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Generalized net model of intercriteria analysis

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Abstract: Short remarks on InterCriteria Analysis (ICrA) are given. A generalized net model of functioning and results of the work of the ICrA is described.

Keywords: Generalized net, Intercriteria analysis, Intuitionistic fuzzy pair

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1. Introduction

The InterCriteria Analysis (ICrA; see [6, 7, 8]) was introduced in 2014 as a new Data Mining tool. It is similar to, but different from the correlation analyses [1, 10, 11, 12]. In the present paper, we construct a Generalized Net (GN, see [3, 4]) model of functioning and results of the work of the ICrA.

2. Short remark on intercriteria analysis

Let us have an Index Matrix (IM, see [2, 5, 6]

$$A = \begin{array}{c|ccccccc} & O_1 & \dots & O_k & \dots & O_l & \dots & O_n \\ \hline C_1 & a_{C_1,O_1} & \dots & a_{C_1,O_k} & \dots & a_{C_1,O_l} & \dots & a_{C_1,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_i & a_{C_i,O_1} & \dots & a_{C_i,O_k} & \dots & a_{C_i,O_l} & \dots & a_{C_i,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_j & a_{C_j,O_1} & \dots & a_{C_j,O_k} & \dots & a_{C_j,O_l} & \dots & a_{C_j,O_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_m & a_{C_m,O_1} & \dots & a_{C_m,O_k} & \dots & a_{C_m,O_l} & \dots & a_{C_m,O_n} \end{array},$$

where for every p, q, $(1 \le p \le m, 1 \le q \le n)$:

- C_p is a criterion,
- O_q is an object,

- a_{C_p,O_q} is a real number or another object, that is comparable about relation R with the other a-objects, so that for each i, j, k: $R(a_{C_k,O_i}, a_{C_k,O_j})$ is defined. Let $\overline{R}$ be the dual relation of R in the sense that if R is satified, then $\overline{R}$ is not satisfied and vice versa. For example, if "R" is the relation "$<$", then $\overline{R}$ is the relation "$>$", and vice versa.

If all numbers $a_{C_p,O_q} \in [0,1]$, then we can use the intercriteria analysis to detect relations between the criteria, as well as of the relations between objects. But, when there are numbers $a_{C_p,O_q} \notin [0,1]$, it is difficult to apply intercriteria analysis in both directions. The intercriteria analysis can be used without restriction to search relations between the criteria, because the method compares homogenious data. But, it cannot be applied to searching of relations between objects.

Now, we can transform all data related to each criterion C_p, for which $a_{C_p,O_q} \notin [0,1]$, using the following procedure.

(1) Determine the minimal element of the set $\{a_{C_p,O_q} | 1 \leq q \leq n\}$. Let it be $A_{\min,p}$.
(2) Determine the maximal element of the set $\{a_{C_p,O_q} | 1 \leq q \leq n\}$. Let it be $A_{\max,p}$.
(3) Change a_{C_p,O_q} for all $1 \leq q \leq n$ with

$$b_{C_p,O_q} = \frac{a_{C_p,O_q} - A_{\min,p}}{A_{\max,p} - A_{\min,p}}.$$

Of course, we suppose that

$$A_{\max,p} > A_{\min,p} > 0.$$

In the opposite case, all numbers a_{C_p,O_q} will coincide.

Therefore, we obtain the IM

$$B = \begin{array}{c|ccccccc}
 & O_1 & \cdots & O_i & \cdots & O_j & \cdots & O_n \\ \hline
C_1 & b_{C_1,O_1} & \cdots & b_{C_1,O_i} & \cdots & b_{C_1,O_j} & \cdots & b_{C_1,O_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_k & b_{C_k,O_1} & \cdots & b_{C_k,O_i} & \cdots & b_{C_k,O_j} & \cdots & b_{C_k,O_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_l & b_{C_l,O_1} & \cdots & b_{C_l,O_i} & \cdots & b_{C_l,O_j} & \cdots & b_{C_l,O_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_m & b_{C_m,O_1} & \cdots & b_{C_m,O_i} & \cdots & b_{C_m,O_j} & \cdots & b_{C_m,O_n}
\end{array},$$

where $b_{C_k,O_i} \in [0,1]$ for $1 \leq q \leq n$.

