

## Subdivision and semi total point graphs of Archimedean lattices on some topological indices

N. Ananda<sup>(1)</sup>, P.S. Ranjini<sup>(2)</sup>, V. Lokesha<sup>(3)</sup>, S.A. Wazzan<sup>(4),\*</sup>

<sup>(1)</sup> Govt P U College, Akkihebbalu, K R Pet, Mandya, India  
*ananda464@gmail.com*

<sup>(2)</sup> Department of Mathematics, Don Bosco Institute of Technology, Bangalore-74, India  
*ranjinip.s@gmail.com*

<sup>(3)</sup> Department of Studies in Mathematics, Vijayanagara Sri Krishnadevaraya University, Ballari, Karnataka, India  
*v.lokesha@gmail.com*

<sup>(4)</sup> Department of Mathematics, Science Faculty, KAU King Abdulaziz University, Girls Campus, 21589, Jeddah, S. Arabia  
*swazzan@kau.edu.sa*

### Abstract

It is well known that any topological index is a type of molecular descriptor that is calculated based on the molecular graph of a chemical compound. In this paper, we investigate subdivision and semi total point graphs of the Archimedean lattices  $L_{(4,6,12)}$  for the topological indices Nano-Zagreb index, multiplicative Nano-Zagreb index,  $VL$ -index, Atom bond connectivity index, Inverse sum indeg index, Forgotten index, Augmented Zagreb index, Geometric-Arithmetic index,  $SK$ -index and Hyper Zagreb index.

2000 *Mathematics Subject Classification*: 05C12, 05C35, 05C90.

*Keywords and Phrases*: Topological indices, subdivision graph, semi total point graph, Archimedean lattices.

## 1 Introduction and Preliminaries

Topological indices are numerical values computed from a graph which characterize its topology and are usually graph invariants. In fact, the topological index of a molecule is a non-empirical numerical quantity that quantifies the structure and the branching pattern of the molecule. Therefore the topological analysis of a molecule involves translating its molecular structure into a characteristic

---

\*Corresponding author

unique number (or index) that may be considered a descriptor of the molecule under examination. Such indices based on the distances in graph are widely used for establishing relationships between the structure of molecular graph and their physiochemical properties (cf. [17]).

A molecular graph is a simple graph such that its vertices corresponds to the atoms and the edges to the bonds, as a useful tool of research chemical graph is applied to reveal the relationships between various physical characteristics and chemical structures such as biological activity, chemical reactivity. A major part of the current research in mathematical chemistry, chemical graph theory and quantitative structure-activity property relationship studies involves topological indices.

As the classical way, we notate by  $G$  as graph, where  $E(G)$  and  $V(G)$  as the edge and vertex sets of  $G$ , respectively. We also use the notation  $E_{x,y}$  to express the set of edges that the degrees of end vertices  $x$  and  $y$ , mathematically,  $E_{x,y} = \{uv/\{x,y\} = \{d(u), d(v)\}\}$ , where the notations  $d(u)$  and  $d(v)$  denote the degree of vertices  $u$  and  $v$  in the graph  $G$ .

The *subdivision graph*  $S(G)$  is the graph obtained from  $G$  by replacing each of its edge by a path of length 2 or equivalently by inserting an additional vertex into each edge of  $G$  ([14]). On the other hand, the *semi total point graph*  $R(G)$  is the graph obtained from  $G$  by adding a new vertex corresponding to each edge of  $G$  and by joining each new vertex to the end vertices of the edge corresponding to it ([5]).

In the following definitions, we will remind the meanings of some topological indices that will be needed for our results in this paper.

**Definition 1.** In [3], the *Nano-Zagreb and multiplicative Nano-Zagreb indices* have been defined by

$$NZ(G) = \sum_{uv \in E(G)} (d^2(u) - d^2(v)) \quad \text{and} \quad N^*Z(G) = \prod_{uv \in E(G)} (d^2(u) - d^2(v)),$$

respectively, where  $d(u) \geq d(v)$ .

**Definition 2.** In [6], the *Veerabhadraiah Lokesha index*, shortly  $VL(G)$ , has been recently defined as

$$VL(G) = \frac{1}{2} \sum_{uv \in E(G)} (d_e + d_f + 4),$$

where  $d_e = d(u) + d(v) - 2$  and  $d_f = [d(u) \times d(v)] - 2$ .

