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A CASE STUDY ON THE EFFECT OF REPEATED READING IN LEARNING NUMBER THEORY

BONGJU LEE

ABSTRACT. This study aimed to highlight repeated reading as a teaching and learning method in math education by analyzing the effects of the technique. It also aimed to explore more efficient repeated reading strategies by classifying the repeat cycle into four types. Math reading was defined as reading to understand and learn mathematics content presented in textbooks, rather than simply reading text or content aloud. I applied repeated reading to the number theory learning of eight 4^{th} year college students who were majoring in economics or business administration. Then I investigated the changes in their achievement and collected their opinions on repeated reading after learning. It was also found that repeated reading in university math education had positive effects in the point of both cognitive and affective aspects, regardless of repeat cycles. In addition, although there were differences according to individual student dispositions, the shorter the repeat cycle, the more positive it was. On the other hand, this study suggested that it would be more effective to provide students with feedback on what they read when teachers apply repeated reading in their math classrooms.

1. INTRODUCTION

Math reading is a basic and essential skill in mathematics education, as math learning cannot occur without reading [18, 36]. Math reading differs from linguistic reading because mathematical texts have different characteristics from texts on other subjects. This is because mathematical texts contain more concepts and information than texts in other subject areas, such as numbers, symbols, terms, and graphs to be deciphered [12]. In other words, math reading is a multifaceted task because it requires mathematical understanding when reading numbers, symbols, and verbal expressions [1]. The core elements of math reading are mathematical language deciphering, mathematical language transformation, and understanding [36]. As described above, math reading not only plays an important role in mathematics education, but also has different characteristics from reading in other fields, so studies on math reading need to cover various aspects.

In mathematics education in particular, educators and researchers should take note of the repeated reading teaching and learning method proposed by Samuels [25]. Samuels suggested that repeated reading is effective in raising vocabulary identification and allows learners to free up more cognitive resources to aid their understanding. Yun [37]

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argued that through repeated reading, learners can confirm the information that they may have missed when reading the first time, and that the learning information stored in the first reading can be transformed and reinforced through repeated reading. Blum and Koskinen [4] established that repeated reading is helpful not only in terms of cognitive skills, but also as regards the affective aspect of learners. This means that the more opportunities to input the target learning content, the greater the possibility of learning, thus suggesting the importance of repeated reading, not only in language learning but also when learning math.

However, it is difficult to locate studies on repeated reading in the field of mathematics education except for early childhood mathematics education [11, 15]. Since mathematical knowledge is hierarchical, it has limitations to conclude that it will have the same effect as repeated reading of children's math storybooks. This means that it is also necessary to investigate the effect of repeated reading learning in the upper grades. In addition, according to Yang and Yu [35], who surveyed the current state of math reading education among teachers and students, not only did the reading of math textbooks decrease as students' graders got higher, but also teachers did not consider reading as part of math teaching. These facts show that it is necessary to provide opportunities to utilize repeated reading as a teaching and learning strategy in mathematics education, and that through studies, the effectiveness of the repeated reading strategy should be verified.

Therefore, this study attempts to propose and highlight repeated reading as a math teaching and learning strategy by investigating the learning of number theory among college students and exploring the effects of repeated reading in detail. Since the unique characteristics of mathematical texts, including mathematical symbols, terms, and graphs, etc., are the same in both school and university mathematics, it was considered possible to derive implications for more effective teaching and learning method in school mathematics. In this study, math reading is defined as reading to understand and learn mathematics content presented in textbooks, rather than simply reading text or content aloud.

2. THEORETICAL BACKGROUND

2.1. Reading and Mathematics Education

In the early 1960s, reading in the field of pronunciation education was dominated by simply looking at a text and being able to say the content out loud. However, in the discipline of reading theory in the field of linguistics, this perspective has changed [28]. Goodman [8] defined reading as a psycholinguistic guessing game and explained it as a process in which the reader actively selects information from reading materials. Widdowson [34] defined it as a means of information interpretation rather than information processing, and Smith [31] described it as a method of understanding that is predictive, selective, and purposeful. Since the 1980s, theorists have recognized reading as an active process in which readers construct new meaning by using and manipulating their own knowledge of language and the world [24]. That is to say, reading is not a simple process of interpreting sentences by connecting letters and sounds, but it is a procedure in which a reader's background knowledge and thinking system interact with the text in a particular context [9].