Now, we must mention that there are different algorithms for the cases when the a-elements are different. So, we discuss the following cases.

Case 1: the a-elements are natural numbers or in a particular case, elements of set $\{0,1\}$.

Let $S^{\mu}_{k,l}$ be the number of cases is which $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $R(a_{C_l,O_i}, a_{C_l,O_j})$ are simultaneously satisfied; let $S^{\nu}_{k,l}$ be the number of cases is which $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $\overline{R}(a_{C_l,O_i}, a_{C_l,O_j})$ are simultaneously satisfied, where $R \in \{<, =, >\}$ and $\overline{R} \in \{<, >\}$.

Case 2: the a-elements are rational, real or complex numbers, or in a particular case, elements of interval $[0,1]$.

Let $S^{\mu}_{k,l}$ be the number of cases is which $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $R(a_{C_l,O_i}, a_{C_l,O_j})$ are simultaneously satisfied; let $S^{\nu}_{k,l}$ be the number of cases is which $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $\overline{R}(a_{C_l,O_i}, a_{C_l,O_j})$ are simultaneously satisfied, where $R, \overline{R} \in \{<, >\}$.

In Case 2, we do not discuss the situation when the relation between two objects a_{C_k,O_i} and a_{C_k,O_j}, evaluated by real numbers, was a relation of equality (=) with the degree of uncertainty. The reason is that if we work by the first digit after decimal point and if both objects are evaluated, e.g., as 2.1, then we can adopt them as equal, but if we work with two digits after decimal ponit and if the first object has an estimation 2.15, then the probability of the second object to have the same estimation is only 10 %.

We must note that if we have real numbers in fixed-point format, i.e. precision of k digits after the decimal point, we can multiply all numbers by 10^k and then all a-arguments will be integers.

In the general case for the real numbers in floating-point format we may assume some precision ε and two numbers c_1 and c_2 coinside if $|c_1 - c_2| \leq \varepsilon$.

Case 3: the a-elements are propositions or predicates. In this case we must have an evaluation function V that changdes the a-element a_{C_p,O_q} with $V(a_{C_p,O_q}) = b_{C_p,O_q} \in \{0, 1\}$, after which we go to Case 2.

Case4: the a-elements are IFPs.

Let $S^{\mu}_{k,l}$ be the number of cases in which

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle < \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle < \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle,$$

or

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle > \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle > \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle$$

are simultaneously satisfied.

Let $S^{\nu}_{k,l}$ be the number of cases in which

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle < \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle < \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle,$$

or

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle > \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle < \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle$$

are simultaneously satisfied.

The next steps of the ICrA-algorithm for the first four cases are similar, while for the fifth case they are different. By this reason, we will discuss the fifth case later, while here, we continue with the algorithm for the first four cases.

Obviously,

$$S^{\mu}_{k,l} + S^{\nu}_{k,l} \leq \frac{n(n-1)}{2}.$$

In [9], the pair $\langle a, b \rangle$ for which $a, b, a + b \in [0, 1]$ is called an Intuitionistic Fuzzy Pair (IFP).

Now, for every k, l, such that $1 \leq k < l \leq m$ and for $n \geq 2$, we define

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)},$$

$$\nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{n(n-1)}.$$

Therefore, $\langle \mu_{C_k,C_l}, \nu_{C_k,C_l} \rangle$ is an IFP. Now, we can construct the IM

$$\begin{array}{c|ccc} & C_1 & \dots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \dots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \vdots & \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_m,C_1} \rangle & \dots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array},$$

that determines the degrees of correspondence between criteria $C_1, ..., C_m$.

