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HUB AND GLOBAL HUB NUMBERS OF A GRAPH

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ABSTRACT. A set H of vertices in a graph G is a hub set of G, if for any $u, v \in V(G) \setminus H$, there is a $uv-$path with all intermediate vertices in H. A hub set H is a global hub set of G if it is a hub set of both G and its complement $\overline{G}$. The minimum cardinality of a global hub set is called the global hub number $h_g(G)$ of G. In this paper we determine the global hub number of some standard graphs. Also upper and lower bounds for $h_g(G)$ are obtained. The relationship between $h_g(G)$ and other graph parameters is discussed.

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1. INTRODUCTION

Throughout this paper, all graphs $G = (V, E)$ are assumed to be simple, finite and connected, and G is called (p, q) graph, where $|V| = p$ and $|E| = q$. In general, the degree of a vertex v in a graph G denoted by $deg(v)$ is the number of edges of G incident with v [3]. Also $\delta(G), \Delta(G)$ denote the minimum, maximum degree among the vertices of G, respectively, the girth $g(G)$ of a graph G is the length of it's shortest cycle [3]. Given any vertex $v \in V(G)$, the graph obtained from G by removing the vertex v and all of its incident edges is denoted by $G - v$. In a tree, a leaf is a vertex of degree one. See [3] for terminology and notations not defined here.

Introduced by Walsh [12], a hub set of G is a set H of vertices in G such that any two vertices outside H are connected by a path whose all internal vertices lie in H. The hub number of G, denoted by $h(G)$, is the minimum size of a hub set in G. A hub set H_r of G is a restrained hub set of G if for any two vertices $u, v \notin H_r$, there is a path between them with all intermediate vertices in $V \setminus H_r$, the minimum cardinality of H_r in G is called a restrained hub number of G and is denoted by $h_r(G)$ [7]. For more details on the hub studies we refer to [5, 6, 7, 10, 11, 13]. For graphs G_1 and G_2 having disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2, respectively, their union, $G(V, E) = G_1 \cup G_2$ has as expected, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. Their join is denoted by $G_1 + G_2$ and consists of $G_1 \cup G_2$ and all edges joining V_1 with V_2 [3]. The corona $G \circ F$ of two graphs G and F is the graph obtained by taking one copy of G of order p and p copies of F, and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of F. For every $v \in V(G)$, denote by F^v the copy of F whose vertices are attached one by one to the vertex v [2]. The open neighbourhood of a vertex $v \in V(G)$ is $N(v)$(or $N_G(v)) = \{u \in V(G) : uv \in E(G)\}$, and $N(v) \cup \{v\}$ is the closed neighbourhood of a vertex v denoted by $N[v]$ or $N_G[v]$, the neighborhood of a set of vertices $U \subseteq V$ is the union of their neighborhoods, that is $N_G(U) = \bigcup_{u \in U} N_G(u)$. The contraction of a vertex v in G (denoted by G/v) is the graph obtained by deleting v and putting a clique on the (open) neighbourhood of v, (note that this operation does not create multiple edges, if two neighbours of v are already adjacent, then they remain simply adjacent) [12].

A set D of vertices in a graph G is called dominating set of G if every vertex in $V \setminus D$ is adjacent to some vertex in D, the minimum cardinality of a dominating set in G is called the domination number $\gamma(G)$ of a graph G [4].

The growing in attention of the researchers in the hub theory as a new branch of graph theory, motivated us to introduce a new hub parameters. One of these parameters is the global hub number of a graph, this parameter is close to the concept of global domination number of a graph. Moreover there are various applications of the hub theory. When we see these applications and by the definition of hub set we can see that the main application should be concerned to communications network. This network is an arrangement establishing communication links connecting a fixed set of locations.

The following results will be useful in the proof of our results.

Theorem 1.1. [12] *Let T be a tree with p vertices and l leaves. Then*

$$h(T) = p - l.$$

Proposition 1.1. [13] *For any connected non tree graph G,*

$$h(G) \geq g(G) - 3.$$

Theorem 1.2. [1] *For any connected graphs G and F such that $|V(G)| \geq 2$,*

$$h(G \circ F) = |V(G)|.$$

2. Characterization of graph G with $h(G) = 2$

Walsh obtained a characterization of graph G with $h(G) = h_c(G) = 1$, in the following result.

