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FURTHER RESULTS ON MEAN CORDIAL LABELING FOR THREE STAR GRAPH

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ABSTRACT. A vertex labeling $h : V(G) \longrightarrow \{0,1,2\}$ is said to be a mean cordial labeling of G if it induces an edge labeling h^* given by $\left\lceil \frac{h(r)+h(s)}{2} \right\rceil$ such that $|v_h(b) - v_h(p)| \leq 1$ and $|e_h(b) - e_h(p)| \leq 1$, $b, p \in \{0,1,2\}$, where $v_h(r)$ and $e_h(r)$ denote the number of vertices and edges respectively labeled with $r\,(r = 0,1,2)$. A graph G is said to be a mean cordial graph if it admits a mean cordial labeling. In this paper we proved that three star graph is a mean cordial labeling. If $r = s < t$, the three star graph $K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ is a mean cordial graph if and only if $|s-t| \leq 3r+6$ for $3r+s-7 \leq t \leq 3r+s+6$; $r = 1,2,3,\cdots$. and $s = 1,2,3,\cdots$.

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KEYWORDS AND PHRASES. Wedge, Three star graph and Mean cordial labeling.

1. Introduction

All graphs $G = (V(G), E(G))$ considered here are simple, connected, finite, undirected with no loops and multiple edges [2]. In [4] Raja Ponraj et.al., introduced mean cordial labeling and they investigated the behavior for some graphs. R. Ponraj and S. Sathish narayanan further investigated mean cordial labeling behavior of prism, $K_2 + K_m$, $K_n + 2K_2$, book B_m and some snake graphs [5]. Moreover, Albert William et al., [1] studied mean cordial labeling behavior of several graphs like banana tree, caterpillar, subdivision of ladder, $S(B_n, n)$. In [6] and [7], Balaji et al proved that the two star is a mean cordial graph if and only if $|2g - h| \leq 4$ for $g \leq h$ and also they have proved that three star is a mean cordial graph if and only if $|\beta_2 - \beta_3| \leq 3\beta_1 - 7$ for $3\beta_1 + \beta_2 - 7 \leq \beta_3 \leq 3\beta_1 + \beta_2 + 7$ if $\beta_1 < \beta_2 < \beta_3$. Then the symbol $\lfloor x \rfloor$ for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. A general reference for ideas of graph theory can be seen from [3].

2. Preliminaries

Definition 2.1. Wedge
When a disconnected graph's are connected by an edge in order to form a single connected graph is known as wedge. It is denoted by the symbol $\wedge$, $\omega(G\wedge) < \omega(G)$.

Definition 2.2. Mean cordial graph
A vertex labeling $h : V(G) \longrightarrow \{0,1,2\}$ is said to be a mean cordial labeling of G if it induces an edge labeling h^* given by $\left\lceil \frac{h(r)+h(s)}{2} \right\rceil$ such that $|v_h(b) - v_h(p)| \leq 1$ and $|e_h(b) - e_h(p)| \leq 1$, $b, p \in \{0,1,2\}$, where $v_h(r)$ and $e_h(r)$ denote the number

Dedicated to Prof. Chandrashekar Adiga on his 62nd Birthday.

of vertices and edges respectively labeled with $r\,(r = 0, 1, 2)$. A graph G is said to be a mean cordial graph if it admits a mean cordial labeling.

Note: In this paper, we have been specifically discussing about the star graph with wedge.

Definition 2.3. Star Graph
A graph of form $K_{1,n}$ is said to be star graph.

Definition 2.4. Two Star Graph
Two star graph is a graph obtained by joining the wedge to two copies of star $K_{1,n}$ by an edge.