**Definition 3.** Estrada et al. [7] introduced *Atom-bond connectivity index*, shortly  $(ABC)$ -index, as  $ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d(u)+d(v)-2}{d(u)d(v)}}$ . This index has been applied until now to study the stability of alkanes and the strain energy of cyclo alkanes.

**Definition 4.** Vukicevic et al. ([21]) introduced the *Inverse sum indeg index* as

$$ISI(G) = \sum_{uv \in E(G)} \frac{d(u)d(v)}{d(u) + d(v)}.$$

**Definition 5.** *The Forgotten topological index or F-index of a graph  $G$  has more applications in the analysis of drug molecular structures which was firstly introduced by Furtula and Gutman in [11]. This index can be formulated as  $F(G) = \sum_{uv \in E(G)} (d(u)^2 + d(v)^2)$ .*

**Definition 6.** *The Augmented Zagreb index is defined as  $AZI(G) = \sum_{uv \in E(G)} [\frac{d(u)d(v)}{d(u)+d(v)-2}]^3$  which was introduced in [10].*

**Definition 7.** *The Geometric-Arithmetic index (GA-index) defined in [20] as*

$$GA(G) = \sum_{uv \in E(G)} \frac{2[\sqrt{d(u)d(v)}]}{d(u) + d(v)}.$$

**Definition 8.** *It has been introduced  $SK, SK_1$  and  $SK_2$  indices ([13, 15]) which are defined as*

$$SK(G) = \sum_{uv \in E(G)} \frac{d(u) + d(v)}{2}, \quad SK_1(G) = \sum_{uv \in E(G)} \frac{d(u)d(v)}{2},$$

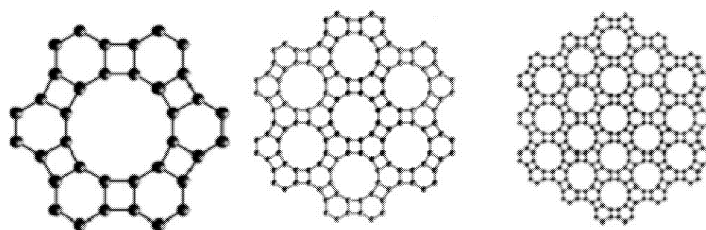
$$SK_2(G) = \sum_{uv \in E(G)} [\frac{d(u) + d(v)}{2}]^2.$$

**Definition 9.** *The Hyper Zagreb index  $HM(G)$  introduced by Shirdel et. al in [16] and is actually defined as  $HM(G) = \sum_{uv \in E(G)} ([d(u) + d(v)])^2$ . For some various study of this index, we may refer the references [4, 8, 9, 19].*

## 2 Main Results

The Archimedean lattices are uniform tilings of the plane in which all the faces are regular polygons and the symmetry group acts transitively on the vertices. It follows that all vertices are equivalent and have the same co-ordination number [1].

In this paper, we define an Archimedean lattice and name it as  $L_{(4,6,12)}(n)$  circumference. In geometry, the great rhombitrihexagonal contains one dodecagon, two hexagons, and three squares on each edge, as shown in Figure 1.



**Figure 1:** The Archimedean lattice  $L_{(4,6,12)}(n)$ .

We note that while the number of vertices of subdivision of  $L_{(4,6,12)}(n)$  is  $90n^2 - 6n$ , the number of edges of subdivision of  $L_{(4,6,12)}(n)$  is  $108n^2 - 12n$ . Similarly, the number of vertices of semi total point of  $L_{(4,6,12)}(n)$  is  $90n^2 - 6n$  and the number of edges of semi total point of  $L_{(4,6,12)}(n)$  is  $162n^2 - 18n$ .