Theorem 2.1. [12] *$G = (V, E)$ is a graph with $h(G) = h_c(G) = 1$ if and only if G has the following structure:*

(1) *G is not a complete graph.*
(2) *$V(G) = L \cup M \cup \{u\}$, where L, M and $\{u\}$ are disjoint.*
(3) *u is adjacent to every vertex in L and no vertex in M.*
(4) *Every vertex in L is adjacent to every vertex in M.*
(5) *$G[M]$ is a complete graph.*

We will call the structure of a graph in Theorem 2.1, by structure A. Now we obtain characterization of graph G with $h(G) = 2$.

Theorem 2.2. *Let G be a graph. Then $h(G) = 2$ if and only if G has the following structure:*

(1) *G does not have a structure A.*
(2) *$V(G) = R \cup S \cup \{v, w\}$, where R, S and $\{v, w\}$ are disjoint sets.*
(3) *$R = N(v) \cup N(w)$, and v, w are not adjacent to any vertex of S. If v is not adjacent to w, then for any two vertices $x \in N(w) \setminus N(v)$ and $y \in N(v) \setminus N(w)$, x is adjacent to y.*
(4) *Every vertex in R is adjacent to every vertex in S.*
(5) *$G[S]$ is a complete graph.*

Proof. Suppose that $h(G) = 2$, and let $\{v, w\}$ be the hub set of G. Now let R and S be the sets of neighbours and non-neighbours of $\{v, w\}$ respectively. Since $R = N(\{v, w\})$, then $R = N(v) \cup N(w)$, let $x, y \in R$, if $x, y \in N(v)$ or $x, y \in N(w)$, then the paths xvy or xwy are in G and these paths are unaffected by adjacency of v and w. If $x \in N(v) \setminus N(w)$ and $y \in N(w) \setminus N(v)$, and v is adjacent to w, then the path $xvwy$ is in G. So if v is not adjacent to w, then x and y must be adjacent, now the first three conditions are satisfied. Since $S = V(G) \setminus N[\{v, w\}]$, then for any vertex $a \in S$ there is no $vw-$path between a and any other vertex in $V(G) \setminus \{v, w\}$. Hence every vertex of S must be adjacent to every vertex of $V(G) \setminus \{v, w\}$. The converse is trivial. □

3. Global hub number of some standard graphs

Definition 3.1. A hub set H is a global hub set of G if it is a hub set of both G and its complement $\overline{G}$. The minimum cardinality of a global hub set is called the global hub number $h_g(G)$ of G.

It is clear that $h_g(G)$ is well-defined for any graph G, since $V(G)$ is a global hub set of G. Also it is obvious that $h_g(G) \geq h(G)$ since any global hub set of G is a hub set of G. Moreover, for any graph G, $h_g(G) = h_g(\overline{G})$.
In this section, we determine the value of global hub number for some standard graphs, the following theorem is an immediate consequence of the definition.

Theorem 3.1. (1) *For any cycle C_p,*

$$h_g(C_p) = \begin{cases} 2, & \text{if } p = 3, 4 \text{ ;} \\ p - 3, & \text{if } p \geq 5. \end{cases}$$

(2) *For any path P_p,*

$$h_g(P_p) = \begin{cases} 1, & \text{if } p = 2 \text{ ;} \\ p - 2, & \text{if } p \geq 3. \end{cases}$$

(3) *For the star $K_{1,p-1}$,*

$$h_g(K_{1,p-1}) = 1.$$

Theorem 3.2. *Let T be a tree with order p and l leaves. Then $h_g(T) = p - l$.*

Proof. Let T_n be a tree of n internal vertices. Using the mathematical induction on n, we have
1) $n = 1$, the graph is star and the result is done.
2) For $n = k$ assume that the set of all internal vertices is the minimum global hub set of T_n.
3) Let $n = k + 1$, we have to show that the set of all internal vertices of T_{k+1} is the minimum global hub set of T_{k+1}. Let v be an internal vertex adjacent to end vertex, and let $S = \{w \in N(v) : deg(w) = 1\}$, it follows that $T' = T_{k+1} - S$ is a tree of k internal vertices, so by hypothesis the minimum global hub set of T' is the set of k internal vertices denoted by H_g. Note that, $h(T_{k+1}) = k + 1 > k$, so we must add at least one vertex to H_g to get a minimum global hub set of T_{k+1}. Hence $H_g \cup \{v\}$ is the minimum global hub set of T_{k+1}. Therefore the assertion follows. □

Proposition 3.1. *Let G be a disconnected graph having $N_1, N_2, ..., N_c$ components. Then $h_g(G) = \min_{1 \leq t \leq c} \{k_t\}$, where $k_t = h(N_t) + \sum_{j=1, j \neq t}^{c} |V(N_j)|$.*