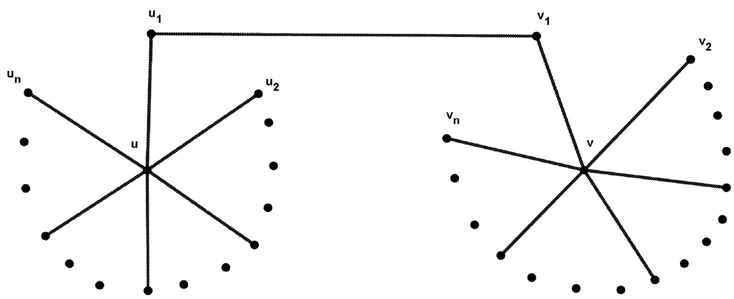

Figure 1. General representation of two star graph

Definition 2.5. Three Star Graph
Three star graph is a graph obtained by joining the wedge to three copies of star $K_{1,n}$ by an edge.

3. Results

Lemma 3.1. [7] *The two star $K_{1,g} \wedge K_{1,h}$ is mean cordial graph if and only if $|2g - h| \leq 4$ for $g \leq h$ and $g = 1, 2, 3, .$*

Lemma 3.2. [6] *If $\beta_1 < \beta_2 < \beta_3$, the three star graph $K_{1,\beta_1} \wedge K_{1,\beta_2} \wedge K_{1,\beta_3}$ is a mean cordial graph if and only if $|\beta_2 - \beta_3| \leq 3\beta_1 + 7$ for $3\beta_1 + \beta_2 - 7 \leq \beta_3 \leq 3\beta_1 + \beta_2 + 7$; $\beta_1 = 1, 2, ...$ and $\beta_2 = 2, 3, ...$*

Theorem 3.3. *If $r = s < t$, the three star graph $K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ is a mean cordial graph if and only if $|s - t| \leq 3r + 6$ for $3r + s - 7 \leq t \leq 3r + s + 6$; $r = 1, 2, 3, \cdots$. and $s = 1, 2, 3, \cdots$.*

Proof. Let the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$.
Let $V(G)$ be the node set of G and $E(G)$ be the link set of G. Then G is given by,

$$V(G) = \{u, q, w\} \cup \{u_\psi : 1 \leq \psi \leq r\} \cup \{q_\psi : 1 \leq \psi \leq s\} \cup \{w_\psi : 1 \leq \psi \leq t\}$$

$$\begin{aligned} E(G) = &\{uu_\psi : 1 \leq \psi \leq r\} \cup \{qq_\psi : 1 \leq \psi \leq s\} \cup \{ww_\psi : 1 \leq \psi \leq t\} \\ &\cup \{u_\psi q_\psi \text{ for any } \psi, u_\psi : 0 \leq \psi \leq r \text{ and } q_\psi : 0 \leq \psi \leq s\} \\ &\cup \{q_\psi w_\psi \text{ for any } \psi, q_\psi : 0 \leq \psi \leq s \text{ and } w_\psi : 0 \leq \psi \leq t\}. \end{aligned}$$

Then G has $r+s+t+3$ nodes and $r+s+t+2$ links.
To prove that G is a mean cordial graph for all $r=s<t$; $h:V(G)\rightarrow\{0,1,2\}$ and $h^*:E(G)\rightarrow\{0,1,2\}$.
Then, we shall consider the following cases:
Case (i): $t=3r+s+6$
Consider the graph $G=K_{1,r}\wedge K_{1,s}\wedge K_{1,t}$ where $r=s<t$.
Defining the vertex labeling as follows:
$h(u)=0$; $h(q)=0$; $h(w)=1$
$h(u_\psi)=0$ for $1\leq\psi\leq r$
$h(q_\psi)=0$ for $1\leq\psi\leq s$
$h(w_{2\psi-1})=1$ for $1\leq\psi\leq\frac{t}{2}-1$
$h(w_{2\psi})=2$ for $1\leq\psi\leq\frac{t}{2}$
$h(w_{n\psi})=0$.
Then, we obtain the link label as follows:
$uu_\psi=0$ for $1\leq\psi\leq r$; $qq_\psi=0$ for $1\leq\psi\leq s$; $ww_{2\psi-1}=1$ for $1\leq\psi\leq\frac{t}{2}-1$; $ww_{2\psi}=2$ for $1\leq\psi\leq\frac{t}{2}$; $ww_{n\psi}=1$.
The wedge labeling of $u_\psi q_\psi$ is 0 for any ψ.
The wedge labeling of $q_1w_{n\psi}$ is 0.

