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New results on the F-index of graphs based on corona-type products of graphs

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Dedicated to Prof. Chandrashekar Adiga on His 62nd Birthday

Abstract

In mathematical chemistry, graph operations act in very essential roles since some chemical graphs can be derived from some simpler graphs by applying different graph operations. Among all products of graphs, the corona product of two graphs is one of the most useful product. In this paper the explicit interpretation for F-index of different forms of corona products involving Zagreb indices, graph size

and order are obtained.
Keywords: F-Index, Zagreb index, corona graph

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1 Introduction and preliminaries

In chemical graph theory, different chemical structures are usually modeled by a molecular graph to understand different properties of the chemical compound theoretically. A graph invariant that correlates the physico-chemical properties of a molecular graph with a number is called a molecular structure index. The application of molecular structure indices is a standard procedure in structure-property relations, i.e, in QSPR/QSAR studies [3]. As studied in [2], some chemically interesting graphs can be obtained by different graph operations applied onto some general or particular graphs, and so it is essential to study such graph operations to understand how they are related to the corresponding topological indices of the original graphs.

The first and second Zagreb indices $M_1(G) = \sum_{uv\in E(G)}[d(u) + d(v)]$ and $M_2(G) = \sum_{uv\in E(G)}[d(u)d(v))]$ of a graph G are among the oldest and extremely studied vertex degree based topological indices (cf. [4, 7, 10]). These two indices were introduced to study the structure-dependency of the total π-electron energy in 1972. But a similar index which is under consideration in this paper was not further studied till a recent article by Furtula and Gutman [5] where this index and some basic properties of it were reinvestigated. They conclude that the predictive ability of this index is almost near to that of first Zagreb index and for the entropy and acentric factor, both of them yield correlation coefficients greater than 0.954. It was defined by $F(G) = \sum_{u\in V(G)} d(u)^3$. In [1], F-index of the subdivision graphs have been calculated and in [11], inverse problem for F-index was completely solved.

The corona of two graphs was introduced by Frucht and Harary in [6] with the goal of constructing a graph whose automorphism group is the wreath product of two component automorphism groups. Let G, H be two simple connected graphs such that k_1, k_2 denote the number of vertices and l_1, l_2 denote the number of edges, respectively. The corona product $G \circ H$ of these graphs is obtained by taking one copy of G and k_1 copies of H and by joining each vertex of the i^{th} copy of H to the i^{th} vertex of G, where $1 \leq i \leq k$. In fact the product $G \circ H$ has total $k_1(k_2+1)$ vertices and $l_1+k_1(l_2+k_2)$ edges. In

[8], several types of corona products of two given graphs were studied. Here we present explicit expressions of the F-index of different corona products, namely R-vertex, R-edge, R-vertex neighborhood and R-edge neighborhood coronas which are much stronger than the ones in [8].

Recall that the semitotal point graph $R(G)$ is obtained from G by adding a new vertex for each edge of G and joining such each new vertex to both end vertices of the corresponding edge, [9]. Here we study the F-index of four types of corona products by means of the semitotal point graph.

2 R-vertex corona

Let H_1 and H_2 be two vertex disjoint graphs, $\alpha_1) \in V(H_1)$ and $\beta_1 \in V(H_2)$. The *R-vertex corona* of H_1 and H_2 is denoted by $H_1 \odot_R H_2$ and obtained from $R(H_1)$ and $|V(H_1)| = k_1$ copies of H_2 by joining the i^{th} vertex of $V(H_1)$ to every vertex in the i^{th} copy of H_2. Hence we observe that the degrees of the vertices of $H_1 \odot_R H_2$ are given by $d_{H_1 \odot_R H_2}(x_i) = 2d_{H_1}(x_i) + k_2$ for each vertex x_i where $i = 1, 2, \dots, k_1$; $d_{H_1 \odot_R H_2}(e_i) = 2$ for each newly added vertex e_i where $i = 1, 2, \dots, l_1$ and $d_{H_1 \odot_R H_2}(y_j) = d_{H_2}(y_j) + 1$ for $j = 1, 2, \dots, k_2$.