Proof. Any hub set H of a graph G must contain all the vertices of $c-1$ components, and the vertices of the hub set of the remaining component. It remains to show that H is minimum. By taking the arbitrary union of all components except one, and taking the hub set of the remaining component, all possible hub sets of G can be computed. This means that any hub set of G is of the form $H = [\bigcup_{j=1, j \neq t}^{c} N_j] \cup H_t$, where H_t is a hub set of N_t, let $k_t = h(N_t) + \sum_{j=1, j \neq t}^{c} |V(N_j)|$. Then $\min_{1 \leq t \leq c} \{k_t\} = h(G)$. Let v be a vertex of any component N_j, $j \neq t$, then in $\overline{G}$, clearly v is adjacent to every vertex of $V(N_t) \setminus H_t$, hence v is adjacent to every vertex of $V(\overline{G}) \setminus H$, and $v \in H$. Therefore, any hub set H of G is a hub set of $\overline{G}$, so $h_g(G) = \min_{1 \leq t \leq c} \{k_t\}$. □

Theorem 3.3. *Let $G = K_{k_1,k_2,...,k_m}$ be a complete $m-$partite graph, $k_1 \leq k_2 \leq ... \leq k_m$. Then $h_g(G) = |V(G)| - k_m$.*

Proof. Suppose that $G = K_{k_1,k_2,...,k_m}$ is a complete $m-$partite graph, $k_1 \leq k_2 \leq ... \leq k_m$. Then $\overline{G}$ is disconnected with m complete components each of order k_i, $1 \leq i \leq m$, so by Proposition 3.1, and since the hub number of a complete graph is zero, it follows that $h_g(G) = |V(G)| - k_m$. □

Corollary 3.1. *For any complete graph K_p,*

$$h_g(K_p) = p - 1.$$

4. Global hub number of join graph and corona

Proposition 4.1. *Let G and F be any two graphs. Then $h_g(G + F) = \min_{1 \leq r \leq c} \{k_r\}$, where $k_r = h(N_r) + \sum_{j=1, j \neq r}^{c} |V(N_j)|$, for the components $N_1, N_2, ..., N_c$, of $\overline{G + F}$.*

Proof. Clearly $\overline{G + F}$ is disconnected, since no vertex of G is adjacent to any vertex of F in $\overline{G + F}$, let $N_1, N_2, ..., N_c$ be the components of $\overline{G + F}$. Then by Proposition 3.1, $h_g(\overline{G + F}) = \min_{1 \leq r \leq c} \{k_r\}$, where $k_r = h(N_r) + \sum_{j=1, j \neq r}^{c} |V(N_j)|$. Since $h_g(\overline{G + F}) = h_g(G + F)$, the result follows. □

Proposition 4.2. *Let F be any non complete graph. Then $h_g(K_1 \circ F) = 1 + h(\overline{F})$.*

Proof. Clearly $h(K_1 \circ F) = 1$, and $\overline{K_1 \circ F}$ is disconnected with at least two components. Since $|V(K_1)| = 1$, at least one of these components is of order 1. Therefore, $h(\overline{G \circ F}) = 1 + h(\overline{F})$. □

Theorem 4.1. *Let G be a connected graph of order $p \geq 2$, and F be any graph. Then $h_g(G \circ F) = p$.*

Proof. By Theorem 1.2, $h(G \circ F) = |V(G)| = p$. Now in $\overline{G \circ F}$, let $H = V(G)$ and $x, y \in V(\overline{G \circ F}) \setminus H$, then we discuss the following cases:

Case 1: $x, y \in V(F^{v_i})$ for some $v_i \in V(G)$. Since $p \geq 2$, and by definition of corona of two graphs, there exists a vertex $v_j \in V(G)$ such that $j \neq i$, and xv_jy is a path in $\overline{G \circ F}$. Hence there is an $H-$path between x and y in $\overline{G \circ F}$.

Case 2: $x \in V(F^{v_i})$ and $y \in V(F^{v_j})$ for some distinct vertices $v_i, v_j \in V(G)$. By definition of corona of two graphs x and y are adjacent in $\overline{G \circ F}$.

