t)\rangle | t \in T\}$$

(see Fig. 5).

It is directly seen that $T(x)$ is an IFS, but now over universe T.

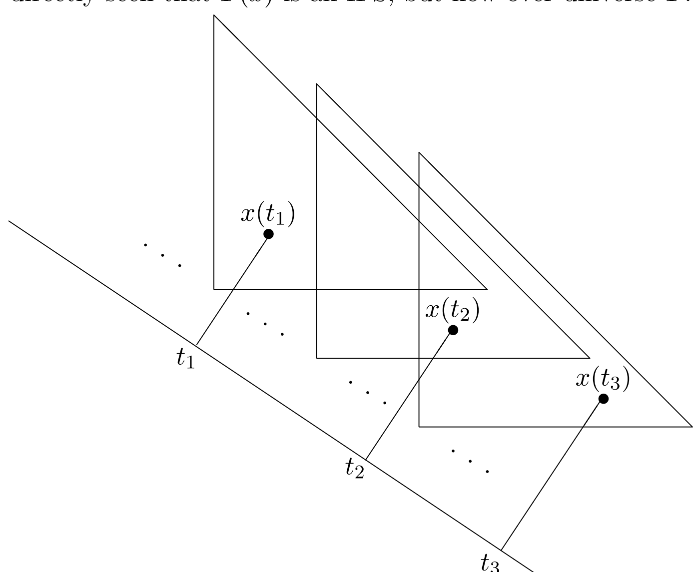


Fig. 5.

8. Conclusion

In the present paper, we transformed the definitions of the basic relations, operations and operators from the intuitionistic fuzzy logics (propositionsl, predicate and modal) to the concept of a TIFP.

In future, we will introduce analogues of all intuitionistic fuzzy implications over TIFPs and the generated by them intuitionistic fuzzy negations over TIFPs. Also, by analogy with [1, 2, 3], we will introduce the intuitionistic fuzzy conjunctions and disjunctions over TIFPs.

We will give definition of new types of IFPs, related to the rest IFS-extensions.

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