Using the above values for pairs $\langle M_{C_k,C_l}, N_{C_k,C_l} \rangle$, we can construct the final form of the IM that determines the degrees of correspondence between criteria $C_1, ..., C_m$:

$$\begin{array}{c|ccc} & C_1 & \dots & C_m \\ \hline C_1 & \langle M_{C_1,C_1}, N_{C_1,C_1} \rangle & \dots & \langle M_{C_1,C_m}, N_{C_1,C_m} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_m & \langle M_{C_m,C_1}, N_{C_m,C_1} \rangle & \dots & \langle M_{C_m,C_m}, N_{C_m,C_m} \rangle \end{array}.$$

Let $\alpha, \beta \in [0, 1]$ be given, so that $\alpha + \beta \leq 1$. We call that criteria C_k and C_l are in

- (α, β)-positive consonance, if

$$\mu_{C_k,C_l} > \alpha \text{ and } \nu_{C_k,C_l} < \beta;$$

- (α, β)-negative consonance, if

$$\mu_{C_k,C_l} < \beta \text{ and } \mu_{C_k,C_l} > \alpha;$$

- (α, β)-dissonance, otherwise.

Analogically, we can compare the objects, determining which of them are in (α, β)-positive or (α, β)-negative consonance, or in (α, β)-dissonance.

If we know which criteria are more complex, or that their evaluation is more expensive, or it needs longer time, then we can omit these criteria keeping the simpler, cheaper or those requiring less time.

3. On two geometrical interpretations of the results of ICrA

In [7], a geometrical interpretation of the results of ICrA is given in the form from Fig. 1.

Now, a new geometrical interpretation of the results of ICrA will given (see Fig. 2).

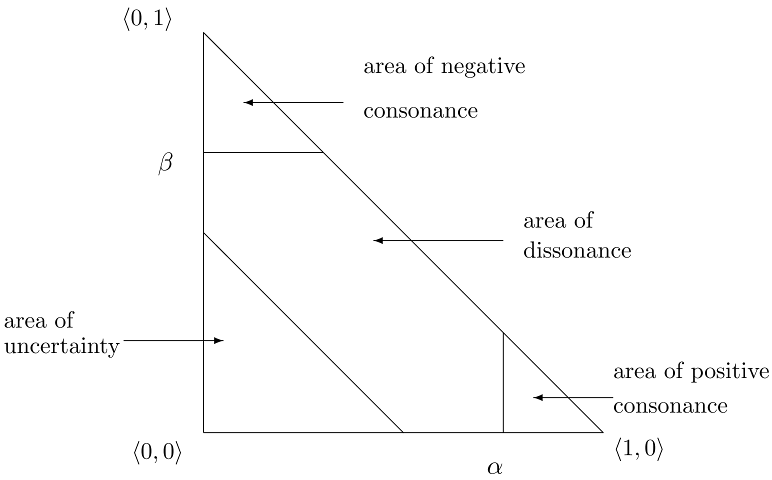

Fig. 1.

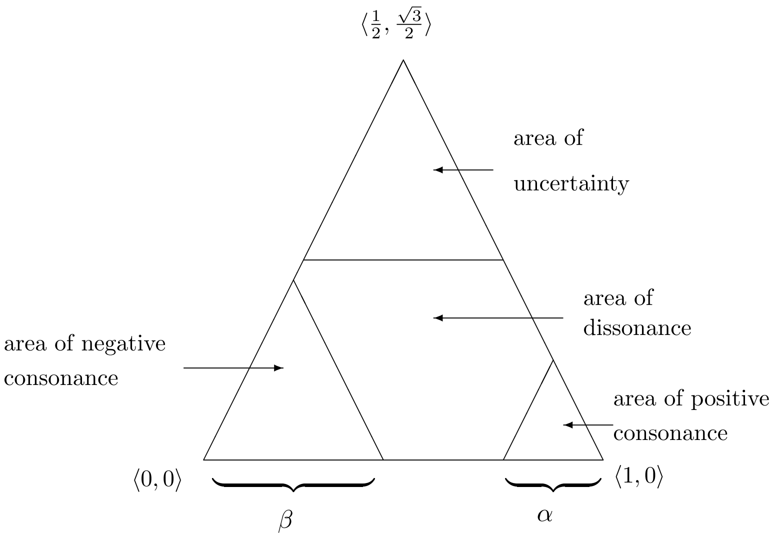

Fig. 2.