Through these modifications, the concept of reading has expanded to involve reading as a way of thinking and learning [6]. Readers integrate their existing knowledge structures with new information to create meaning during the process of reading [7]. Reading researchers concur that every reading activity involves both reading and understanding

symbols [28]. This is because in the early stages of learning to read, the process of decoding symbols takes precedence and gradually moves to the skill of understanding, but these two processes cannot be separated from each other when reading.

Reading is not only an important part of everyday life but also an essential way of acquiring knowledge and understanding the world, although it is difficult to define reading in simple terms as explained above [6, 7, 28]. In school education, reading plays a highly important role as a basic tool for learning the content of the curriculum [23, 35]. As textbooks and reference books must be read in all subjects, reading is an essential part of learning and helps to improve thinking skills in all areas [14]. In particular, learners who lack the basic ability to read are negatively affected in other subjects [11].

The importance of reading is not an exception in mathematics education. Many researchers [1, 16, 27] have reported that successful math learning requires the ability to read and interpret math well. Adams and Lowery [2] observed that doing math is the result of reading math and emphasized the importance of reading in math learning.

However, many students experience difficulties in reading math and this may be one of the obstacles to math learning. According to Yang and Yu [35], many students made errors or suffered learning difficulties because of misunderstandings and misuse of literal, symbolic, and graphic language in the math classrooms. Shepherd, Selden, and Selden [28] asserted that students might have difficulties in reading mathematics because of the peculiar way of describing the processes. Simonson [29] specified four features of mathematical texts, and described a strategy for reading mathematics that is based on these features. Examples of these features are as follows: (1) math reading should happen from right to left, top to bottom, bottom to top, or diagonally; (2) mathematics textbooks contain more concepts in sentences, units, and paragraphs than textbooks in other subjects; (3) mathematical concepts are abstract and sometimes require an effort to visualize; and (4) mathematics textbooks use concise descriptions with little redundancy and this helps students to understand the meaning. In outlining these features, Simonson explained that understanding the content of math requires cross-references, scans, pauses, returns, and so on.

Despite these difficulties, National Council of Teachers of Mathematics [20] emphasized the use of reading in math learning. This is because they know that for effective math learning, reading to accurately understand and interpret math content is necessary [5, 32]. Therefore, research on reading in mathematics education needs to be carried out from various aspects.

2.2. Repeated Reading and Mathematics Education

The repeated reading method was suggested by Samuels [25] for training learners to read. Lee [17] explained the advantages of repeated reading from a linguistic, anthropological perspective. He said that because acquiring a language happens firsthand, the more opportunities to input the same material, the higher the chance of learning. Another advantage of repeated reading is that the learner can convert and reinforce the learning information stored during the first reading [3]. Min and Lee [19] also revealed that repeated reading helps to improve comprehension by providing an opportunity to understand and master the structure of the text.

Repeated reading has a positive effect not only on a student's cognitive abilities, but also on affective aspect. Blum and Koskinen [4] proved that repeated reading not only improves reading fluency and comprehension, but also helps students to become confident and motivated. Kim [13] reported that if learners read the same paragraph repeatedly, they become experts in that particular section, and learners who are satisfied with these results are motivated to

increase their reading experience. On the other hand, Rasinski [21] was concerned that repetitions could lead to students becoming tired or indifferent. However, in a 2003 study, Rasinski [22] concluded that learners should be provided with reasons to expand their knowledge and participate in their learning.