Then H is a hub set of $\overline{G \circ F}$. Therefore, by Theorem 1.2, H is a minimum global hub set of $G \circ F$. □

5. Some properties of global hub number

Theorem 5.1. *For a graph G with order $p \geq 3$, $h_g(G) = p-1$ if and only if $G \cong K_p$ or $G \cong \overline{K_p}$.*

Proof. Suppose that $h_g(G) = p - 1$ and $G \ncong K_p$ and $G \ncong \overline{K_p}$. Then we discuss the following cases:
Case 1: G is connected. Since G is not complete, there are at least two non adjacent vertices say u, v in G. Since G is connected, there is a path between u and v with all internal vertices in $H = V(G) \setminus \{u, v\}$, hence H is a hub set of G. Now in $\overline{G}$, u and v are adjacent, therefore H is a hub set of $\overline{G}$. Then H is a global hub set of G with cardinality $p - 2$, which is a contradiction since $h_g(G) = p - 1$.
Case 2: G is disconnected. Since $G \ncong \overline{K_p}$, there are at least two adjacent vertices u, v in one component of G and a vertex w in the other component. Let $H = V(G) \setminus \{u, v\}$, clearly H is a hub set of G. In $\overline{G}$, w is adjacent to u and v, so there is a path uwv in $\overline{G}$. Since $w \in H$ and H is a hub set of $\overline{G}$, hence $h_g(G) \leq p - 2$, which is a contradiction.
From all of the above cases, if $h_g(G) = p-1$, then $G \cong K_p$ or $G \cong \overline{K_p}$. The converse is trivial. □

Corollary 5.1. *Let G be any nontrivial graph with order $p \geq 3$.*

(1) *If G is connected, then $h_g(G) \leq p - 1$.*
(2) *If G is disconnected which is not a totaly disconnected, then $h_g(G) \leq p - 2$.*

By Proposition 1.1, and using this inequality $h_g(G) \geq h(G)$, for any graph G, we get the following observation.

Observation 5.1. *For any connected, non tree graph G,*

$$h_g(G) \geq g(G) - 3.$$

Theorem 5.2. *Let H be a minimum hub set of G, if there exists a vertex $v \in V(G) \setminus H$, such that $N(v) \subseteq H$. Then*

$$h(G) \leq h_g(G) \leq h(G) + 1.$$

Proof. Let H be a minimum hub set of G, and $v \in V(G) \setminus H$ be a vertex with $N(v) \subseteq H$. Then in $\overline{G}$, $[V(G) \setminus H] \subseteq N[v]$, therefore $H \cup \{v\}$ is a hub set of $\overline{G}$, and $h_g(G) \leq h(G) + 1$. □

Proposition 5.1. *Let H be any hub set of a graph G, if there exists a vertex $v \in H$, such that $N(v) \subseteq H$. Then H is a global hub set of G.*

Proof. Let H be any hub set of a graph G, suppose that there exists a vertex $v \in H$, such that $N(v) \subseteq H$. Then $[V(G) \setminus H] \subseteq N[v]$ in $\overline{G}$, so H is a hub set of $\overline{G}$. Hence we obtain the assertion. □

Lemma 5.1. *For any graph G, $h_g(G) \leq h(G) + h(\overline{G})$.*

Proof. Let H be a minimum hub set of G, and M be a minimum hub set of $\overline{G}$, if $M \cap H \neq \phi$, then there are vertices $v_1, v_2, ..., v_k \in H \cap M$. Let H_g be a minimum global hub set of G, then $|H_g| = |H \cup M| = |H| + |M| - |H \cap M|$, hence $h_g(G) = h(G) + h(\overline{G}) - k$, since $k \geq 1$, $h_g(G) < h(G) + h(\overline{G})$. If $H \cap M = \phi$, then $|H_g| = |H \cup M| = |H| + |M|$. So, $h_g(G) = h(G) + h(\overline{G})$. This yields the desired conclusion. □

Proposition 5.2. *Let G be any graph with $\Delta(G) = p - 1$ and $p \geq 3$. Then $h_g(G) \leq \delta(G) + 1$.*

Proof. Let $v \in V(G)$ such that $deg_G(v) = \Delta(G) = p - 1$. Then uvw is a path in G for any two vertices $u, w \in V(G)$, hence $h(G) = 1$, let $x \in V(G)$ such that $deg_G(x) = \delta(G)$, then in $\overline{G}$, x is adjacent to every vertex in $N_{\overline{G}}(x)$. Hence $H' = V(\overline{G}) \setminus N_{\overline{G}}(x)$ is a hub set of $\overline{G}$, since $N_{\overline{G}}(x) = p - 1 - \delta(G)$, then $|H'| = p - (p - 1 - \delta(G)) = \delta(G) + 1$. Since $deg_G(v) = p - 1$, $v \notin N_{\overline{G}}(x)$ and $v \in H'$. Now by the previous lemma any global hub set H_g of G is a subset of H', so $h_g(G) \leq \delta(G) + 1$. □

Corollary 5.2. *For any graph G, $h(\overline{G}) \leq \delta(G) + 1$.*

Let A be a subset of $V(G)$, and let $A^* = \{v \in V \setminus A : A \subseteq N(v)\}$. If the induced subgraph of A in $\overline{G}$, $\overline{G}[A]$ is connected and if $A^* = \phi$, then A is a hub set of $\overline{G}$.