Theorem 1. *The F-index of the R-vertex corona of H_1 and H_2 is given by*

$$\begin{aligned} F(H_1 \odot_R H_2) &= 8F(H_1) + 12k_2M_1(H_1) + 12l_1k_2^2 + 8l_1 \\ &\quad + k_1\big[F(H_2) + k_2 + 3M_1(H_2) + 6l_2 + k_2^3\big]. \end{aligned}$$

Proof. By considering the definition of F-index, we get

$$\begin{aligned} F(H_1 \odot_R H_2) &= \sum_{i=1}^{k_1}\big[2d_{H_1}(\alpha_i) + k_2\big]^3 + \sum_{i=1}^{l_1} 2^3 + \sum_{i=1}^{k_1}\sum_{j=1}^{k_2}(d_{H_2}(\beta_j) + 1)^3 \\ &= \sum_{i=1}^{k_1}\big[8(d_{H_1}(\alpha_i))^3 + k_2^3 + 12k_2(d_{H_1}(\alpha_i))^2 + 6k_2^2 d_{H_1}(\alpha_i)\big] \\ &\quad + 8l_1 + k_1\sum_{j=1}^{k_2}\big[(d_{H_2}(\beta_j))^3 + 1^3 + 3(d_{H_2}(\beta_j))^2 + 3d_{H_2}(\beta_j)\big] \\ &= 8F(H_1) + 12k_2M_1(H_1) + 12l_1k_2^2 + 8l_1 \\ &\quad + k_1\big[F(H_2) + k_2 + 3M_1(H_2) + 6l_2 + k_2^3\big]. \end{aligned}$$

□

If we apply Theorem 1 for a path and a cycle graph, we obtain

$$F(P_{k_1} \odot_R C_{k_2}) = 72k_1 - 72k_2 + 75k_1k_2 + k_2^2(k_1k_2 + 12k_1 - 12) - 120$$

and

$$F(C_{k_1} \odot_R P_{k_2}) = 34k_1 + 75k_1k_2 + k_1k_2^2(k_2 + 12).$$

3 R-edge corona

Let $I(G)$ denote the set of new vertices added to G to get $R(G)$. Let H_1 and H_2 be two vertex disjoint graphs. The *R-edge corona* of H_1 and H_2 is denoted by $H_1 \ominus_R H_2$ and obtained using $R(H_1)$ and $|I(H_1)| = l_1$ copies of H_2 by joining the i^{th} vertex of the set $I(H_1)$ to every vertex in the i^{th} copy of H_2. We observe that the degrees of the vertices of $H_1 \ominus_R H_2$ are given by $d_{H_1 \ominus_R H_2}(x_i) = 2d_{H_1}(x_i) + k_2$ for each vertex x_i where $i = 1, 2, \ldots, k_1$; $d_{H_1 \ominus_R H_2}(e_i) = 2$ for each newly added vertex e_i where $i = 1, 2, \ldots, l_1$ and $d_{H_1 \ominus_R H_2}(y_j) = d_{H_2}(y_j) + 1$ for $j = 1, 2, \ldots, k_2$.

Theorem 2. *The F-index of the R-edge corona of H_1 and H_2 is given by*

$$F(H_1 \ominus_R H_2) = 8F(H_1) + l_1\big[F(H_2) + 3M_1(H_2) + 6l_2 + 8 + k_2^3 + 12k_2 + 6k_2^2 + k_2\big].$$

