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SOME MODIFIED REDUCTION FORMULAS FOR THE GAUSS AND CLAUSEN HYPERGEOMETRIC FUNCTIONS

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ABSTRACT. In this paper, by using the series rearrangement technique, we derive closed forms of some reduction formulas for the following two Clausen hypergeometric functions:

$$ {}_3F_2\left[\begin{array}{c} a,\ 3a-1, 3a-\frac{3}{2};\\ \\ a-1, 6a-2; \end{array} z\right]$$

and

$$ {}_3F_2\left[\begin{array}{c} 2a, 3a-1, 3a-\frac{3}{2};\\ \\ 2a-1,\ 6a-2; \end{array} z\right].$$

We also obtain some reduction formulas of the following four Gauss hypergeometric functions:

$$ {}_2F_1\left[\begin{array}{c} a, a\pm\frac{1}{2};\\ \\ 2a; \end{array} \frac{4z^3}{(1-3z)^2}\right]$$

and

$$ {}_2F_1\left[\begin{array}{c} a, a\pm\frac{1}{2};\\ \\ 2a; \end{array} -\frac{4z^3}{(1-z)^2(1-4z)}\right].$$

Several appropriately modified forms of a known reduction formula for the Clausen hypergeometric function are also appropriately derived here.

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KEY WORDS AND PHRASES. Hypergeometric transformations; Summation, transformation and reduction formulas; Gauss and Clausen hypergeometric functions.

1. INTRODUCTION, DEFINITIONS AND PRELIMINARIES

In our investigation here, we use the following standard notations:

$$\mathbb{N} := \{1, 2, 3, \cdots\},\ \ \mathbb{N}_0 := \mathbb{N}\cup\{0\} \quad \text{and} \quad \mathbb{Z}_0^- := \mathbb{Z}^-\cup\{0\} = \{0, -1, -2, \cdots\}.$$

Also, as usual, the symbols $\mathbb{C}$, $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}^+$ and $\mathbb{R}^-$ denote the sets of complex numbers, real numbers, natural numbers, integers, positive and negative real numbers, respectively.

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The general Pochhammer symbol (or the *shifted* factorial) $(\lambda)_\nu \quad (\lambda, \nu \in \mathbb{C})$ is defined by (see, for example, [10] and [14])

$$(\lambda)_\nu := \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu = 0;\ \lambda \in \mathbb{C}\setminus\{0\}) \\ \prod_{j=0}^{n-1}(\lambda+j) & (\nu = n \in \mathbb{N};\ \lambda \in \mathbb{C}) \\ \dfrac{(-1)^k\, n!}{(n-k)!} & (\lambda = -n;\ \nu = k;\ n,k \in \mathbb{N}_0;\ 0 \leqq k \leqq n) \\ 0 & (\lambda = -n;\ \nu = k;\ n,k \in \mathbb{N}_0;\ k > n) \\ \dfrac{(-1)^k}{(1-\lambda)_k} & (\nu = -k;\ k \in \mathbb{N};\ \lambda \in \mathbb{C}\setminus\mathbb{Z}), \end{cases}$$

it being understood conventionally that $(0)_0 := 1$ and assumed tacitly that the Gamma quotient exists.

The generalized hypergeometric function ${}_pF_q$ with p numerator parameters $\alpha_j \in \mathbb{C} \quad (j = 1, \cdots, p)$ and q denominator parameters $\beta_j \in \mathbb{C}\setminus\mathbb{Z}_0^- \quad (j = 1, \cdots, q)$ is defined by (see [2], [10] and [11]) is defined by

$$\begin{aligned} {}_pF_q\left[\begin{array}{c} \alpha_1, \alpha_2, \cdots, \alpha_p; \\ \\ \beta_1, \beta_2, \cdots, \beta_q; \end{array} z\right] &= {}_pF_q\left[\begin{array}{c} (\alpha_p); \\ \\ (\beta_q); \end{array} z\right] \\ &= \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}(\alpha_j)_n}{\prod_{j=1}^{q}(\beta_j)_n} \frac{z^n}{n!} \end{aligned} \tag{1.1}$$