As seen above, studies on repeated reading in language education have been conducted from various perspectives. Since mathematical texts have different characteristics from linguistic texts, limited conclusions can be drawn from these. The literature on math reading prior to 2008 mostly covered topics related to math reading strategies that focused on the importance of math reading [33]. On the other hand, Schickedanz [26] argued that the teaching and learning method of interdisciplinary integration through storybook reading should promote learning in all subjects, and suggested a teaching strategy of repeated reading as an alternative. This teaching strategy is not simply repeated reading, but a way of telling a story by reading a math storybook three times and then shifting the focus from story-centered exploration to a mathematical situation. This proposal presented a valid theory for the repeated reading approach of mathematical storybooks involving various mathematical strategies. Based on this, the effectiveness of the repeated reading of math storybooks among young children was verified, and positive results were reported [10, 15]. Additionally, the math storybooks used in these studies were not only written in everyday language suitable for young children, but also the books were read by the teacher while the children listened. Consequently, it is difficult to say whether the results of these studies would produce the same effect when students repeatedly read mathematics textbooks themselves in school mathematics education. This implies that it is necessary to provide an opportunity to use repeated reading as a teaching and learning method in the math classrooms through studies that can verify the effectiveness of the approach.

3. METHODS

3.1. Design

This selected eight 4^{th} year college students majoring in economics or business administration to examine the effect of repeated reading in math from two aspects: 1) change of scores in pre- and post-tests, and 2) students' perceptions of repeated reading after learning. In order to diversify the repeated reading learning process, the cycle consisted of four cycles of 1-day, 3-day, 5-day, and 7-day intervals, and the number of readings for all cycles was fixed to three rounds. Two students were assigned to each reading cycle group, and each student was not limited as regards reading time in subsequent readings, but they were guided to read for more than 30 minutes during the first reading. That is, students could autonomously adjust their learning time and decide to end their reading session. Repeated reading learning was conducted independently according to students' individual schedules, and the learning material was designed to be displayed on a computer screen for learning only by reading. Students could read the content while navigating the material on the computer screen. The overall research process was designed to be carried out in the following order; development of reading material, development of pre- and post-test tools and open questionnaire, open recruitment and selection of research targets, guidance of research and learning procedures, pre-test, repeated reading, post-test and questionnaire survey, data arrangement and analysis. Figure 1 is a schematic diagram of the research procedure.

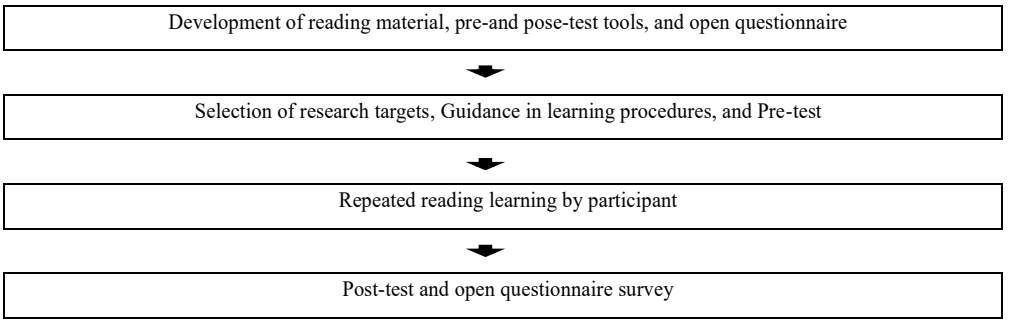

FIGURE 1. Research procedure

3.2. Reading Material

The reading material was intended to be content that students could learn by themselves through reading using their prerequisite knowledge. The main reason is that, according to Sindelar, Monda, & O'Shea [30], it is more effective to use the appropriate level of text so that students do not feel that it is too difficult to understand. Therefore, this study selected the reading material by considering the level at which students could learn through reading based on the high school mathematics curriculum. Thus, relatively easy content was extracted from the number theory textbook which students had not learned previously and it was then reorganized into reading material. Reading material consists of two definitions, five theorems and their proofs, one corollary, and three examples. The content order is as follows; Definition 1 (definition of divisor and multiple and their notations), Theorems 1 and 2 (two propositions related to Definition 1 and their proofs), Definition 2 (definition of the modulus congruence and its notation), Example 1 (examples relating to Definition 2), Theorems 3 and 4 (two propositions related to Definition 2 and their proofs), Corollary (proposition related to Theorem 4), Theorem 5 (proposition about the modulus congruence of an integer and the remainder and its proof), Example 2 (simple examples on Theorem 5), and Example 3 (complex examples on Theorem 5).