Proof. Again, by the definition of F-index, it is easy to observe

$$F(H_1 \ominus_R H_2) = \sum_{i=1}^{k_1}(2d_{H_1}(\alpha_i))^3 + \sum_{i=1}^{l_1}(2+k_2)^3 + \sum_{i=1}^{l_1}\sum_{j=1}^{k_2}(d_{H_2}(\beta_j)+1)^3$$

such that

$$\begin{aligned} F(H_1 \ominus_R H_2) &= 8\textstyle\sum_{i=1}^{k_1} d_{H_1}(\alpha_i)^3 + \sum_{i=1}^{l_1}\big[8 + k_2^3 + 12k_2 + 6k_2^2\big] \\ &+ l_1 \textstyle\sum_{j=1}^{k_2}\big[d_{H_2}(\beta_j)^3 + 1^3 + 3d_{H_2}(\beta_j)^2 + 3d_{H_2}(\beta_j)\big] \\ &= 8F(H_1) + l_1\left[F(H_2) + 3M_1(H_2) + 6l_2 + 8 + k_2^3 \right. \\ &\left. + 12k_2 + 6k_2^2 + k_2\right]. \end{aligned}$$

□

Theorem 2 gives the following result for path and cycle graphs:

$$F(P_{k_1} \ominus_R C_{k_2}) = 72k_1 - 39k_2 + 39k_1k_2 + k_2^3(k_2+6)(k_1-1) - 120$$

and

$$F(C_{k_1} \ominus_R P_{k_2}) = 34k_1 + k_1k_2[k_2(k_2+6) + 39].$$

4 R-vertex neighborhood corona

Let H_1 and H_2 be two vertex disjoint graphs. The *R-vertex neighborhood corona* of H_1 and H_2 is denoted by $H_1 \odot_{NR} H_2$ and obtained using $R(H_1)$

and $|V(H_1)| = k_1$ copies of H_2, by joining the neighbors i^{th} vertex of $V(H_1)$ to every vertex in the i^{th} copy of H_2. By the definition, we see that the degrees of the vertices of $H_1 \odot_{NR} H_2$ are given by $d_{H_1 \odot_{NR} H_2}(x_i) = 2d_{H_1}(x_i)$ for each vertex x_i where $i = 1, 2, \ldots, k_1$, $d_{H_1 \odot_{NR} H_2}(e_i) = 2k_2 + 2$ for each newly added vertex e_i where $i = 1, 2, \ldots, l_1$ and $d_{H_1 \odot_{NR} H_2}(y_j) = d_{H_1}(x_i) + d_{H_2}(y_j)$ for $i = 1, 2, \ldots, l_1$ and $j = 1, 2, \ldots, k_2$.

Theorem 3. *The F-index of the R-vertex neighborhood corona of H_1 and H_2 is given by*

$$\begin{aligned} F(H_1 \odot_{NR} H_2) &= 8F(H_1) + k_2\left(k_2^2\left(F(H_1) + 32l_1\right) + 6k_2F(H_1)\right. \\ &+ \left. 24l_1 + 20F(H_1)\right) + 8l_1 \\ &+ k_1F(H_2) + 24l_2M_1(H_1) + 12l_1M_1(H_2). \end{aligned}$$

Proof. By taking into account

$$\begin{aligned} F(H_1 \odot_{NR} H_2) &= \textstyle\sum_{i=1}^{k_1}(2d_{H_1}(\alpha_i) + k_2d_{H_1}(\alpha_i))^3 + \sum_{i=1}^{l_1}(2k_2+2)^3 \\ &\quad + \textstyle\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}(2d_{H_1}(\alpha_i) + d_{H_2}(\beta_j))^3, \end{aligned}$$

we easily obtain

$$\begin{aligned} F(H_1 \odot_{NR} H_2) &= \textstyle\sum_{i=1}^{k_1}\left[8(d_{H_1}(\alpha_i))^3 + k_2^3(d_{H_1}(\alpha_i))^3 + 12k_2(d_{H_1}(\alpha_i))^3\right. \\ &\quad \left. +6k_2^2(d_{H_1}(\alpha_i))^3 + \textstyle\sum_{i=1}^{l_1}\left[8k_2^3 + 24k_2^2 + 24k_2 + 8\right]\right] \\ &\quad + \textstyle\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[8d_{H_1}(\alpha_i)^3 + (d_{H_2}(\beta_j))^3\right. \\ &\quad \left. +12d_{H_1}(\alpha_i)^2d_{H_2}(\beta_j) + 6d_{H_1}(\alpha_i)(d_{H_2}(\beta_j))^2\right] \\ &= 8F(H_1) + k_2\left(k_2^2\left(F(H_1) + 32l_1\right) + 6k_2F(H_1) + 24l_1\right. \\ &\quad \left. +20F(H_1)\right) + 8l_1) + k_1F(H_2) + 24l_2M_1(H_1) + 12l_1M_1(H_2), \end{aligned}$$

