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Special Case on Rogers-Ramanujan Type Continued Fraction Identity

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Dedicated to Prof. C. Adiga on the occasion of his 62^{nd} birthday.

Abstract: In this article, we derive a new continued fraction $d(q)$ by using a general continued fraction in Ramanujan's lost notebook. By using this continued fraction $d(q)$, we establish modular relation between $d(q)$ and $d(q^n)$, where $n = 2, 3, 4, 5, 7, 11$. We also establish some explicit values of $d(q)$ by using Ramanujan's class invariant.

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1. Introduction

In Chapter 16 of his second notebook [3], Ramanujan developed the theory of theta-function, defined by

$$f(a,b) := \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}}, \mid ab \mid < 1, \tag{1.1}$$

$$= (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty$$

where $(a;q)_0 = 1$,

(1.2)
$(a;q)_n = (1-a)(1-aq)(1-aq^2)\cdots(1-aq^{n-1})$ *and* $(a;q)_\infty = (1-a)(1-aq)(1-aq^2)\cdots$.

Following Ramanujan, we define

$$\varphi(q) := f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = \frac{(-q;-q)_\infty}{(q;-q)_\infty}, \tag{1.3}$$

$$\psi(q) := f(q,q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}, \tag{1.4}$$

$$f(-q) := f(-q,-q^2) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.5}$$

and

$$\chi(q) := (-q;q^2)_\infty. \tag{1.6}$$

Now we define a modular equation in brief. The ordinary hypergeometric series ${}_2F_1(a,b;c;x)$ is defined by

$$ {}_2F_1(a,b;c;x) := \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n n!} x^n, $$

where $(a)_0 = 1, (a)_n = a(a+1)(a+2)\cdots(a+n-1)$ for any positive integer n, and $\mid x \mid < 1$.
Let

$$ z := z(x) := {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; x\right) \tag{1.7} $$

and

$$ q := q(x) := \exp\left(-\pi \frac{{}_2F_1(\frac{1}{2}, \frac{1}{2}; 1; 1-x)}{{}_2F_1(\frac{1}{2}, \frac{1}{2}; 1; x)}\right), \tag{1.8} $$

where $0 < x < 1$.
Let r denote a fixed natural number and assume that the following relation holds:

$$ r\frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; 1-\alpha\right)}{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \alpha\right)} = \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; 1-\beta\right)}{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \beta\right)}. \tag{1.9} $$

Then a modular equation of degree r in the classical theory is a relation between α and β induced by (1.9). We often say that β is of degree r over α and $m := \dfrac{z(\alpha)}{z(\beta)}$ is called the multiplier. We also use the notations $z_1 := z(\alpha)$ and $z_r := z(\beta)$ to indicate that β has degree r over α.

The function $\chi(q)$ is intimately connected to Ramanujan's class invariants G_n an g_n which are defined by

$$ G_n = 2^{-1/4}q^{-1/24}\chi(q), \quad g_n = 2^{-1/4}q^{-1/24}\chi(-q), \tag{1.10} $$

where $q = e^{-\pi\sqrt{n}}$ and n is a positive rational number. Since from [3, Entry 124(v),(vi), p.56]

$$ \chi(q) = 2^{1/6}\left\{\alpha(1-\alpha)q^{-1}\right\}^{-1/24} = (-q;q^2)_\infty, \tag{1.11} $$

$$ \chi(-q) = 2^{1/6}(1-\alpha)^{1/12}\alpha^{-1/24}q^{1/24} = (q;q^2)_\infty. \tag{1.12} $$

The most famous of them is the celebrated Rogers-Ramanujan continued fraction $R(q)$, defined as

$$ R(q) := \cfrac{q^{1/5}}{1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{1+\cdots,}}}}, \quad |q| < 1. \tag{1.13} $$

In his Lost Notebook [7,pp 365] as well as in his letters to Hardy [6], he provided five beautiful identities connecting the continued fraction $R(q)$ with the five continued fractions $R(-q), R(q^2), R(q^3), R(q^4), and R(q^5)$. He also provided some explicit values of the continued fraction R(q). An account of this can be found in [2] and [5].