3.3. Test Tool and Open Questionnaire

The test tools for investigating the difference in scores on the pre-test and post-test are identical. The test tool consisted of ten questions, comprising five questions to judge true or false and five descriptive problems that were asked both the answer and the solution. For the true or false question, students were asked not to guess the answer but only to answer when they were convinced whether it is true or false. For the descriptive problems, they were asked to leave all what they wrote without erasing. One example of the true or false questions was "Let $a, b, c \in Z$. If $a|c$ and $b|c$, then $c|(a+b)$." One of the descriptive problems was "Find the least positive integer m which is congruent to $3 \times 10^3 + 4 \times 10$ modulo 9."

A survey of students' perceptions of repeated reading was conducted immediately after the post-test using a structured open questionnaire. This intended to minimize the impact on learning by repeated reading. The open questions were as follows: 1) what did you focus on in the learning process? 2) what did you learn through repeated reading? 3) feel free to provide any other comments.

3.4. Participants

This study was conducted after IRB (2018-0103) approval. Eight 4th year college students majoring in economics or business administration in a national university were openly recruited. They were informed that I would follow research ethics throughout the experiment and that they could withdraw at any time even if they applied to participate. Each student filled out a written consent form.

The students were divided into four groups, with two assigned to each group. The groups were organized so that the gender and pre-test scores of the participants were distributed as evenly as possible, and their available schedule was also considered. Therefore, two out of three students whose pre-test score was not 0 were assigned to the same group that the repeat cycle is 5-day. They were each provided with a description of the entire repeated reading process.

They were told that learning should be limited to the experiment time, and that they could not discuss the experimental processes, including the learning content, with each other. Table 1 provides detailed information regarding the eight students including their participant dates.

TABLE 1. Participants and data collection dates

Cycle	Participants			Dates (mm/dd)				
	ID	Gender	Major	Pre-test	First reading	Second reading	Third reading	Post-test
1-day	1-NH	F	Economics	01/30	02/02	02/03	02/04	02/07
	1-SB	F	Economics	02/06	02/09	02/10	02/11	02/14
3-day	3-HJ	F	Economics	02/04	02/07	02/10	02/13	02/16
	3-WJ	M	Economics	02/04	02/07	02/10	02/13	02/16
5-day	5-DJ	F	Business ad.	02/01	02/04	02/09	02/14	02/17
	5-UH	M	Economics	02/01	02/04	02/09	02/14	02/17
7-day	7-YN	F	Business ad.	02/04	02/07	02/14	02/21	02/24
	7-JH	M	Economics	01/30	02/02	02/09	02/16	02/19

3.5. Data Collection and Analysis

For each student, the reading time during the repeated reading session was measured. Before and after repeated reading, all students' achievements were tested. The pre-test was conducted three days before the start of the repeated reading learning, and the post-test was conducted three days after the learning had finished. This was to minimize the influence between tests and readings. The scores of the students' pre-test and post-test were calculated, and the effect of repeated reading was analyzed based on the differences between the scores.

Once the post-test was finished, students answered open questions that explored their thoughts about repeated reading. Students' responses to the open questions were analyzed in terms of the effectiveness of repeated reading and the implications for more efficient repeated reading. After the analysis, the results were described and discussed by presenting the students' actual responses and evaluating them.

4. FINDINGS AND DISCUSSION

4.1. Reading Time

Table 2 shows the reading times per round and the total reading time of the eight students. In the case of the first reading, they were guided to learn for more than 30 minutes, and the students in the 1-day, 3-day, and 5-day cycle groups spent about two to four minutes longer than 30 minutes. The students in the 7-day cycle group spent almost 30 minutes. However, the total reading times during the three repetitions were relatively longer for the students in the 7-day cycle group than for the other students except for 1-NH. As can be seen from Table 2, this indicates that the students in the 7-day cycle group spent more time on the second and third readings than the other students, except for 1-NH. On the other hand, when comparing the times of the second reading and the third reading, six participants, except for 1-SB and 5-DJ, spent more time on the second reading than they did on the third. These findings suggest that when learning new content, the time it takes to repeatedly read varies depending on the student.