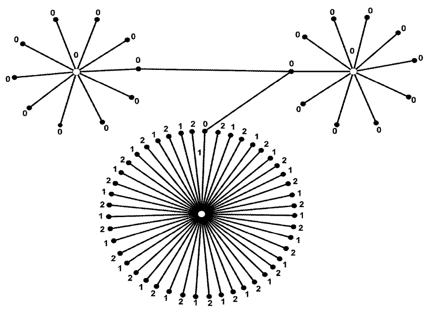

Figure 2. $t=3r+s+6$ *when* $r=10,s=10,$ *then* $t=46$

Case (ii): $t=3r+s+5$
Consider the graph $G=K_{1,r}\wedge K_{1,s}\wedge K_{1,t}$ where $r=s<t$.
Defining the vertex labeling as follows:
$h(u)=0$; $h(q)=0$; $h(w)=1$
$h(u_\psi)=0$ for $1\leq\psi\leq r$
$h(q_\psi)=0$ for $1\leq\psi\leq s$
$h(w_{2\psi-1})=1$ for $1\leq\psi\leq\left\lfloor\frac{t}{2}\right\rfloor$
$h(w_{2\psi})=2$ for $1\leq\psi\leq\left\lfloor\frac{t}{2}\right\rfloor$
$h(w_{n\psi})=0$.
Then, we obtain the link label as follows:
$uu_\psi=0$ for $1\leq\psi\leq r$; $qq_\psi=0$ for $1\leq\psi\leq s$; $ww_{2\psi-1}=1$ for $1\leq\psi\leq\left\lfloor\frac{t}{2}\right\rfloor$; $ww_{2\psi}=2$ for $1\leq\psi\leq\left\lfloor\frac{t}{2}\right\rfloor$; $ww_{n\psi}=1$.
The wedge labeling of $u_\psi q_\psi$ is 0 for any ψ.
The wedge labeling of $w_{n\psi}q_1$ is 0.