Ramanujan recorded the following general continued fraction in his Lost Notebook [7] or [1, p. 144, Entry 6.2.1]: For any complex numbers a, b, λ, and q with $|q| < 1$,

$$\frac{G(aq,\lambda q;b;q)}{G(a,\lambda;b;q)} = \cfrac{1}{1+\cfrac{(aq+\lambda q)}{1+\cfrac{(bq+\lambda q^2)}{1+\cfrac{(aq^2+\lambda q^3)}{1+\cfrac{(bq^2+\lambda q^4)}{1+\cdots,}}}}}, \tag{1.14}$$

where

$$G(a,\lambda;b;q) := \sum_{n=0}^{\infty} \frac{(-\lambda/a;q)_n (a)^n}{(q;q)_n(-bq;q)_n} q^{n(n+1)/2}. \tag{1.15}$$

In [1] C. Adiga and D. D. Somashekara gave some special result on Rogers-Ramanujan type continued fraction identities.Recently [10] Nipen Saika and Chanyanika Boruah gave special case of a general continued fraction of Rogers-Ramanujan type. They established several new modular identities and also proved general theorems for the explicit evaluation of the continued fraction by using Ramanujans class invariants.

In this paper, we derive a new special case $d(q)$ of the general continued fraction (1.2), which is defined by

$$d(q) := \cfrac{1}{q^{1/8}+\cfrac{q^{1/8}}{1+\cfrac{2q}{1+\cfrac{q^2}{1+\cfrac{q^2+q^3}{1+\cfrac{q^4}{1+\cdots,}}}}}}, \quad |q|<1. \tag{1.16}$$

and prove some results analogous to those of $R(q)$. In Sect. 2 we record some preliminary results which will be used in the subsequent sections. In Sect. 3 we establish several new modular identities of the continued fraction of $d(q)$ and in Sect. 4 we establish some explicit values of $d(q)$ by using Ramanujans class invariants.

2. Preliminary results

Lemma 2.1. [2, Entry 17.3.1, p.385].*If β is of degree 2 over α, then*

$$(1-\sqrt{1-\alpha})(1-\sqrt{\beta}) = 2\sqrt{\beta(1-\alpha)}. \tag{2.1}$$

Lemma 2.2. [3, Entry 5(xiii), p.231] *If $P = (\alpha\beta)^{1/8}$ and $Q = (\beta/\alpha)^{1/4}$ and β has degree 3 over α, then*

$$Q - \frac{1}{Q} = 2\left(P - \frac{1}{P}\right). \tag{2.2}$$

Lemma 2.3. [2, Entry 17.3.2, p.385] *If β has degree 4 over α, then*

$$(1-\sqrt[4]{1-\alpha})(1-\sqrt[4]{\beta}) = 2\sqrt[4]{\beta(1-\alpha)}. \tag{2.3}$$

Lemma 2.4. [3, Entry 13(xv), p.282] *If $P=(\alpha\beta)^{1/4}$ and $Q=(\beta/\alpha)^{1/8}$ and β has degree 5 over α, then*

$$\left(Q-\frac{1}{Q}\right)^3 + 8\left(Q-\frac{1}{Q}\right) = 4\left(P-\frac{1}{P}\right). \tag{2.4}$$

Lemma 2.5. [3, Entry 19(xv), p.315] *If $P=(\alpha\beta)^{1/2}$ and $Q=(\beta/\alpha)^{1/2}$ and β has degree 7 over α, then*

$$\left(P+\frac{1}{P}\right) = \left(Q+\frac{1}{Q}\right) + \left(P^{1/8}-P^{-1/8}\right)^8. \tag{2.5}$$

Lemma 2.6. [5, Entry 7, p.363].*If β is of degree 11 over α, then*

$$(\alpha\beta)^{1/4} + \{(1-\alpha)(1-\beta)\}^{1/4} + 2\{16\alpha\beta(1-\alpha)(1-\beta)\}^{1/12} = 1. \tag{2.6}$$