$(s, t)$	(2,2)	(2,3)
$ E_{s,t} $	$24n$	$108n^2 - 36n$

**Table1:** The subdivision of the partition of the edge set of  $L_{(4,6,12)}(n)$  into  $E_{s,t}$ .

$(s, t)$	(2,4)	(4,4)	(4,6)	(2,6)	(6,6)
$ E_{s,t} $	$24n$	$6n$	$12n$	$108n^2 - 36n$	$54n^2 - 24n$

**Table2:** The semi total point graph of the partition of the edge set of  $L_{(4,6,12)}(n)$  into  $E_{s,t}$ .

In the proof of each following results, we will consider the subdivision of the edge partition  $S(L_{(4,6,12)}(n))$  given in Table 1 and the semi total point of the edge partition  $R(L_{(4,6,12)}(n))$  given in Table 2, respectively.

**Theorem 1.** *Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $NZ[S(L_{(4,6,12)}(n))] = 540n^2 - 180n$  and  $NZ[R(L_{(4,6,12)}(n))] = 3456n^2 - 624n$ .*

*Proof.* By Definition 1,

$$\begin{aligned} NZ[S(L_{4,6,12}(n))] &= (9 - 4)|E_{3,2}| = 5(108n^2 - 36n) \\ &= 540n^2 - 180n \end{aligned}$$

and

$$\begin{aligned} NZ[R(L_{4,6,12}(n))] &= (16 - 4)|E_{4,2}| + (36 - 16)|E_{6,4}| + (36 - 4)|E_{6,2}| \\ &= 12(24n) + 20(12n) + 32(108n^2 - 36n) \\ &= 3456n^2 - 624n. \end{aligned}$$

□

**Theorem 2.** *Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $VL[S(L_{(4,6,12)}(n))] = 594n^2 - 102n$  and  $VL[R(L_{(4,6,12)}(n))] = 2376n^2 - 492n$ .*

*Proof.* By Definition 2,

$$\begin{aligned} VL[S(L_{(4,6,12)}(n))] &= \frac{1}{2}\{8|E_{2,2}| + 11|E_{2,3}|\} \\ &= \frac{1}{2}\{8(24n) + 11(108n^2 - 36n)\} = 594n^2 - 102n \end{aligned}$$

and

$$\begin{aligned} VL[R(L_{(4,6,12)}(n))] &= \frac{1}{2}\{14|E_{2,4}| + 24|E_{4,4}| + 34|E_{4,6}| + 20|E_{2,6}| + 48|E_{6,6}|\} \\ &= \frac{1}{2}\{14(24n) + 24(6n) + 34(12n) + 20(108n^2 - 36n) + 48(54n^2 - 24n)\} \\ &= 2376n^2 - 492n. \end{aligned}$$

□

**Theorem 3.** Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $ABC[S(L_{(4,6,12)}(n))] = \sqrt{2}\{54n^2 - 6n\}$  and  $ABC[R(L_{(4,6,12)}(n))] = \frac{(24n+3n\sqrt{3})}{\sqrt{2}} + \frac{12n}{\sqrt{3}} + \frac{(108n^2-36n)}{\sqrt{2}} + \frac{(54n^2-24n)}{6}$ .

*Proof.* By Definition 3,

$$\begin{aligned} ABC[S(L_{(4,6,12)}(n))] &= \sqrt{\frac{2}{4}}|E_{2,2}| + \sqrt{\frac{3}{6}}|E_{2,3}| \\ &= 24n\sqrt{\frac{1}{2}} + \sqrt{2}(54n^2 - 18n) = \sqrt{2}(54n^2 - 6n) \end{aligned}$$

and

$$\begin{aligned} ABC[R(L_{(4,6,12)}(n))] &= \sqrt{\frac{1}{2}}|E_{2,4}| + \sqrt{\frac{6}{16}}|E_{4,4}| + \sqrt{\frac{8}{24}}|E_{4,6}| + \sqrt{\frac{6}{12}}|E_{2,6}| + \sqrt{\frac{10}{36}}|E_{6,6}| \\ &= \sqrt{\frac{1}{2}}24n + \sqrt{\frac{6}{16}}6n + \sqrt{\frac{8}{24}}12n + \sqrt{\frac{6}{12}}(108n^2 - 36n) + \sqrt{\frac{10}{36}}(54n^2 - 24n) \\ &= \frac{(24n + 3n\sqrt{3})}{\sqrt{2}} + \frac{12n}{\sqrt{3}} + \frac{(108n^2 - 36n)}{\sqrt{2}} + \frac{(54n^2 - 24n)}{6}. \end{aligned}$$

□

**Theorem 4.** Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $ISI[S(L_{(4,6,12)}(n))] = \frac{648n^2-96n}{5}$  and  $ISI[R(L_{(4,6,12)}(n))] = \frac{2}{5}(810n^2 - 133n)$ .