4. Generalized net model of intercriteria analysis process

The constructted here GN is reduced one. Its transitions and places do not have any priorities, while tokens β and γ will have higher priorities than the α-tokens. Its places and arcs have infinite capacities (but at each time-moment, in them only finite number of tokens will stay or will be transfered through them), transitions will be activated in each time-moment with duration of the active state one elementary time-step (t^0), tokens will keep their history and the global time-parameters will be $T = 0$ and $t^* = \infty$, but for each concrete simulation t^* will be a finite number.

In place l_1, the token $\alpha_{m,m}$ enters with the initial characteristic

"IM with elements being evaluations of n objects by m criteria and these elements have the form of natural/$\{0,1\}$-/rational/real numbers or IFPs".

The GN contains 12 transitions and 27 places that have the following forms (see Fig. 5.1).

$$Z_1 = \langle \{l_1\}, \{l_2, l_3, l_4, l_5\}, \begin{array}{c|cccc} & l_2 & l_3 & l_4 & l_5 \\ \hline l_1 & W_{1,2} & W_{1,3} & W_{1,4} & W_{1,5} \end{array} \rangle,$$

where
$W_{1,2}$ = "the IM-elements are in the form of natural numbers or elements of set $\{0,1\}$",
$W_{1,3}$ = "the IM-elements are in the form of rational/real numners",
$W_{1,4}$ = "the IM-elements are in the form of logical or linguistic variables, or predicates",
$W_{1,5}$ = "the IM-elements are in the form of intuitionistic fuzzy pairs".

In places l_2 or l_3, if this is necessity for the current data, token $\alpha_{m,m}$ obtains the characteristic

"normalization of the IM-elements, following the algorithm from Section 5.1".

Token $\alpha_{m,m}$ does not obtain any characteristic in places l_4 and l_5.

Let variables k and l are defined so that $1 \le k \le l \le m$.

For $s = 2, 3, 5$:

$$Z_s = \langle \{l_s, l_{2s+4}\}, \{l_{2s+3}, l_{2s+4}\}, \begin{array}{c|cc} & l_{2s+3} & l_{2s+4} \\ \hline l_s & false & true \\ l_{2s+4} & W_{2s+4,2s+3} & W_{2s+4,2s+4} \end{array} \rangle,$$

where
$W_{2s+4,2s+3}$ = "$i \le m$",
$W_{2s+4,2s+4}$ = "$i < m$.

Initially, token $\alpha_{m,m}$ from place l_s enters place l_{2s+4} without any characteristic and after this, step-by-step it splits to two tokens – the same token $\alpha_{m,m}$ that stays in place l_{2s+4} and the tokens $\alpha_{1,1}, \alpha_{1,2}, \ldots, \alpha_{1,m}, \alpha_{2,2}, \ldots, \alpha_{m-1,m}$ that enter place l_{2s+3}. On the time-moment, when the predicate $W_{2s+4,2s+4}$ is false, but the predicate $W_{2s+4,2s+3}$ is true, the token $\alpha_{m,m}$ also enters place l_{2s+3}.

Entering place l_{2s+3}, token $\alpha_{k,l}$ (for $1 \le k \le l \le m$) receives the characteristic

"$\langle C_k, a_{C_k,O_1}, a_{C_k,O_2}, \ldots, a_{C_k,O_n}\rangle, \langle C_l, a_{C_l,O_1}, a_{C_l,O_2}, \ldots, a_{C_l,O_n}\rangle, S^{\mu}_{k,l} = S^{\mu}_{k,l} = 0$".

The token β enters the GN from place l_6 with initial characteristic

"numerical function V for evaluation of the linguistic variables being elements of initial $\alpha_{m,m}$-token's characteristic".