Lemma 2.7. [6, 7, Baily's and Daum formula].*If $|q|<1$, $|\frac{q}{b}|<1$ and $|q|<min\{1,|b|\}$, then*

$$\sum_{n=0}^{\infty} \frac{(a;q)_n(b;q)_n}{(q;q)_n\left(\frac{aq}{b};q\right)_n}\left(\frac{-q}{b}\right)^n = \frac{(aq;q^2)_\infty(-q;q)_\infty\left(\frac{aq^2}{b^2};q^2\right)_\infty}{\left(\frac{aq}{b};q\right)_\infty\left(\frac{-q}{b};q\right)_\infty}. \tag{2.7}$$

Lemma 2.8. [3, Entry 12(vii), p.124] *We have*

$$\chi(-q^2) = 2^{1/3}(1-\alpha)^{1/24}\alpha^{1/12}q^{12} = (q^2;q^4)_\infty. \tag{2.8}$$

Lemma 2.9. [8, Theorem 3.5, p.107] *We have*

$$4G_n^8 g_{4n}^8 - 4G_n^{-8} g_{4n}^{16} - 4 = 0. \tag{2.9}$$

Lemma 2.10. [9, Theorem 3.5, p.107] *We have*

$$G_4^8 = \sqrt{2}\left(\frac{1+\sqrt{2}}{2}\right)^2. \tag{2.10}$$

3. General Theorems on $d(q)$

Proposition 3.1. *For $|q|<1$, We have*

$$d(q) := \frac{1}{2}\frac{G(1,1;0;q)}{G(1,q;0;q)}q^{-1/8} = \cfrac{1}{q^{1/8}+\cfrac{q^{1/8}}{1+\cfrac{2q}{1+\cfrac{q^2}{1+\cfrac{q^2+q^3}{1+\cfrac{q^4}{1+\cdots,}}}}}} \tag{3.1}$$

Proof. Putting $a=1$,$\lambda=1$ and $b=0$ in (1.14), we obtain

$$\frac{G(q,q;0;q)}{G(1,1;0;q)}=\cfrac{1}{1+\cfrac{2q}{1+\cfrac{q^2}{1+\cfrac{q^2+q^3}{1+\cfrac{q^4}{1+\cdots,}}}}}\quad |q|<1. \tag{3.2}$$

where by putting $a=q,\lambda=q,b=0$ and $a=1,\lambda=1,b=0$ in (1.15), we obtain

$$G(q,q;0;q):=\sum_{n=0}^{\infty}\frac{(-1;q)_nq^n}{(q;q)_n}q^{n(n+1)/2}, \tag{3.3}$$

and

$$G(1,1;0;q):=\sum_{n=0}^{\infty}\frac{(-1;q)_n}{(q;q)_n}q^{n(n+1)/2}. \tag{3.4}$$

respectively, and also putting $a=1,\lambda=q,b=0$ in (1.15), we get

$$G(1,q;0;q):=\sum_{n=0}^{\infty}\frac{(-q;q)_n}{(q;q)_n}q^{n(n+1)/2}. \tag{3.5}$$

Equation (1.2) can be written as

$$2(-q;q)_n=(-1;q)_nq^n+(-1;q)_n. \tag{3.6}$$

Multiplying (3.6) by $\dfrac{q^{\frac{n(n+1)}{2}}}{(q;q)_n}$, we get

$$2\frac{(-q;q)_n}{(q;q)_n}q^{\frac{n(n+1)}{2}}=\frac{(-1;q)_nq^n}{(q;q)_n}q^{\frac{n(n+1)}{2}}+\frac{(-1;q)_n}{(q;q)_n}q^{\frac{n(n+1)}{2}}. \tag{3.7}$$