*Proof.* By Definition 4,

$$\begin{aligned} ISI[S(L_{(4,6,12)}(n))]n &= \frac{4}{4}|E_{2,2}| + \frac{6}{5}|E_{2,3}| \\ &= \frac{4}{4}(24n) + \frac{6}{5}(108n^2 - 36n) = \frac{648n^2 - 96n}{5} \end{aligned}$$

and

$$\begin{aligned} ISI[R(L_{(4,6,12)}(n))] &= \frac{8}{6}|E_{2,4}| + \frac{16}{8}|E_{4,4}| + \frac{24}{10}|E_{4,6}| + \frac{12}{8}|E_{2,6}| + \frac{36}{12}|E_{6,6}| \\ &= \frac{8}{6}(24n) + \frac{16}{8}(6n) + \frac{24}{10}(12n) + \frac{12}{8}(108n^2 - 36n) + \frac{36}{12}(54n^2 - 24n) \\ &= \frac{2}{5}(810n^2 - 133n). \end{aligned}$$

□

**Theorem 5.** Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $F[S(L_{(4,6,12)}(n))] = 1404n^2 - 276n$  and  $F[R(L_{(4,6,12)}(n))] = 8208n^2 - 1872n$ .

*Proof.* By Definition 5,

$$\begin{aligned} F[S(L_{(4,6,12)}(n))] &= (4+4)|E_{2,2}| + (4+9)|E_{2,3}| \\ &= (4+4)(24n) + (4+9)(108n^2 - 36n) = 1404n^2 - 276n \end{aligned}$$

and

$$\begin{aligned} F[R(L_{(4,6,12)}(n))] &= (4+16)|E_{2,4}| + (16+16)|E_{4,4}| + (16+36)|E_{4,6}| + (4+36)|E_{2,6}| + \\ &\quad (36+36)|E_{6,6}| \\ &= (4+16)(24n) + (16+16)(6n) + (16+36)(12n) + (4+36)(108n^2 - 36n) + \\ &\quad (36+36)(54n^2 - 24n) \\ &= 8208n^2 - 1872n. \end{aligned}$$

□

**Theorem 6.** Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $AZI[S(L_{(4,6,12)}(n))] = 864n^2 - 96n$  and  $AZI[R(L_{(4,6,12)}(n))] = \frac{5668}{9}n + 8(108n^2 - 36n) + \frac{5832}{125}(54n^2 - 24n)$ .

*Proof.* By Definition 6,

$$\begin{aligned} AZI[S(L_{(4,6,12)}(n))] &= \frac{4}{2}|E_{2,2}| + \frac{6}{3}|E_{2,3}| \\ &= \frac{4}{2}(24n) + \frac{6}{3}(108n^2 - 36n) = 864n^2 - 96n \end{aligned}$$

and

$$\begin{aligned} AZI[R(L_{(4,6,12)}(n))] &= \frac{8}{4}|E_{2,4}| + \frac{16}{6}|E_{4,4}| + \frac{24}{8}|E_{4,6}| + \frac{12}{6}|E_{2,6}| + \frac{36}{10}|E_{6,6}| \\ &= \frac{8}{4}(24n) + \frac{16}{6}(6n) + \frac{24}{8}(12n) + \frac{12}{6}(108n^2 - 36n) + \frac{36}{10}(54n^2 - 24n) \\ &= \frac{5668}{9}n + 8(108n^2 - 36n) + \frac{5832}{125}(54n^2 - 24n). \end{aligned}$$

□

**Theorem 7.** *Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $GA[S(L_{(4,6,12)}(n))] = 2[12n + \frac{\sqrt{6}}{5}(108n^2 - 36n)]$  and  $GA[R(L_{(4,6,12)}(n))] = 2[8\sqrt{2}n + \frac{12}{5}n\sqrt{6} + \frac{\sqrt{3}}{4}(108n^2 - 36n) + 27n^2 - 9n]$ .*