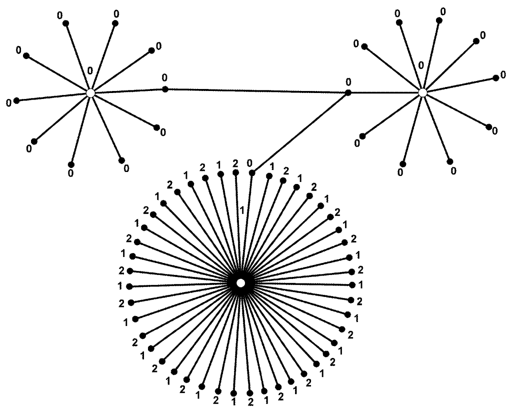

Figure 3. $t = 3r + s + 5$ *when* $r = 10, s = 10,$ *then* $t = 45$

Case (iii): $t = 3r + s + 4$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 0$; $h(q) = 0$; $h(w) = 1$
$h(u_\psi) = 0$ for $1 \le \psi \le r$
$h(q_\psi) = 0$ for $1 \le \psi \le s$
$h(w_{2\psi-1}) = 1$ for $1 \le \psi \le \frac{t}{2} - 1$
$h(w_{2\psi}) = 2$ for $1 \le \psi \le \frac{t}{2}$
$h(w_{n\psi}) = 0$.
Then, we obtain the link label as follows:
$uu_\psi = 0$ for $1 \le \psi \le r$; $qq_\psi = 0$ for $1 \le \psi \le s$; $ww_{2\psi-1} = 1$ for $1 \le \psi \le \frac{t}{2} - 1$; $ww_{2\psi} = 2$ for $1 \le \psi \le \frac{t}{2}$; $ww_{n\psi} = 1$.
The wedge labeling of $u_\psi q_\psi$ is 0 for any ψ.
The wedge labeling of $q_1 w_{n\psi}$ is 0.
For Example: Refer Figure 4.
Case (iv): $t = 3r + s + 3$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \le \psi \le r$
$h(q_\psi) = 2$ for $1 \le \psi \le s$
$h(w_{2\psi-1}) = 1$ for $1 \le \psi \le \left\lceil \frac{t}{2} \right\rceil$
$h(w_{2\psi}) = 0$ for $1 \le \psi \le \left\lfloor \frac{t}{2} \right\rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \le \psi \le r$; $qq_\psi = 2$ for $1 \le \psi \le s$; $ww_{2\psi-1} = 1$ for $1 \le \psi \le \left\lceil \frac{t}{2} \right\rceil$; $ww_{2\psi} = 0$ for $1 \le \psi \le \left\lfloor \frac{t}{2} \right\rfloor$.
The wedge labeling of $u_\psi q_\psi$ is 2 for any ψ.
The wedge labeling of $q_1 w_1$ is 2.
Case (v): $t = 3r + s + 2$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.

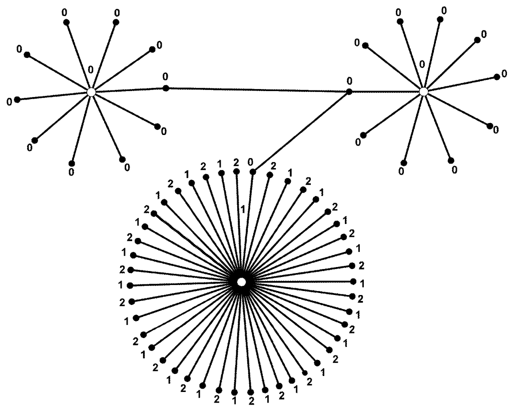

FIGURE 4. $t = 3r + s + 4$ *when* $r = 10, s = 10,$ *then* $t = 44$

Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \leq \psi \leq r$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s$
$h(w_{2\psi-1}) = 1$ for $1 \leq \psi \leq \frac{t}{2}$
$h(w_{2\psi}) = 0$ for $1 \leq \psi \leq \frac{t}{2}$
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \leq \psi \leq r$; $qq_\psi = 2$ for $1 \leq \psi \leq s$; $ww_{2\psi-1} = 1$ for $1 \leq \psi \leq \frac{t}{2}$; $ww_{2\psi} = 0$ for $1 \leq \psi \leq \frac{t}{2}$.
The wedge labeling of $u_\psi q_\psi$ is 2 for any ψ.
The wedge labeling of $q_1 w_1$ is 2.
Case (vi): $t = 3r + s + 1$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \leq \psi \leq r$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s - 1$
$h(q_{n\psi}) = 1$
$h(w_{2\psi-1}) = 0$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \leq \psi \leq r$; $qq_\psi = 2$ for $1 \leq \psi \leq s - 1$; $qq_{n\psi} = 2$; $ww_{2\psi-1} = 0$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$.
The wedge labeling of $u_\psi q_\psi$ is 2 for any ψ.
The wedge labeling of $q_{n\psi} w_2$ is 1.
Case (vii): $t = 3r + s$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \leq \psi \leq r$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s - 1$