TABLE 2. Reading time per round (min:s)

Round	1-NH	1-SB	3-HJ	3-WJ	5-DJ	5-UH	7-YN	7-JH
First	34:07	31:42	32:06	32:38	32:34	31:49	30:13	30:20
Second	25:51	11:00	18:05	22:13	12:53	15:14	20:52	21:49
Third	17:41	11:24	15:01	09:48	17:35	14:30	14:19	12:46
Total	77:39	54:06	65:12	64:39	63:02	61:33	65:24	64:55

4.2. Pre-Test and Post-Test Scores

Each question in the test tools scored 1 point if correct and 0 if incorrect, so 10 points was the total score. Table 3 shows the pre-test and post-test results of the eight students. Excluding 5-DJ, the post-test scores of seven students improved by more than 5 points compared to the pre-test scores. In particular, the students in the 1-day cycle and 3-day cycle groups all improved by more than 6 points. These results show the potential of a self-directed method of learning math with repeated reading, especially showing that relatively short repeat cycles have a more positive effect on math learning than long repeat cycles.

TABLE 3. Pre-test and post-test scores

Test		1-NH	1-SB	3-HJ	3-WJ	5-DJ	5-UH	7-YN	7-JH
Pre		0	0	0	3[(1)]	2[(2)]	2[(3)]	0	0
Post	T/F	4	3	3	5	4	3	4	2
	Descriptive	2	5	3	5	1	4[(4)]	2	3
	Total	6	8	6	10	5	7	6	5

[(1), (2)] Correct answers to the T/F questions.

[(3)] Q9 and Q10 were solved by the corresponding learning content.

[(4)] Q9 and Q10 were solved by pattern and calculation.

4.3. Students' Focus While Learning

In this section, let us look at students' focus while learning through repeated reading. What all eight students had in common is that during learning, they tried to understand the content rather than memorizing it. Some students responded that they memorized content that they did not understand. Some students also mentioned that they tried to memorize content. From these responses, it can be seen that most students strive to understand when learning math.

First, let us look at the two students whose achievements improved quite a lot after learning. 3-WJ, who received a perfect score in the post-test, was found to have read more repeatedly about concepts that he was not familiar with (see ①). In addition, he made an effort to repeatedly read to check if he had understood correctly (see ②). In the case of 1-SB who obtained the second highest score, in the first reading, she studied the entire content focusing on understanding the concepts (see ③), and in the subsequent reading, she confirmed the solving of examples based on the concepts she had understood (see ④). These cases show that while reading over and over again, two students were making good use of the repeated reading strategy by revisiting their previous learning and verifying that they had understood correctly.

> *I focused on grasping and understanding the principles of the concept of congruence. To this end, ① I tried to read the textbook repeatedly until the congruent formula became familiar, and after reading the overall solving process, I focused on the parts where I could not understand intuitively. Finally, ② I tried to verify that I had correctly grasped the concepts and development process while comparing those in my mind with those in the textbook. (The response of 3-WJ)*

> *Since there were concepts I had encountered for the first time, I learned based on definition and proof. ③ In the first reading, I learned the later content after studying the concepts, and ④ in the subsequent reading, I focused on following the solutions to examples presented at the end based on the memorized concepts. (The response of 1-SB)*

Next, let us take a closer look at the two students who had relatively low achievements. Both students improved by 5 points. 5-DJ focused on whether she could read and understand, despite her lack of mathematical knowledge (see ⑥), and she tried to memorize simple mathematical formulas (see ⑤). The improvement in her score suggests that students can learn at a certain level through repeated reading, even if they feel that they lack mathematical knowledge. 7-JH tried to confirm that he understood the learning content correctly by reading the examples repeatedly (see ⑧). This shows that he used the repeated reading strategy appropriately as high-scoring students demonstrate. Meanwhile, 7-JH mentioned that he memorized content that he could not understand, despite his efforts (see ⑦). This shows that direct feedback from instructors about what students do not understand is required.