□

For a path and a cycle graph, Theorem 3 gives

$$F(P_{k_1} \odot_{NR} C_{k_2}) = (8k_1-14)(k_2^3+6k_2^2+12k_2+8)+8(k_1-1)(k_2^3+3k_2^2+3k_2+1)+8k_2(27k_1-38)$$

and

$$F(C_{k_1} \odot_{NR} P_{k_2}) = 2k_1(8k_2^3 + 36k_2^2 + 168k_2 - 55).$$

5 R-edge neighborhood corona

We now study the F-index of the R-edge neighborhood corona of two vertex disjoint graphs H_1 and H_2. The *R-edge neighborhood corona* of H_1 and H_2 is denoted by $H_1 \ominus_{NR} H_2$ and obtained by using $R(H_1)$ and $|I(H_1)| = l_1$ copies of H_2 joining the neighbors i^{th} vertex of the set $I(H_1)$ to every vertex in the i^{th} copy of H_2. The vertex degrees of $H_1 \ominus_{NR} H_2$ are given by $d_{H_1 \ominus_{NR} H_2}(x_i) = 2d_{H_1}(x_i)(k_2+1)$ for each vertex x_i where $i = 1, 2, \ldots, k_1$; $d_{H_1 \ominus_{NR} H_2}(e_i) = 2$ for each newly added vertex e_i where $i = 1, 2, \ldots, l_1$ and $d_{H_1 \ominus_{NR} H_2}(y_j) = d_{H_1}(x_i) + d_{H_2}(y_j)$ for $i = 1, 2, \ldots, k_1$ and $j = 1, 2, \ldots, k_2$.

Theorem 4. *The F-index of the R-edge neighborhood corona of H_1 and H_2 is given by*

$$F(H_1 \ominus_{NR} H_2) = F(H_1)(k_2^3+6k_2^2+12k_2+8)+l_1F(H_2)+6l_1M_1(H_2)+8l_1(1+k_2+4l_2).$$

Proof. We have

$$\begin{aligned} F(H_1 \ominus_{NR} H_2) &= \textstyle\sum_{i=1}^{k_1}(2d_{H_1}(\alpha_i) + k_2 d_{H_1}(\alpha_i))^3 + \sum_{i=1}^{l_1} 2^3 \\ &\quad + \textstyle\sum_{i=1}^{l_1}\sum_{j=1}^{k_2}(d_{H_2}(\beta_j)+2)^3 \\ &= F(H_1)(k_2^3+6k_2^2+12k_2+8) + l_1F(H_2) + 6l_1M_1(H_2) \\ &\quad +8l_1(1+k_2+4l_2), \end{aligned}.$$

□

Theorem 4 gives the following values for a path and a cycle graph $F(P_{k_1} \ominus_{NR} C_{k_2}) = (8k_1-14)(k_2^3+6k_2^2+12k_2+8) + (k_1-1)(64k_2+8)$ and $F(C_{k_1} \ominus_{NR} P_{k_2}) = 8k_1(k_2^3+6k_2^2+12k_2+8) + k_1(64k_2-66)$.

6 Conclusion

The main idea in this paper is to obtain the explicit interpretation for the F-index of different forms of corona products involving Zagreb indices, F-index, size and order. These results are useful to stumble on $QSAR$ and $QSPR$ of chemical compounds. For the future study, one may apply similar ideas to other graph products to get other useful results related to the above chemical compounds.

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