$h(q_{n\psi}) = 1$
$h(w_{2\psi-1}) = 1$ for $1 \leq \psi \leq \frac{t}{2}$
$h(w_{2\psi}) = 0$ for $1 \leq \psi \leq \frac{t}{2}$
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \leq \psi \leq r$; $qq_\psi = 2$ for $1 \leq \psi \leq s-1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 1$ for $1 \leq \psi \leq \frac{t}{2}$; $ww_{2\psi} = 0$ for $1 \leq \psi \leq \frac{t}{2}$.
The wedge labeling of $u_\psi q_\psi$ is 2 for any ψ.
The wedge labeling of $q_{n\psi} w_1$ is 1.
Case (viii): $t = 3r + s - 1$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 0$; $h(q) = 0$; $h(w) = 1$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s-1$
$h(q_{n\psi}) = 1$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 0$ for $1 \leq \psi \leq r$; $qq_\psi = 0$ for $1 \leq \psi \leq s-1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$.
The wedge labeling of $u_\psi q_1$ is 0.
The wedge labeling of $q_{n\psi} w_2$ is 1.
Case (ix): $t = 3r + s - 2$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \leq \psi \leq r-1$
$h(u_{n\psi}) = 1$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s-1$
$h(q_{n\psi}) = 0$
$h(w_{2\psi-1}) = 1$ for $1 \leq \psi \leq \frac{t}{2}$
$h(w_{2\psi}) = 0$ for $1 \leq \psi \leq \frac{t}{2}$
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \leq \psi \leq r-1$; $uu_{n\psi} = 1$; $qq_\psi = 2$ for $1 \leq \psi \leq s-1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 1$ for $1 \leq \psi \leq \frac{t}{2}$; $ww_{2\psi} = 0$ for $1 \leq \psi \leq \frac{t}{2}$.
The wedge labeling of $u_1 q_1$ is 2.
The wedge labeling of $q_{n\psi} w_2$ is 0.
Case (x): $t = 3r + s - 3$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 0$; $h(q) = 0$; $h(w) = 1$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r-1$
$h(u_{n\psi}) = 1$
$h(q_\psi) = 0$ for $1 \leq \psi \leq s-1$
$h(q_{n\psi}) = 2$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$

Then, we obtain the link label as follows:
$uu_\psi = 0$ for $1 \leq \psi \leq r-1$; $uu_{n\psi} = 1$; $qq_\psi = 0$ for $1 \leq \psi \leq s-1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
The wedge labeling of u_1q_1 is 0.
The wedge labeling of $q_{n\psi}w_2$ is 2.
Case (xi): $t = 3r + s - 4$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \leq \psi \leq r-1$
$h(u_{n\psi}) = 1$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s-1$
$h(q_{n\psi}) = 0$
$h(w_{2\psi-1}) = 1$ for $1 \leq \psi \leq \frac{t}{2}$
$h(w_{2\psi}) = 0$ for $1 \leq \psi \leq \frac{t}{2}$
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \leq \psi \leq r-1$; $uu_{n\psi} = 2$; $qq_\psi = 2$ for $1 \leq \psi \leq s-1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 1$ for $1 \leq \psi \leq \frac{t}{2}$; $ww_{2\psi} = 0$ for $1 \leq \psi \leq \frac{t}{2}$.
The wedge labeling of u_1q_1 is 2.
The wedge labeling of $q_{n\psi}w_2$ is 0.
Case (xii): $t = 3r + s - 5$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 0$; $h(q) = 0$; $h(w) = 1$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r-1$
$h(u_{n\psi}) = 2$
$h(q_\psi) = 0$ for $1 \leq \psi \leq s-1$
$h(q_{n\psi}) = 1$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 0$ for $1 \leq \psi \leq r-1$; $uu_{n\psi} = 1$; $qq_\psi = 0$ for $1 \leq \psi \leq s-1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
The wedge labeling of u_1q_1 is 0.
The wedge labeling of $q_{n\psi}w_1$ is 2.
Case (xiii): $t = 3r + s - 6$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 2$; $h(q) = 2$; $h(w) = 0$
$h(u_\psi) = 2$ for $1 \leq \psi \leq r-1$
$h(u_{n\psi}) = 1$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s-2$
$h(q_{n\psi-1}) = 1$
$h(q_{n\psi}) = 0$
$h(w_{2\psi-1}) = 1$ for $1 \leq \psi \leq \frac{t}{2}$
$h(w_{2\psi}) = 0$ for $1 \leq \psi \leq \frac{t}{2}$
Then, we obtain the link label as follows:

$uu_\psi = 2$ for $1 \leq \psi \leq r-1$; $uu_{n\psi} = 2$; $qq_\psi = 2$ for $1 \leq \psi \leq s-2$; $qq_{n\psi-1} = 2$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 1$ for $1 \leq \psi \leq \frac{t}{2}$; $ww_{2\psi} = 0$ for $1 \leq \psi \leq \frac{t}{2}$.
The wedge labeling of $u_{n\psi}q_{n\psi}$ is 1.
The wedge labeling of $q_{n\psi}w_2$ is 0.
Case (xiv): $t = 3r + s - 7$
Consider the graph $G = K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ where $r = s < t$.
Defining the vertex labeling as follows:
$h(u) = 0$; $h(q) = 0$; $h(w) = 1$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r-1$
$h(u_{n\psi}) = 1$
$h(q_\psi) = 0$ for $1 \leq \psi \leq s-2$
$h(q_{n\psi-1}) = 1$
$h(q_{n\psi}) = 2$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 0$ for $1 \leq \psi \leq r-1$; $uu_{n\psi} = 1$; $qq_\psi = 0$ for $1 \leq \psi \leq s-2$; $qq_{n\psi-1} = 1$; $qq_{n\psi} = 1$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$.
The wedge labeling of u_1q_1 is 0.
The wedge labeling of $q_{n\psi}w_2$ is 2.

Nature of t	$v_f(0)$	$v_f(1)$	$v_f(2)$	$e_f(0)$	$e_f(1)$	$e_f(2)$
$t = 3r+s+6$	$r+s+3$	$r+s+3$	$r+s+3$	$r+s+2$	$r+s+3$	$r+s+3$
$t = 3r+s+5$	$r+s+3$	$r+s+3$	$r+s+2$	$r+s+2$	$r+s+3$	$r+s+2$
$t = 3r+s+4$	$r+s+3$	$r+s+2$	$r+s+2$	$r+s+2$	$r+s+2$	$r+s+2$
$t = 3r+s+3$	$r+s+2$	$r+s+2$	$r+s+2$	$r+s+1$	$r+s+2$	$r+s+2$
$t = 3r+s+2$	$r+s+2$	$r+s+1$	$r+s+2$	$r+s+1$	$r+s+1$	$r+s+2$
$t = 3r+s+1$	$r+s+2$	$r+s+1$	$r+s+1$	$r+s+1$	$r+s+1$	$r+s+1$
$t = 3r+s+0$	$r+s+1$	$r+s+1$	$r+s+1$	$r+s$	$r+s+1$	$r+s+1$
$t = 3r+s-1$	$r+s+1$	$r+s+1$	$r+s$	$r+s$	$r+s+1$	$r+s$
$t = 3r+s-2$	$r+s+1$	$r+s$	$r+s$	$r+s$	$r+s$	$r+s$
$t = 3r+s-3$	$r+s$	$r+s$	$r+s$	$r+s-1$	$r+s$	$r+s$
$t = 3r+s-4$	$r+s$	$r+s-1$	$r+s$	$r+s-1$	$r+s-1$	$r+s$
$t = 3r+s-5$	$r+s$	$r+s-1$	$r+s-1$	$r+s-1$	$r+s-1$	$r+s-1$
$t = 3r+s-6$	$r+s-1$	$r+s-1$	$r+s-1$	$r+s-2$	$r+s-1$	$r+s-1$
$t = 3r+s-7$	$r+s-1$	$r+s-1$	$r+s-2$	$r+s-2$	$r+s-1$	$r+s-2$