Combining the above equations (3.3),(3.4),(3.5) and (3.7), we obtain

$$2G(1,q;0;q)=G(q,q;0;q)+G(1,1;0;q). \tag{3.8}$$

Equivalently

$$2\frac{G(1,q;0;q)}{G(1,1;0;q)}=1+\frac{G(q,q;0;q)}{G(1,1;0;q)}. \tag{3.9}$$

Employing (3.2) in (3.9), we obtain

$$2\frac{G(1,q;0;q)}{G(1,1;0;q)}=1+\cfrac{1}{1+\cfrac{2q}{1+\cfrac{q^2}{1+\cfrac{q^2+q^3}{1+\cfrac{q^4}{1+\cdots,}}}}}\quad |q|<1. \tag{3.10}$$

Taking the reciprocal of (3.10), we obtain

$$\frac{1}{2}\frac{G(1,1;0;q)}{G(1,q;0;q)} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{2q}{1+\cfrac{q^2}{1+\cfrac{q^2+q^3}{1+\cfrac{q^4}{1+\cdots,}}}}}}, \quad |q|<1. \tag{3.11}$$

Multiplying (3.11) by $q^{1/8}$, we arrive at the desired result. □

Lemma 3.1. *Let* $d(q)$,$\chi(q)$ *and* $\chi(-q^2)$ *are as defined in* (1.6), (1.11), (2.6) *and* (1.16) *respectively. Then, we have*

$$d(q) = \frac{\chi(q)\chi(-q^2)}{2q^{1/8}} = \frac{1}{\sqrt{2}\alpha^{1/8}}. \tag{3.12}$$

Proof. Baily's formula (2.7) can also be written as

$$\sum_{n=0}^{\infty} \frac{(a;q)_n\left(\frac{1}{b}-1\right)\left(\frac{1}{b}-q\right)\cdots\left(\frac{1}{b}-q^{n-1}\right)}{(q;q)_n\left(\frac{aq}{b};q\right)_n}(-q)^n = \frac{(aq;q^2)_\infty(-q;q)_\infty\left(\frac{aq^2}{b^2};q^2\right)_\infty}{\left(\frac{aq}{b};q\right)_\infty\left(\frac{-q}{b};q\right)_\infty}. \tag{3.13}$$

Letting b tends to ∞ in the above equation (3.13), we obtain

$$\sum_{n=0}^{\infty}\frac{(a;q)_n}{(q;q)_n}q^{n(n+1)/2} = (aq;q^2)_\infty(-q;q)_\infty. \tag{3.14}$$

Putting $a=-q$ in (3.14) and employing (3.5), we get

$$G(1,q;0;q) := \sum_{n=0}^{\infty}\frac{(-q;q)_n}{(q;q)_n}q^{n(n+1)/2} = (-q^2;q^2)_\infty(-q;q)_\infty. \tag{3.15}$$

Again, putting $a=-1$ in (3.14) and employing (3.4), we get

$$G(1,1;0;q) := \sum_{n=0}^{\infty}\frac{(-1;q)_n}{(q;q)_n}q^{n(n+1)/2} = (-q;q^2)_\infty(-q;q)_\infty. \tag{3.16}$$

Employing (3.15) and(3.15) in Proposition (3.17) and simplifying , we obtain

$$d(q) := \frac{1}{2}\frac{G(1,1;0;q)}{G(1,q;0;q)}q^{-1/8} = \frac{1}{2}\frac{(-q;q^2)_\infty}{(-q^2;q^2)_\infty}q^{-1/8} = \frac{1}{2}(-q;q^2)_\infty(q^2;q^4)_\infty q^{-1/8}. \tag{3.17}$$

Using (1.11) and (2.6) in (3.17), we obtain required result (3.12). □

Theorem 3.1. *If* $x=d(q)$ *and* $y=d(q^2)$, *then*

$$y^2+\frac{1}{4y^2} = 4x^4. \tag{3.18}$$

Proof. Employing the Lemma's 2.1 and 3.1 we get

$$(-4y^4-1+16y^2x^4)(4y^4+1+16y^2x^4)=0. \tag{3.19}$$

By examining the behavior of the above factors near $q=0$, we can find a neighborhood about the origin, where the first factor is zero; whereas other factors are not zero in this neighborhood. By the Identity Theorem second factor vanishes identically. This completes the proof. □