Lemma 5.2. *If H is a hub set of G, and $H' \subseteq H$ is a hub set for $\overline{G}$, then $h_g(G) \leq |H|$.*

Proposition 5.3. *Let G be a graph, then $h(G) = h_g(G)$ if and only if there exists a minimum hub set H of G containing a set H' such that H' is a hub set of $\overline{G}$.*

Proof. Let $h(G) = h_g(G)$ and suppose that for every minimum hub set H of G, any $H' \subseteq H$, H' is not a hub set of $\overline{G}$. Then for any minimum hub set B of $\overline{G}$, $B \nsubseteq H$ so there is a vertex $a \in B$ and $a \notin H$, hence for any minimum global hub set H_g of G, $H \cup \{a\} \subseteq H_g$, therefore $h_g(G) > h(G)$ which is a contradiction. The converse is trivial by Lemma 5.2. □

Theorem 5.3. *Let G be a (p, q) graph with cut vertex v. Then $h_g(G) \leq h_r(G) + 1$, with equality for G with $h_r(G) = 1$.*

Proof. Let G be a graph with cut vertex v, and let H_r be a minimum restrained hub set of G. Suppose that $G - v = \bigcup_{i=1}^{r} G_i$, where $G_1, G_2, ..., G_r$ are the components of $G - v$. Then H_r contains all $G_i's$ except one say, $G_k = Z$, let $B = \bigcup_{i=1, i\neq k}^{r} G_i$, thus $B \subseteq H_r$. Now in $\overline{G}$ every vertex of B is adjacent to every vertex of Z, so $B \cup \{v\}$ is a hub set of $\overline{G}$, since $B \subseteq H_r$, $H_r \cup \{v\}$ is a hub set of $\overline{G}$, then it follows that $H_r \cup \{v\}$ is a global hub set of G, therefore $h_g(G) \leq h_r(G) + 1$. Suppose that $h_r(G) = 1$, then $B = \{u\}$, where u is an end vertex in G adjacent only to v. So in $\overline{G}$, u is adjacent to every vertex except v, and we conclude that $\{u, v\}$ is a minimum hub set of $\overline{G}$, hence $h_g(G) = 2$. So the desired equality is hold. □

Corollary 5.3. *Let G be a (p, q) graph with a bridge e. Then $h_g(G) \leq h_r(G)$.*

Proposition 5.4. *For any graph G, $h_g(G) = 1$ if and only if $G \cong K_{1,p-1}$ or $G \cong K_1 \cup K_{p-1}$.*

Proof. Suppose that $h_g(G) = 1$, then either $h(G) = 1$ or $h(G) = 0$, so we have the following cases:

Case 1: $h(G) = 1$ and G is connected. Then G must have the structure A and the set $\{u\}$ is the hub set of G, as we have $h_g(G) = 1$ then $\{u\}$ is also hub set of $\overline{G}$. If $|V(G)| \geq 2$, the connectivity of G implies that $L \neq \phi$, so $\overline{G}$ is disconnected. Because $\{u\}$ is a hub set of $\overline{G}$, it follows that the component of $\overline{G}$ which contains u must contain u only, but $N_{\overline{G}}(u) = M$, therefore $M = \phi$. Since u is not adjacent to any vertex in L, we deduce that $\overline{G}[L]$ must be complete. Thus, $G[L]$ is totaly disconnected, and $G \cong K_{1,p-1}$.

Case 2: $h(G) = 1$ and G is disconnected. Then G must have the structure A and the set $\{u\}$ is the hub set of G. Clearly, $L = \phi$, hence $V(G) = \{u\} \cup M$ and $G \cong K_1 \cup K_{p-1}$.

Case 3: $h(G) = 0$. Then G is a complete graph, since $h_g(G) = 1$, $h(\overline{G}) = 1$, so G must be K_2.

Hence, $G \cong K_{1,p-1}$ or $G \cong K_1 \cup K_{p-1}$. The converse is clear. □

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