When $s = 4$, the transition Z_4 has the form

$$Z_4 = \langle \{l_4, l_6, l_{12}\}, \{l_{11}, l_{12}\}, \begin{array}{c|cc} & l_{11} & l_{12} \\ \hline l_4 & false & true \\ l_6 & false & W_{6,12} \\ l_{12} & W_{12,11} & W_{12,12} \end{array} \rangle,$$

where
$W_{6,12}$ = "the IM-elements are in the form of logical or linguistic variables, or predicates",
$W_{12,11}$ = "$i \leq m$",
$W_{12,12}$ = "$i < m$.

As in the three previous cases, initially, the token $\alpha_{m,m}$ from place l_4 enters place l_{12} without any characteristic, but in difference with the previous cases, it unites with token β that enters first this place, because it has a higher priority. After this, step-by-step the token $\alpha_{m,m}$ splits to two tokens – the same token $\alpha_{m,m}$ that stays in place l_{2s+4} and the tokens $\alpha_{1,1}, \alpha_{1,2}, \ldots, \alpha_{1,m}, \alpha_{2,2}, \ldots, \alpha_{m-1,m}$ that enter place l_{2s+3}. On the time-moment, when the predicate $W_{2s+4,2s+4}$ is false, but the predicate $W_{2s+4,2s+3}$ is true, the token $\alpha_{m,m}$ also enters place l_{2s+3}.

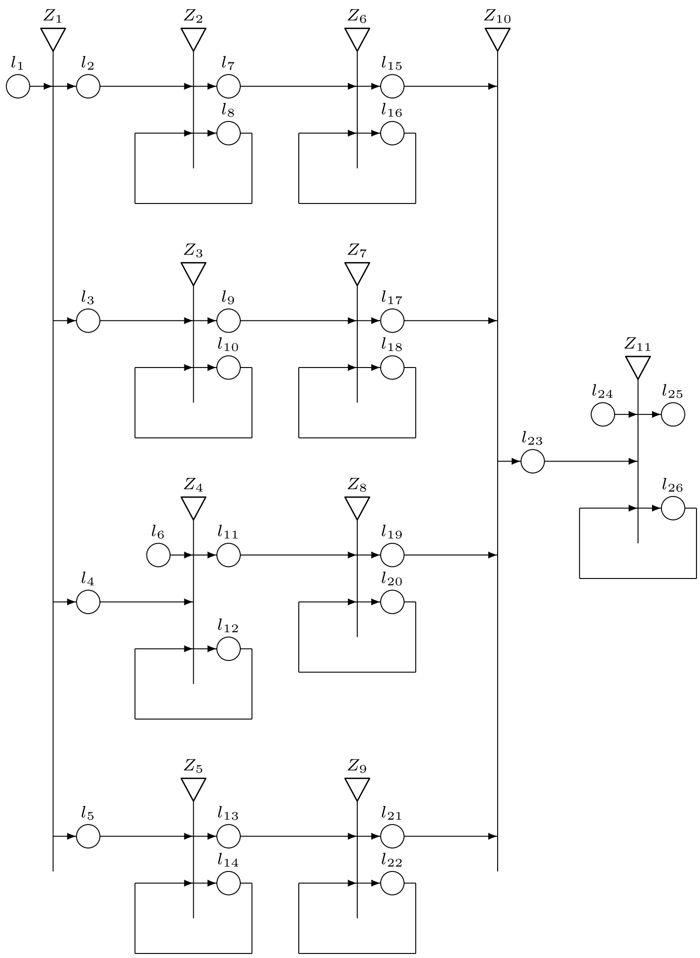

Fig. 5.1.