*Proof.* By Definition 7,

$$\begin{aligned} GA[S(L_{(4,6,12)}(n))] &= 2 \times \frac{\sqrt{4}}{4}|E_{2,2}| + 2 \times \frac{\sqrt{6}}{5}|E_{2,3}| \\ &= 2 \times \frac{\sqrt{4}}{4}(24n) + 2 \times \frac{\sqrt{6}}{5}(108n^2 - 36n) \\ &= 2[12n + \frac{\sqrt{6}}{5}(108n^2 - 36n)] \end{aligned}$$

and

$$\begin{aligned} GA[R(L_{(4,6,12)}(n))] &= 2 \times \frac{\sqrt{8}}{6}|E_{2,4}| + 2 \times \frac{\sqrt{16}}{8}|E_{4,4}| + 2 \times \frac{\sqrt{24}}{10}|E_{4,6}| + 2 \times \frac{\sqrt{12}}{8}|E_{2,6}| + \\ &2 \times \frac{\sqrt{36}}{12}|E_{6,6}| \\ &= 2 \times \frac{\sqrt{8}}{6}(24n) + 2 \times \frac{\sqrt{16}}{8}(6n) + 2 \times \frac{\sqrt{24}}{10}(12n) + 2 \times \frac{\sqrt{12}}{8}(108n^2 - 36n) + \\ &2 \times \frac{\sqrt{36}}{12}(54n^2 - 24n) \\ &= 2[8\sqrt{2}n + \frac{12}{5}n\sqrt{6} + \frac{\sqrt{3}}{4}(108n^2 - 36n) + 27n^2 - 9n]. \end{aligned}$$

□

**Theorem 8.** *Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then*

$$SK[S(L_{(4,6,12)}(n))] = 270n^2 - 42n, \quad SK[R(L_{(4,6,12)}(n))] = 756n^2 - 132n, \quad (2.1)$$

$$SK_1[S(L_{(4,6,12)}(n))] = 324n^2 - 60n, \quad SK_1[R(L_{(4,6,12)}(n))] = 1620n^2 - 360n, \quad (2.2)$$

$$SK_2[S(L_{(4,6,12)}(n))] = \frac{2700n^2 - 516n}{4}, \quad SK_2[R(L_{(4,6,12)}(n))] = 3672n^2 - 828n. \quad (2.3)$$

*Proof.* We will just prove the case in (2.1), since equations in (2.2) and (2.3) can be proved quite similarly.

Therefore, by Definition 8, we get

$$\begin{aligned} SK[S(L_{(4,6,12)}(n))] &= \frac{4}{2}|E_{2,2}| + \frac{5}{2}|E_{2,3}| \\ &= \frac{4}{2}(24n) + \frac{5}{2}(108n^2 - 36n) \\ &= 270n^2 - 42n \end{aligned}$$

and

$$\begin{aligned} SK[R(L(4, 6, 12)(n))] &= \frac{6}{2}|E_{2,4}| + \frac{8}{2}|E_{4,4}| + \frac{10}{2}|E_{4,6}| + \frac{8}{2}|E_{2,6}| + \frac{12}{2}|E_{6,6}| \\ &= \frac{6}{2}(24n) + \frac{8}{2}(6n) + \frac{10}{2}(12n) + \frac{8}{2}[(108n^2 - 36n) + \frac{12}{2}(54n^2 - 24n)] \\ &= 756n^2 - 132n, \end{aligned}$$

as required.  $\square$

The final result is the following.

**Theorem 9.** *Let  $G$  be an  $L_{(4,6,12)}(n)$  circumference. Then  $HM[S(L_{(4,6,12)}(n))] = 2700n^2 - 516n$  and  $HM[R(L_{(4,6,12)}(n))] = 14688n^2 - 3312n$ .*