> *(1) When reading, can I read while understanding? (2) ⑤ Can I memorize simple mathematical formulas? (3) ⑥ Can I understand just by reading texts without mathematical knowledge?(The response of 5-DJ)*

> *I tried to understand the overall process and flow. I was able to understand most of the definitions and theorems, but ⑦ I chose to just memorize the parts that I could not understand even by repeated reading. ⑧ I also focused on checking whether what I understood was correct by repeating the last example. (The response of 7-JH)*

4.4. Students' Perceptions of Repeated Reading

This study aimed to examine the effects of the repeated reading strategy. In this section, I try to examine the effect in detail based on the perception of students who have experienced repeated reading. This is because, from their opinions, the positive and negative aspects of repeated reading as a learning strategy could be examined in more detail. Four students in the 1-day and the 3-day cycles presented only positive aspects of repeated reading. Students in the 5-day and 7-day cycle groups mentioned both positive and negative aspects. I would suggest the potential of using repeated reading strategies based on their opinions of positive aspects, and I would derive implications for a more efficient repeated reading strategy based on their opinions of negative aspects.

First, let us take a look at the cases that mentioned positive effects. 1-NH, who improved by 6 points in the post-test, mentioned the following three positive effects of repeated reading; (1) what was not understood in the first reading could be understood in the next reading (see ⓐ), (2) as the number of readings increased, the reading speed became faster (see ⓑ), and (3) the application of mathematical knowledge that is used in other areas was noticed (see ⓒ). (1) and (2) correspond to the typical effects of repetitive reading, and (3) can be said to be simply a learning effect.

> *When I read "the definition of* $a|b$ *" on the first day, I didn't know what it means, and when I read it twice in a row, it seemed that I could partially understand it. ⓐ On the second day, I was able to understand the part that I did not understand on the first day. I think this happened due to the memory of the first day. On the last day, ⓑ I could read more quickly compared to the first and second days. And now that I think about it, ⓒ I was able to get the remainder by using the* mod *function in Excel, and it seems that the* mod *is this* mod. *When I read the first day, I couldn't understand it, so I didn't even think about it. (The response of 1-NH)*

In the case of 1-SB, the following two positive effects of repeated reading were mentioned: (1) The more focus there is on the concepts and if the content is read repeatedly, the shorter the learning time will be (see ⓓ), and (2) there is a possibility for reading strategies as a learning method (see ⓔ). (1) is consistent with the effect of repeated reading that 1-NH suggested earlier (see ⓑ), and (2) can be said to show the potential of repeated reading as a learning method.

> *I felt that even 30 minutes was not enough on the first reading, but ⓓ just focusing on the concepts and reading them over and over again feeling like the learning time was shortening. When I was learning normally, I studied while writing, but ⓔ realized that learning is possible just by reading. (The response of 1-SB)*

The responses of the two students in the 3-day cycle are as follows. In the case of 3-HJ, she mentioned one of the effects mentioned by 1-NH. Even if the content was not understood in the first reading, it would be understood by reading it again later (see ⓕ and ⓐ). For 3-WJ, he revealed from his own experience that repeated reading is useful as a learning method (see ⓖ).

> *ⓕ When I first read it, it was difficult to understand, and it seemed that I could not understand the very difficult part at all. But on the second reading, I understood it. That was an amazing experience. Even if I look at it once and I don't understand it, I think I will understand well if I read it over and over again. (The response of 3-HJ)*

ⓖ I feel that the learning content stays in my head if I constantly acquire knowledge through repeated reading. Of course, it is difficult to guarantee how long it will remain in my head. (The response of 3-WJ)

For the students in the 5-day and 7-day cycles, 5-DJ and 7-YN had positive perceptions of repeated reading. 5-DJ felt that she lacked mathematical knowledge, but she thought her understanding of mathematics improved through repeated reading (see ⓘ). In particular, she said that in the first reading, it was difficult to understand the content, but that in the subsequent reading, the reading time was reduced and the content was a little more understandable (see ⓗ). This shows that although she lacked confidence in her own mathematical knowledge, and actually received a relatively low score on the post-test, she became confident and thought that her mathematical comprehension had improved through repeated reading. This implies that, to some extent, it is she had the opportunity to understand math where she lacked confidence, and to solve some questions in the post-test.