$$\big(p,q \in \mathbb{N}_0;\ p \leqq q+1;\ p \leqq q \ \text{ and } \ |z| < \infty;\ p = q+1 \ \text{ and } \ |z| < 1;$$
$$p = q+1,\ |z| = 1 \ \text{ and } \ \Re(\omega) > 0;$$
$$p = q+1,\ |z| = 1\ (z \neq 1) \ \text{ and } \ -1 < \Re(\omega) \leqq 0\big),$$

where, by convention, a product over an empty set is interpreted as 1, and

$$\omega := \sum_{j=1}^{q}\beta_j - \sum_{j=1}^{p}\alpha_j, \tag{1.2}$$

$\Re(\omega)$ being the real part of complex number ω.

Remark 1. Throughout the remainder of this paper, the applicable parametric and argument constraints, which would correspond appropriately to the above-mentioned parametric and argument constraints, will be tacitly assumed to be satisfied appropriately. Moreover, exceptional values of the parameters and the arguments, which are involved in any equation, are also *tacitly* excluded. For example, it is understood for the denominator parameters $\beta_1, \beta_2, \cdots, \beta_q$ that, in general,

$$\beta_j \neq 0, -1, -2, \cdots \qquad (j = 1, \cdots, q).$$

Remark 2. If none of the numerator and denominator parameters is zero or a negative integer, we note that the ${}_pF_q$ series defined by the equation (1.1) satisfied the following constraints:

(i) It converges for $|z| < \infty$ if $p \leqq q$;
(ii) It converges for $|z| < 1$ if $p = q + 1$;
(iii) It diverges for all z $(z \neq 0)$ if $p > q + 1$;
(iv) It converges absolutely for $|z| = 1$ if $p = q + 1$ and $\Re(\omega) > 0$;
(v) It converges conditionally for $|z| = 1$ $(z \neq 1)$ if

$$p = q + 1 \qquad \text{and} \qquad -1 < \Re(\omega) \leqq 0;$$

(vi) It diverges for $|z| = 1$ if $p = q + 1$ and $\Re(\omega) \leqq -1$,

where ω is given, as before, by (1.2).

Each of the following results will be needed in our present study.

Pfaff-Kummer Linear Transformations (see [8, p. 247, Eqs. (9.5.1) and (9.5.2)]; see also [1, p. 68, Eq. (2.2.6)]):

$$(1.3) \qquad {}_2F_1\left[\begin{array}{c}\alpha, \beta; \\ \\ \gamma;\end{array} z\right] = (1-z)^{-\alpha}\, {}_2F_1\left[\begin{array}{c}\alpha, \gamma-\beta; \\ \\ \gamma;\end{array} -\frac{z}{1-z}\right]$$

$$\left(\gamma \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ |\arg(1-z)| < \pi\right)$$

and

$$(1.4) \qquad {}_2F_1\left[\begin{array}{c}\alpha, \beta; \\ \\ \gamma;\end{array} z\right] = (1-z)^{-\beta}\, {}_2F_1\left[\begin{array}{c}\gamma-\alpha, \beta; \\ \\ \gamma;\end{array} -\frac{z}{1-z}\right]$$

$$\left(\gamma \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ |\arg(1-z)| < \pi\right).$$

Euler's Linear Transformation (see [8, p. 248, Eq. (9.5.3)]; see also [1, p. 68, Eq. (2.2.7)]):

$$(1.5) \qquad {}_2F_1\left[\begin{array}{c}\alpha, \beta; \\ \\ \gamma;\end{array} z\right] = (1-z)^{\gamma-\alpha-\beta}\, {}_2F_1\left[\begin{array}{c}\gamma-\alpha, \gamma-\beta; \\ \\ \gamma;\end{array} z\right]$$

$$\left(\gamma \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ |\arg(1-z)| < \pi\right).$$