The $\alpha_{k,l}$-token's characteristic in place l_{11} is:

$$\text{“}\langle C_k, V(a_{C_k,O_1}), V(a_{C_k,O_2}), \ldots, V(a_{C_k,O_n})\rangle,\ \langle C_l, V(a_{C_l,O_1}), V(a_{C_l,O_2}), \ldots, V(a_{C_l,O_n})\rangle,$$
$$S^{\mu}_{k,l} = S^{\mu}_{k,l} = 0\text{”}.$$

Now, for $s = 2, 3, 4, 5$, the transitions Z_6, Z_7, Z_8 and Z_9 have the form:

$$Z_{s+4} = \langle \{l_{2s+3}, l_{2s+12}\}, \{l_{2s+11}, l_{2s+12}\}, \begin{array}{c|cc} & l_{2s+11} & l_{2s+12} \\ \hline l_{2s+3} & false & true \\ l_{2s+12} & W_{2s+12,2s+11} & W_{2s+12,2s+12} \end{array} \rangle,$$

where
$W_{2s+12,2s+11}$ = “first projection of the current α-token is smaller or equal to m”,
$W_{2s+12,2s+12}$ = “first projection of the current α-token is smaller to m”.

Each one of tokens $\alpha_{k,l}$ from place l_{2s+3} enters place l_{2s+12}, where it obtains as a characteristic with the form:

$$\text{“}\langle p, u, v\rangle\text{”},$$

where $p = 1, 2, \ldots, m$ corresponds to the time-steps of the current α-token staying in place l_{2s+12} and variables u and v increase step-by-step by the formulas from Section 5.1 for $S^{\mu}_{C_k,C_l}$ and $S^{\nu}_{C_k,C_l}$, respectively. So, in the last time-moment, when the $\alpha_{k,l}$-token leaves place l_{2s+12} and enters place l_{2s+11} it characteristic is

$$\text{“}S^{\mu}_{C_k,C_l}, S^{\nu}_{C_k,C_l}\text{”}.$$

$$Z_{10} = \langle \{l_{15}, l_{17}, l_{19}, l_{21}\}, \{l_{23}\}, \begin{array}{c|c} & l_{23} \\ \hline l_{15} & true \\ l_{17} & true \\ l_{19} & true \\ l_{21} & true \end{array} \rangle.$$

In place l_{23}, token $\alpha_{k,l}$ obtains the characteristic

$$\text{“}\langle \mu_{C_k,C_l}, \nu_{C_k,C_l}\rangle\text{”},$$

where, as is described in Section 5.1,

$$\mu_{C_k,C_l} = \frac{2S^{\mu}_{C_k,C_l}}{n(n-1)},\ \nu_{C_k,C_l} = \frac{2S^{\nu}_{C_k,C_l}}{n(n-1)}\text{”}.$$

From the beginning of the GN-functioning, token γ stays in place l_{24} with initial characteristic

“IM with row- and column-indices $C_1, C_2, \ldots, C_m$ and without elements;

constants $\alpha, \beta \in [0, 1]$ described in Section 5.1”.

$$Z_{11} = \langle \{l_{23}, l_{24}, l_{26}\}, \{l_{25}, l_{26}\}, \begin{array}{c|cc} & l_{25} & l_{26} \\ \hline l_{23} & false & true \\ l_{24} & false & true \\ l_{26} & W_{26,25} & W_{26,26} \end{array} \rangle,$$

where
$W_{26,25}$ = “all α-tokens in the net merge with the γ-token”,
$W_{26,26} = \neg W_{26,25}$
and $\neg P$ is the negation of predicate P.

The token γ has highest priority and by this reason, it first enters place l_{26}.

Tokens $\alpha_{1,1}, \alpha_{1,2}, ..., \alpha_{m,m}$ enter sequentially place l_{26} and each one of them unites with token γ that step-by-step obtains its elements, so, when the predicate $W_{26,25}$ obtains value *true*, the token γ enters place l_{25} with the final characteristic

$$\text{“}\begin{array}{c|ccc} & C_1 & \dots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \dots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \vdots & \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_m,C_1} \rangle & \dots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array},$$

lists of the elements that are in positive consonance, of the negative consonance and of dissonance”.

5. Conclusion

In all research on ICrA by the moment, the evaluations of objects by criteria are homogeneous, i.e. they are from only one of the above discussed four types. In future research, we will discuss a GN in which the evaluations of the object by criteria to be simultaneously from more than one type, i.e., from two, three or four types. Also, evaluations can have IVIF-forms, while the IM can be not only 2-dimensional, but n-dimensional, for $n \geq 3$.

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