At first, it was difficult to understand formulas, theorems, proofs, etc. Even when I read a single sentence, I had to read it several times, and there were many moments when I was dazed. ⓗ However, in the second reading, the speed of understanding gradually increased and the overall flow and formula became easier to understand, if not completely memorized. The third reading took a little longer than the second reading, but the formulas came into my mind. ⓘ I lack mathematical skills, but it seems that my comprehension has improved through repeated reading. (The response of 5-DJ)

7-YN, who learned with the longest repeat cycle in this experiment, also mentioned similar aspects that other students saw as being positive. He said that it was somewhat difficult to understand at first, but his comprehension improved as he could repeat reading (see ⓙ). Slightly different from what other students mentioned, he said that the more he read, the more detail he could perceive in the content (see ⓚ). This can be interpreted as naturally recognizing the content that was read at first but was not retained in his memory. In addition, it can be seen that repeated reading allows students to identify the parts that they missed at first in the later reading, giving them an opportunity to check the content more carefully.

As a result of repeated reading at regular intervals, I felt that the amount of information I had acquired became larger. ⓙ At first, I accepted the unfamiliar terminology and the process of proof rather intuitively, but gradually understood the process of deriving the result over time. ⓚ I started reading more meticulously by reading over and over again, and I could easily infer the correct answer to the problem as a result of the review. (The response of 7-YN)

Next, let us look at the students who mentioned negative effects, and identify points that need to be examined. For those in the 5-day and 7-day cycles, 5-UH and 7-JH negatively perceived the repeated reading. 5-UH understood the content at first, but recognized that the efficiency of repeated reading was low due to his learning style of not memorizing the content (see ⓛ). However, his opinion seems to be based on his perception that he did not memorize what he understood, rather than it being a negative aspect of repeated reading itself. Generally, it is necessary to memorize a certain part while learning, and it is a natural phenomenon that some of the content is forgotten over time, so this factor would appear in any learning method, not just repeated reading. Nevertheless, it can be confirmed once again that it is necessary to use a variety of teaching and learning strategies rather than just one strategy in the math

classroom because students' dispositions and learning styles are so diverse.

> *I got to know my learning style. I prefer understanding rather than memorizing, and in particular, I tend to skim over the easy parts and come back and look again. I understand the concept first and then learn the solution through practicing problems, so I am weak at memorizing formulas. In the end, ⓘ I understood it for the first time, but I felt that repeated learning is not very efficient. (The response of 5-UH)*

In the case of 7-JH, he did not understand at first, but it turned out that he tried to understand the material by reading it over and over again (see ⓜ). However, he added that he was not sure if he read it and understood it correctly (see ⓝ). His opinion shows that there is a limit to what students learn the first time they encounter material and also by reading repeatedly on their own. In other words, this suggests that teachers should provide feedback on what students have learned through repeated reading.

> *When encountering a very unfamiliar field, at first I simply read and did not understand the content accurately. ⓜ However, I tried to understand the content in my own way while encountering the same content repeatedly. ⓝ Nevertheless, I am still not sure if what I learned and understood on my own is correct, and I think that over time I will not be able to remember specific solutions. (The response of 7-JH)*

On the other hand, all students except 5-UH responded positively to question 3 which is asking how they felt about participating in the experiment. 5-UH replied that he thought how important the eyes are. This appears to be linked to the learning content on the computer screen. In particular, 7-JH, who expressed a negative opinion of repeated reading in question 2, also expressed positive feelings (see ⓞ). Like 7-JH, five other students (1-NH, 3-HJ, 3-WJ, 5-DJ, and 7-YN) also said they felt happy to learn new knowledge that was not in their major field.