A Set of Closed Forms (see [10, p. 70, Exercise 10]; see also [6, p. 101, Eqs. 2.8 (6)], [11, p. 19, Eqs. (1.5.19) and (1.5.20)]):

$$(1.6) \qquad \left(\frac{2}{1+\sqrt{1-z}}\right)^{2\lambda-1} = {}_2F_1\left[\begin{array}{c}\lambda, \lambda-\frac{1}{2}; \\ \\ 2\lambda;\end{array} z\right]$$

$$\left(2\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ |\arg(1-z)| < \pi\right)$$

and

$$(1.7) \qquad \frac{1}{\sqrt{1-z}}\left(\frac{2}{1+\sqrt{1-z}}\right)^{2\lambda-1} = {}_2F_1\left[\begin{array}{c}\lambda, \lambda+\frac{1}{2}; \\ \\ 2\lambda;\end{array} z\right]$$

$$\left(2\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ |\arg(1-z)| < \pi\right).$$

Various families of summation, transformation and reduction formulas for hypergeometric functions in one, two and more variables are available in the remarkably vast literature due, in part, to their potential for usefulness in several diverse areas of the mathematical, physical, statistical and engineering sciences (see, for example, [2], [3], [6], [7], [10], [11], [13], [15], [16], [17] and [18]; see also the citations to related earlier works which are cited in each of these references). In particular, in the year 2005, the following reduction formula for the Clausen hypergeometric series ${}_3F_2$ (see [5]) was given by Joshi and Vyas [7, p. 1921, Eq. (6.19)]:

$$ {}_3F_2\left[\begin{array}{c} a, 3a-1, 3a-\frac{3}{2}; \\ \\ 2a-1, 6a-2; \end{array} z\right] \mathrel{\overset{\circ}{=}} \frac{1}{(1-z)\sqrt{1-4z}} \left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1}, \tag{1.8} $$

where the symbol $\overset{\circ}{=}$ exhibits the fact that the above reduction formula (1.8) does not hold true as stated. In fact, it cannot be verified numerically. The left-hand side of the equation (1.8) is given by

$$ \begin{aligned} {}_3F_2\left[\begin{array}{c} a, 3a-1, 3a-\frac{3}{2}; \\ \\ 2a-1, 6a-2; \end{array} z\right] &= \sum_{m=0}^{\infty} \frac{(a)_m\ (3a-1)_m\ \left(3a-\frac{3}{2}\right)_m}{(2a-1)_m\ (6a-2)_m}\ \frac{z^m}{m!} \\ &= \sum_{m=0}^{\infty} \frac{(3a-1)_m\ \left(3a-\frac{3}{2}\right)_m}{(6a-2)_m}\ \frac{\Gamma(a+m)\ \Gamma(2a-1)}{\Gamma(a)\ \Gamma(2a-1+m)}\ \frac{z^m}{m!}. \end{aligned} \tag{1.9} $$

The plan of this paper is as follows. In Section 2, we propose to develop the appropriately modified forms (2.3) of the reduction formula (1.8) by considering the right-hand side of the equation (1.8) in conjunction with the closed form (1.6), the Pfaff-Kummer linear transformations (1.3) and (1.4) and Euler's linear transformation (1.5). In Section 3, we give the proofs of these reduction formulas by using the series rearrangement technique (see, for details, [14, Chapter 2]). Finally, in Section 4, several concluding remarks and observations are presented.

2. Modified Forms of the Reduction Formula (1.8)

In what follows, all values of the parameters and the arguments, which would render the results invalid or non-existent, are tacitly excluded.

Our modified forms of the reduction formula (1.8) are now stated below.

$$ {}_3F_2\left[\begin{array}{c} a, 3a-1, 3a-\frac{3}{2}; \\ \\ a-1, 6a-2; \end{array} z\right] = \left(\frac{2}{1+\sqrt{1-z}}\right)^{6a-3} \left(1+\frac{3(2a-1)z}{2(a-1)\left(1-z+\sqrt{1-z}\right)}\right) \tag{2.1} $$

$$ \left(a-1, 6a-2 \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ |z|<1\right) $$

and

$$ {}_3F_2\left[\begin{matrix} 2a, 3a-1, 3a-\frac{3}{2}; \\ 2a-1, 6a-2; \end{matrix} \; z\right] = \left(\frac{2}{1+\sqrt{1-z}}\right)^{6a-3}\left(1+\frac{3z}{2\left(1-z+\sqrt{1-z}\right)}\right) \tag{2.2} $$

$$ \left(2a-1, 6a-2 \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ |z|<1\right). $$