Hence, G is mean cordial graph if $|s-t| \leq 3r+6$ for $3r+s+6 \leq t \leq 3r+s-7$.
Conversely, we have to prove that $|s-t| > 3r+6$ is not a mean cordial graph.
Let us take the first graph $K_{1,1} \wedge K_{1,1} \wedge K_{1,11}$.
Then, G has 16 nodes and 15 links.
Case (i): Let us fix the node label as given below:
$h(u) = 0$; $h(q) = 0$ and $h(w) = 0$
Then, $h(u_\psi) = 2$ for $1 \leq \psi \leq r$
$h(q_\psi) = 2$ for $1 \leq \psi \leq s$
$h(w_{2\psi-1}) = 0$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 1$ for $1 \leq \psi \leq r$; $qq_\psi = 1$ for $1 \leq \psi \leq s$; $ww_{2\psi-1} = 0$ for $1 \leq \psi \leq \left\lceil \frac{t}{2} \right\rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \left\lfloor \frac{t}{2} \right\rfloor$.
The wedge labeling of $u_\psi q_\psi = 2$ for any ψ.
The wedge labeling of $q_\psi w_2 = 2$ for any ψ.
Hence, we get $|v_h(b) - v_h(p)| > 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$, which is a contradiction.

Case (ii): Let us fix the node label as given below:
$h(u) = 1$; $h(q) = 1$ and $h(w) = 1$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r$
$h(q_\psi) = 0$ for $1 \leq \psi \leq s$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$
Then, we obtain the link label as follows:
$uu_\psi = 0$ for $1 \leq \psi \leq r$; $qq_\psi = 0$ for $1 \leq \psi \leq s$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
The wedge labeling of $u_\psi q_\psi = 0$ for any ψ.
The wedge labeling of $q_\psi w_1 = 1$ for any ψ.
Hence, we get $|v_h(b) - v_h(p)| > 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$.
which is a contradiction.
Case (iii): Let us fix the node label as given below:
$h(u) = 2$; $h(q) = 2$ and $h(w) = 2$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r$
$h(q_\psi) = 0$ for $1 \leq \psi \leq s$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
Then, we obtain the link label as follows:
$uu_\psi = 1$ for $1 \leq \psi \leq r$; $qq_\psi = 1$ for $1 \leq \psi \leq s$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$; $ww_{2\psi} = 2$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
The wedge labeling of $u_\psi q_\psi = 0$ for any ψ.
The wedge labeling of $q_\psi w_1 = 1$ for any ψ.
Hence, we get $|v_h(b) - v_h(p)| > 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$,
which is a contradiction.
Case (iv): Let us fix the node label as given below:
$h(u) = 0$; $h(q) = 1$ and $h(w) = 2$
$h(u_\psi) = 1$ for $1 \leq \psi \leq r$
$h(q_\psi) = 1$ for $1 \leq \psi \leq s$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$
$h(w_{2\psi}) = 0$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
Then, we obtain the link label as follows:
$uu_\psi = 1$ for $1 \leq \psi \leq r$; $qq_\psi = 1$ for $1 \leq \psi \leq s$; $ww_{2\psi-1} = 2$ for $1 \leq \psi \leq \lceil \frac{t}{2} \rceil$; $ww_{2\psi} = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$.
The wedge labeling of $u_\psi q_\psi = 1$ for any ψ.
The wedge labeling of $q_\psi w_1 = 2$ for any ψ.
Hence, we get $|v_h(b) - v_h(p)| > 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$,
which is a contradiction.
Case (v): Let us fix the node label as given below:
$h(u) = 0$; $h(q) = 0$ and $h(w) = 2$
$h(u_\psi) = 0$ for $1 \leq \psi \leq r$
$h(q_\psi) = 0$ for $1 \leq \psi \leq s$
$h(w_{2\psi-1}) = 2$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$
$h(w_{2\psi}) = 1$ for $1 \leq \psi \leq \lfloor \frac{t}{2} \rfloor$
$h(w_{n\psi}) = 0$.
Then, we obtain the link label as follows:

$uu_\psi = 0$ for $1 \le \psi \le r$; $qq_\psi = 0$ for $1 \le \psi \le s$; $ww_{2\psi-1} = 2$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{2\psi} = 2$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{n\psi} = 1$.
The wedge labeling of $u_\psi q_\psi = 0$ for any ψ.
The wedge labeling of $q_\psi w_{n\psi} = 0$.
Hence, we get $|v_h(b) - v_h(p)| \le 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$, which is a contradiction.
Case (vi): Let us fix the node label as given below:
$h(u) = 1$; $h(q) = 1$ and $h(w) = 0$
$h(u_\psi) = 1$ for $1 \le \psi \le r$
$h(q_\psi) = 1$ for $1 \le \psi \le s$
$h(w_{2\psi-1}) = 0$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$
$h(w_{2\psi}) = 2$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$
$h(w_{n\psi}) = 1$.
Then, we obtain the link label as follows:
$uu_\psi = 1$ for $1 \le \psi \le r$; $qq_\psi = 1$ for $1 \le \psi \le s$; $ww_{2\psi-1} = 0$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{2\psi} = 1$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{n\psi} = 1$.
The wedge labeling of $u_\psi q_\psi = 1$.
The wedge labeling of $q_\psi w_2 = 2$ for any ψ.
Hence, we get $|v_h(b) - v_h(p)| \le 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$, which is a contradiction.
Case (vii): Let us fix the node label as given below:
$h(u) = 2$; $h(q) = 2$ and $h(w) = 1$
$h(u_\psi) = 2$ for $1 \le \psi \le r$
$h(q_\psi) = 2$ for $1 \le \psi \le s$
$h(w_{2\psi-1}) = 1$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$
$h(w_{2\psi}) = 0$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$
$h(w_{n\psi}) = 2$.
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \le \psi \le r$; $qq_\psi = 2$ for $1 \le \psi \le s$; $ww_{2\psi-1} = 1$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{2\psi} = 1$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{n\psi} = 2$.
The wedge labeling of $u_\psi q_\psi = 2$ for any ψ.
The wedge labeling of $q_\psi w_{n\psi} = 2$ for any ψ.
Hence, we get $|v_h(b) - v_h(p)| \le 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$, which is a contradiction.
Case (viii): Let us fix the node label as given below:
$h(u) = 2$; $h(q) = 2$ and $h(w) = 0$
$h(u_\psi) = 2$ for $1 \le \psi \le r$
$h(q_\psi) = 2$ for $1 \le \psi \le s$
$h(w_{2\psi-1}) = 0$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$
$h(w_{2\psi}) = 1$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$
$h(w_{n\psi}) = 2$.
Then, we obtain the link label as follows:
$uu_\psi = 2$ for $1 \le \psi \le r$; $qq_\psi = 2$ for $1 \le \psi \le s$; $ww_{2\psi-1} = 0$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{2\psi} = 1$ for $1 \le \psi \le \lfloor \frac{t}{2} \rfloor$; $ww_{n\psi} = 1$.
The wedge labeling of $u_\psi q_\psi = 2$ for any ψ.
The wedge labeling of $q_\psi w_{n\psi} = 2$.
Hence, we get $|v_h(b) - v_h(p)| \le 1$ but $|e_h(b) - e_h(p)| > 1$ where $b, p \in \{0, 1, 2\}$,

which is a contradiction.
Since, we have checked in all possible ways by fixing different node labeling to satisfy the condition for mean cordial labeling. The graph fails to satisfy the condition when $t = 3r + s + 7$. So, we conclude that mean cordial graph does not exist from $t = 3r + s + 7$ and above.
We conclude,G is not a mean cordial graph if $|s - t| > 3r + 6$.
Therefore, the three star graph $K_{1,r} \wedge K_{1,s} \wedge K_{1,t}$ is a mean cordial graph iff $|s - t| \leq 3r + 6$ for $3r + s - 7 \leq t \leq 3r + s + 6$; $r = 1, 2, ...$ and $s = 2, 3, ...$ □

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