> *I was often frustrated and dazed by my completely unfamiliar professional mathematical knowledge. ⓞ But this opportunity to encounter mathematics, which is difficult to understand in general, seems to have been a good experience for me. (The response of 7-JH)*

In particular, 3-HJ said that she was confident that she was able to solve some problems or that she was able to try to solve other problems (see ⓟ). It seems that repeated reading has improved her confidence in learning by providing an opportunity to solve math problems after learning alone without the help of others.

> *It seems that I have accumulated mathematical knowledge by participating in the experiment and learning about the mathematical symbols I see for the first time. Also, ⓟ I couldn't solve a single problem at first, but after the experiment, there were some problems that I could solve, and I was proud to be able to try an approach even if the answers were wrong. (The response of 3-HJ)*

5. CONCLUSION AND SUGGESTIONS

This study aimed to highlight repeated reading as a teaching and learning method in math by analyzing the effects of repeated reading. It also aimed to explore more efficient repeated reading strategies by classifying the repeat cycle into four types. I applied repeated reading to the number theory learning of eight 4^{th} year college students who were

majoring in economics or business administration and were openly recruited. Then I investigated the changes in their achievement and collected their opinions on repeated reading after learning. The conclusion and implications based on the findings of the students' changes in achievement and their opinions on repeated reading are as follows.

First, it has been empirically verified that repeated reading can be fully utilized as a teaching and learning method in mathematics education. Although there were slight differences depending on the repeat cycle and individual, all eight students who participated in this study saw improvements in their achievements. In addition, most of the students expressed pleasure in acquiring new mathematical knowledge through repeated reading. In particular, one student clearly stated that she realized that learning math is possible simply by repeated reading.

Second, learning to acquire knowledge through repeated reading can have a positive effect in terms of students' affective aspects and this gives students the confidence and enjoyment that they can study math alone. This is supported by the fact that six out of eight students who experienced repeated reading in math enjoyed this learning method. In particular, a student who thought her mathematical knowledge was lacking also started to think that her understanding of math improved after participating in the experiment. This is due to the confidence we feel when we understand part of the content by reading the same difficult content over and over again.

Third, it is suggested that the effect of reading with relatively short repeat cycles is more positive than with a long repeat cycles. This is based on the fact that the average achievement of students reading every 1 and 3 days is higher than the average achievement of students reading every 5 and 7 days. In addition, students who studied in the 1-day and 3-day cycles reported only positive aspects of repeated reading, while students in the 5- and 7-day cycle groups mentioned both positive and negative aspects. This may be due to differences in individual student dispositions, but there are similar trends and thus more evidence.

Fourth, when teachers plan and use repeated reading as a teaching and learning method in the math classroom, their feedback on what students read and understand is required. This is because, as one student mentioned, students doubt whether they have understood the content correctly. In other words, when teachers apply repeated reading as a teaching and learning method in math, they must carefully consider how to give feedback to students.

Fifth, when teachers use repeated reading as a teaching and learning method in math, they should consider about using different strategies together. This is because students' achievement levels in math vary and the time it takes to understand new mathematical symbols is different too. In particular, one student's perception that his own learning style does not seem to be suitable for repeated reading, which also shows why this suggestion is necessary.

Finally, this study is significant in that it was possible to verify the effect of repeated reading from the students who experienced it directly. They experienced the following positive effects of repeated reading which anyone can predict: (1) what was not understood in the first reading could be understood in the next reading, (2) as the number of readings increased, the reading speed became faster, and (3) the more we read, the more detail we can derive from the content. However, this study is limited in that it was conducted with a small number of college students. Therefore, a follow-up study would be that quantitatively verifies the same effect in mathematics education for a large number of secondary students.

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DEPARTMENT OF MATHEMATICS EDUCATION, KYUNGPOOK NATIONAL UNIVERSITY, DAEGU 41566, REPUBLIC OF KOREA

E-mail address: leebj@knu.ac.kr