Now, if we denote the right-hand side of the equation (1.8) by $\Omega(z)$ and apply the reduction formula (1.6), we observe that

$$ \begin{aligned} \Omega(z) &:= \frac{1}{(1-z)\sqrt{1-4z}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1} \\ &= \frac{(1-3z)^{-2a+1}}{(1-z)\sqrt{1-4z}}\ {}_2F_1\left[\begin{matrix} a, a-\frac{1}{2}; \\ 2a; \end{matrix} \; \frac{4z^3}{(1-3z)^2}\right], \end{aligned} \tag{2.3} $$

which readily yields

$$ {}_2F_1\left[\begin{matrix} a, a-\frac{1}{2}; \\ 2a; \end{matrix} \; \frac{4z^3}{(1-3z)^2}\right] = (1-3z)^{2a-1}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1}. \tag{2.4} $$

Similarly, by using the result (1.3) in (2.3), we have

$$ \begin{aligned} &\frac{1}{(1-z)\sqrt{1-4z}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1} \\ &= \frac{(1-3z)}{(1-z)^{2a+1}(1-4z)^{a+\frac{1}{2}}}\ {}_2F_1\left[\begin{matrix} a, a+\frac{1}{2}; \\ 2a; \end{matrix} \; -\frac{4z^3}{(1-z)^2(1-4z)}\right], \end{aligned} \tag{2.5} $$

so that

$$ {}_2F_1\left[\begin{matrix} a, a+\frac{1}{2}; \\ 2a; \end{matrix} \; -\frac{4z^3}{(1-z)^2(1-4z)}\right] = \frac{(1-4z)^a(1-z)^{2a}}{(1-3z)}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1}. \tag{2.6} $$

In an analogous manner, we can derive the following results:

$$ \begin{aligned} &\frac{1}{(1-z)\sqrt{(1-4z)}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1} \\ &= (1-z)^{-2a}(1-4z)^{-a}\ {}_2F_1\left[\begin{matrix} a-\frac{1}{2}, a; \\ 2a; \end{matrix} \; -\frac{4z^3}{(1-z)^2(1-4z)}\right], \end{aligned} \tag{2.7} $$

which yields

$$
{}_2F_1\left[\begin{array}{c} a-\frac{1}{2}, a; \\ 2a; \end{array} -\frac{4z^3}{(1-z)^2(1-4z)}\right]
= (1-z)^{2a-1}(1-4z)^{a-\frac{1}{2}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1} \tag{2.8}
$$

and

$$
\frac{1}{(1-z)\sqrt{1-4z}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1}
= (1-3z)^{-2a}\ {}_2F_1\left[\begin{array}{c} a, a+\frac{1}{2}; \\ 2a; \end{array} \frac{4z^3}{(1-3z)^2}\right], \tag{2.9}
$$

so that

$$
{}_2F_1\left[\begin{array}{c} a, a+\frac{1}{2}; \\ 2a; \end{array} \frac{4z^3}{(1-3z)^2}\right]
= \frac{(1-3z)^{2a}}{(1-z)\sqrt{1-4z}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1}. \tag{2.10}
$$

Remark 3. Each of the results (2.1) to (2.10) has been verified numerically. Moreover, for simplification purposes, the algebraic identity has been used:

$$(1-3z)^2 - 4z^3 = (1-z)^2(1-4z).$$

3. Demonstration of the Reduction Formulas

Proof of the reduction formula (2.1)**.** First of all, in view of (1.9) and the following identity:

$$\frac{(a)_m}{(a-1)_m} = 1 + \frac{m}{a-1} \qquad (m \in \mathbb{N}_0),$$

we have

$$\begin{aligned}
\Phi(z) &:= {}_3F_2\left[\begin{matrix} a, 3a-1, 3a-\frac{3}{2}; \\ \\ a-1, 6a-2; \end{matrix}\; z\right] \\
&= \sum_{m=0}^{\infty} \frac{(3a-1)_m\,(3a-\frac{3}{2})_m}{(6a-2)_m}\,\frac{z^m}{m!} \\
&\qquad + \sum_{m=1}^{\infty} \frac{1}{a-1}\,\frac{(3a-1)_m\,(3a-\frac{3}{2})_m}{(6a-2)_m}\,\frac{z^m}{(m-1)!} \\
&= {}_2F_1\left[\begin{matrix} 3a-1, 3a-\frac{3}{2}; \\ \\ 6a-2; \end{matrix}\; z\right] \\
&\qquad + \frac{1}{a-1}\sum_{m=1}^{\infty} \frac{(3a-1)_m\,(3a-\frac{3}{2})_m}{(6a-2)_m}\,\frac{z^m}{(m-1)!}.
\end{aligned} \tag{3.1}$$