Theorem 3.2. *If $x=d(q)$ and $y=d(q^3)$, then*

$$\frac{x^2}{y^2}-\frac{y^2}{x^2}=\frac{1}{xy}-4xy. \tag{3.20}$$

Proof. Using the Lemma's 2.2 and 3.1, we obtain required result (3.20). □

Theorem 3.3. *If $x=d(q)$ and $y=d(q^4)$, then*

$$\left(4y^4+\frac{1}{4y^4}\right)+6=2(32x^8-1)\left(4y^2+\frac{1}{y^2}\right). \tag{3.21}$$

Proof. Employing the Lemma's 2.3 and 3.1, we obtain required result (3.21). □

Theorem 3.4. *If $x=d(q)$ and $y=d(q^5)$, then*

$$\left(\frac{x^3}{y^3}-\frac{y^3}{x^3}\right)+5\left(\frac{x}{y}-\frac{y}{x}\right)=\left(\frac{1}{x^2y^2}-16x^2y^2\right). \tag{3.22}$$

Proof. Using the Lemma's 2.4 and 3.1,, we obtain required result (3.22). □

Theorem 3.5. *If $x=d(q)$ and $y=d(q^7)$, then*

$$\left(\frac{x^4}{y^4}+\frac{y^4}{x^4}\right)+70=\left(\frac{1}{(xy)^3}+(4xy)^3\right)-7\left(\frac{1}{(xy)^2}+(4xy)^2\right)+28\left(\frac{1}{xy}+4xy\right). \tag{3.23}$$

Proof. Employing the Lemma's 2.5 and 3.1,, we obtain

$$\begin{aligned}&(28x^3y^3+7x^2y^2+xy+64x^7y^7+112x^5y^5+70x^4y^4+y^8+x^8+112x^6y^6)\\&(-28x^3y^3+7x^2y^2-xy-64x^7y^7-112x^5y^5+70x^4y^4+y^8+x^8+112x^6y^6)\\&(x^{16}-224x^{14}y^6+4096y^{14}x^{14}-1792x^{12}y^{12}+140x^{12}y^4-14x^{10}y^2+448y^{10}x^{10}\\&+70x^8y^8-224y^{14}x^6+28x^6y^6-7x^4y^4+140y^{12}x^4-14y^{10}x^2+x^2y^2+y^{16})=0.\end{aligned} \tag{3.24}$$

By examining the behavior of the above factors near $q=0$, we can find a neighborhood about the origin, where the second factor is zero; whereas other factors are not zero in this neighborhood. By the Identity Theorem second factor vanishes identically. This completes the proof. □

Theorem 3.6. *If $x=d(q)$ and $y=d(q^{11})$ then*

$$\begin{aligned}&\left(\frac{x^6}{y^6}-\frac{y^6}{x^6}\right)+165\left(\frac{x^2}{y^2}-\frac{y^2}{x^2}\right)-44\left(\frac{x^5}{y^3}+\frac{y^5}{x^3}\right)+11\left(\frac{x^3}{y^5}+\frac{y^3}{x^5}\right)\\&=11\left(\frac{1}{x^4}-16x^4\right)-11\left(\frac{1}{y^4}-16y^4\right)+11\left(\frac{1}{x^3y^3}-64x^3y^3\right)+66\left(\frac{1}{xy}-4xy\right).\end{aligned} \tag{3.25}$$