Upon replacing m by $m+1$ in the equation (3.1), we find after some simplification that

$$\Phi(z) = {}_2F_1\left[\begin{matrix} 3a-1, 3a-\frac{3}{2}; \\ \\ 6a-2; \end{matrix}\; z\right] + \frac{3(2a-1)z}{4(a-1)}\,{}_2F_1\left[\begin{matrix} 3a-\frac{1}{2}, 3a; \\ \\ 6a-1; \end{matrix}\; z\right]. \tag{3.2}$$

Now, using the results (1.6) and (1.7) in the equation (3.2), we complete our proof of the reduction formula in the closed form (2.1).

Proof of the reduction formula (2.2)**.** Our proof of the reduction formula (2.2) follows the same lines as in the above proof of the reduction formula (2.1). We, therefore, choose to skip the details involved.

Proofs of the results (2.3), (2.5), (2.7) and (2.9). Let us consider the function $\Xi(z)$ given by

$$\begin{aligned}
\Xi(z) &:= \frac{1}{(1-z)\sqrt{1-4z}}\left(\frac{2}{1-3z+(1-z)\sqrt{1-4z}}\right)^{2a-1} \\
&= \frac{1}{(1-z)\sqrt{1-4z}}\left(\frac{2}{(1-3z)\left[1+\dfrac{(1-z)\sqrt{1-4z}}{1-3z}\right]}\right)^{2a-1} \\
&= \frac{1}{(1-z)(1-3z)^{2a-1}\sqrt{1-4z}}\left(\frac{2}{\left[1+\dfrac{(1-z)\sqrt{1-4z}}{1-3z}\right]}\right)^{2a-1}.
\end{aligned} \tag{3.3}$$

We now apply the closed-form result (1.6) in the equation (3.3). We then obtain the equation (2.3). Also, by using the Pfaff-Kummer linear transformation (1.3) in the equation (2.3), we get the result (2.5). Similarly, by applying the Pfaff-Kummer linear transformation (1.4) in the equation (2.3), we have the result (2.7). Finally, by using Euler's linear transformation (1.5) in the equation (2.3), we are led to the result (2.9).

4. Concluding Remarks and Observations

By making use of the series rearrangement technique, we have successfully derived closed forms of several reduction formulas for two families of the Clausen hypergeometric function ${}_3F_2$. We have also indicated the connections of these reduction formulas with a number of known or new results on transformation and reduction formulas for the Gauss and Clausen hypergeometric functions.

We conclude our present investigation by observing that the several other interesting reduction formulas in closed forms can be derived in an analogous manner. Moreover, the results in closed forms, which we have derived in this paper and which are available in the existing literature (see, for example, [2], [3], [4], [6], [7], [9], [10], [11], [13], [15], [16], [17] and [18]; see also the citations to the related earlier works which are referred to in each of these references), are potentially useful in a wide range of problems in the mathematical, physical, statistical and engineering sciences.

Conflicts of Interests: The authors declare that there have no conflicts of interest.

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$$y = 1 + \frac{\alpha}{1} \cdot \frac{\beta}{\gamma}\, x + \frac{\alpha(\alpha+1)}{1 \cdot 2} \cdot \frac{\beta(\beta+1)}{\gamma(\gamma+1)}\, x^2 + \cdots$$

ein Quadrat von der Form

$$z = 1 + \frac{\alpha'}{1} \cdot \frac{\beta'}{\gamma'} \cdot \frac{\delta'}{\epsilon'}\, x + \frac{\alpha'}{1 \cdot 2} \cdot \frac{\beta'(\beta'+1)}{\gamma'(\gamma'+1)} \cdot \frac{\delta'(\delta'+1)}{\epsilon'(\epsilon'+1)}\, x^2 + \cdots$$

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