Proof. Using the Using the Lemma's 2.6 and 3.1,, we obtain, we obtain

$$\begin{aligned}
&(-3801088x^{20}y^{20} - 29380x^{12}y^{12} + x^2y^2 - 15054x^8y^8 - 3853824x^{16}y^{16}\\
&+ 241359x^{16}y^8 + 241359x^8y^{16} - 58x^4y^4 + 1595x^6y^6 + 25822x^{10}y^{10}\\
&+ 413152x^{14}y^{14} + 6533120x^{18}y^{18} + 1048576x^{22}y^{22} - 90112x^{22}y^{14}\\
&- 90112x^{14}y^{22} - 410688x^{18}y^{10} - 410688x^{10}y^{18} + 242432x^{20}y^{12}\\
&+ 242432x^{12}y^{20} + x^{24} + y^{24} - 25668x^{14}y^6 - 25668x^6y^{14} + 93x^{18}y^2\\
&- 22x^{10}y^2 + 93x^2y^{18} - 22x^2y^{10} - 238x^{20}y^4 + 947x^{12}y^4 - 238x^4y^{20}\\
&+ 947x^4y^{12} + 1488x^{22}y^6 + 1488x^6y^{22})(11xy^9 - 44x^3y^{11} + 704x^9y^9\\
&+ 1024x^{11}y^{11} + 264x^7y^7 - 44x^{11}y^3 - 11y^6x^2 - xy + 11x^9y - y^{12}\\
&+ 176y^6x^{10} - 11x^3y^3 - 165y^8x^4 - 176y^{10}x^6 + 11x^6y^2 - 66x^5y^5\\
&+ x^{12} + 165y^4x^8)(165y^4x^8 - 176y^{10}x^6 - 704x^9y^9 - 1024x^{11}y^{11}\\
&- 264x^7y^7 + 44x^{11}y^3 - 11y^6x^2 - 11x^9y + 176y^6x^{10} - y^{12} + 11x^3y^3\\
&- 165y^8x^4 + x^{12} + xy + 11x^6y^2 + 66x^5y^5 - 11xy^9 + 44x^3y^{11}) = 0
\end{aligned} \tag{3.26}$$

By examining the behavior of the above factors near $q = 0$, we can find a neighborhood about the origin, where the second factor is zero; whereas other factors are not zero in this neighborhood. By the Identity Theorem second factor vanishes identically. This completes the proof. □

4. Explicit Evaluations of $d(q)$

Theorem 4.1. *For any positive real number n, we have*

$$d\left(e^{-\pi\sqrt{n}}\right) = 2^{-1/2}G_n g_{4n}. \tag{4.1}$$

Proof. Setting $q = e^{-\pi\sqrt{n}}$ in Lemma 3.1 and employing the definition of g_n,G_n from (1.10), we obtain required result (4.1). □

Corollary 4.1.

$$d\left(e^{-\pi}\right) = 2^{-3/8}. \tag{4.2}$$

Proof. Setting $n = 1$ in above Theorem 4.1 we obtain,

$$d\left(e^{-\pi}\right) = 2^{-1/2}G_1 g_4. \tag{4.3}$$

Several values of G_n and g_n are listed in [5], [11], and [13]. For example, from [5, p. 189,200], we note that $G_1 = 1$. Employing Lemma 2.9 with $n = 1$, we can find $g_4 = 2^{1/8}$. Substituting the values of G_1 and g_4 in (4.3), we obtain required result. □

Theorem 4.2.

$$G_{4n}^2 g_{16n}^2 + \frac{1}{G_{4n}^2 g_{16n}^2} = 2G_n^4 g_{4n}^4. \tag{4.4}$$

Proof. Employing the theorem (3.1) and (4.1) we get required result. □

Corollary 4.2.

$$(i) g_{16} = \left(\frac{24+17\sqrt{2}}{16\sqrt{2}}\right)^{1/8}, \tag{4.5}$$

$$(ii) g_{1/4} = \left(\frac{24+17\sqrt{2}}{16\sqrt{2}}\right)^{-1/8}. \tag{4.6}$$

Proof. Putting $n = 1$ in the above Theorem 4.2, we obtain

$$G_4^2 g_{16}^2 + \frac{1}{G_4^2 g_{16}^2} = 2G_1^4 g_4^4. \tag{4.7}$$

We know that the values of [5, p. 189] G_1, G_4 from the Lemma 2.10 and already found that the g_4 in the above corollary. Substituting these values in the above equation (4.7), we get g_{16}. We know that $g_{2n}g_{2/n} = 1$ with $n = 8$ from [13], we obtain $g_{1/4}$. □

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