*Proof.* Similarly as in all above theorems, the result is obtained based on the subdivision of the edge partition given in Table 1 and the semi total point of the edge partition given in Table 2, respectively. Using Definition 9, we obtain

$$\begin{aligned} HM[S(L(4, 6, 12)(n))] &= (2 + 2)^2|E_{2,2}| + (2 + 3)^2|E_{2,3}| \\ &= (2 + 2)^2(24n) + (2 + 3)^2(108n^2 - 36n) \\ &= 2700n^2 - 516n \end{aligned}$$

and

$$\begin{aligned} SK_2[R(L(4, 6, 12)(n))] &= (2 + 4)^2|E_{2,4}| + (4 + 4)^2|E_{4,4}| + (4 + 6)^2|E_{4,6}| + (2 + 6)^2|E_{2,6}| + \\ &\quad + (6 + 6)^2|E_{6,6}| \\ &= (2 + 4)^2(24n) + (4 + 4)^2(6n) + (4 + 6)^2(12n) + (2 + 6)^2(108n^2 - 36n) + \\ &\quad + (6 + 6)^2(54n^2 - 24n) \\ &= 14688n^2 - 3312n, \end{aligned}$$

as required.  $\square$

**Conclusion 1.** *In this study, we investigated the subdivision and semi total point graphs of a class of honeycomb network which is covered by  $C_4$ ,  $C_8$  and  $C_{12}$  and also derived some formulas in terms of Nano-Zagreb index, Multiplicative Nano-Zagreb index, VL-index, Atom-bond connectivity index, Inverse sum indeg index, Forgotten index, Augmented Zagreb index, Geometric Arithmetic index, SK-indices and Hyper Zagreb index for those graphs. On the other hand, for a future study, we strictly note that the problem studied in here can also be applied to the indices that have been presented in the papers [2, 12, 18].*



## References

- [1] A. Codello, Exact Curie temperature for the Ising model on Archimedean and Laves lattices, *J. Phys. A: Math. Theor.* 43 (2010), 385002.
- [2] Kinkar Ch. Das, N. Akgunes, M. Togan, A. Yurttas, I.N. Cangul, A.S. Cevik, On the first Zagreb index and multiplicative Zagreb coindices of graphs, *Analele Stiintifice ale Universitatii Ovidius Constanta*, Vol XXIV Fascicola 1 (2016).
- [3] A. Jahanbani and H. Shooshtary, Nano-Zagreb index and Multiplicative Nano-Zagreb index of some graph operations, *Inter. J. Comput. Sci. Appl. Math.*, 5(1), (2019), 15-22.
- [4] B. Basavanagoud and S. Patil, A note on Hyper-Zagreb index of graph operations, *Iranian J. Math. Chem.*, 7(1), (2016), 89-92.
- [5] D. Cvetković, M. Doob and H. Sachs, *Spectra of Graphs - Theory and Application*, Academic Press, New York, 1995.
- [6] T. Deepika, An Emphatic study on  $VL$ -index and their bounds on tensor product of the  $F$ -sum graph, submitted TWMS J.APP. and Eng.Math.
- [7] E. Estrada, L.Torres, L.Rodriguez and I. Gutman, An atom-bond connectivity index: Modeling the enthalpy of formation of alkanes, *Indian J. Chem., Section a* 37(10) (1998), 849-855.
- [8] M.R. Farahani, Computing the Hyper-Zagreb index of hexagonal nanotubes, *J. Chem. Mater. Research* 2(1) (2015), 16-18.
- [9] M.R. Farahani, The Hyper-Zagreb index of  $TUSC_4C_8(S)$  nanotubes, *Int. J. Eng. Tech. Research* 3(1) (2015), 1-6.
- [10] B. Furtula, A. Graovac and D. Vukicevic, Augmented Zagreb index, *J. Math. Chem.* 48 (2010), 370-380.
- [11] B. Furtula and I. Gutman, A Forgotten topological index, *J. Math. Chem.* 86 (2015), 1184-1190.
- [12] I. Gutman, M. Togan, A. Yurttas, A.S. Cevik, I.N. Cangul, Inverse Problem for Sigma Index, *MATCH Communications in Mathematical and in Computer Chemistry*, 79(2) (2018), 491-508.
- [13] M. R. Rajesh Kanna, R. Pradeep Kumar and R. Jagadeesh, Computation of Topological Indices of Dutch Windmill Graph, *Open Journal of Discrete Mathematics*, 6 (2016), 74-81.
- [14] P.S. Ranjini, V. Loksha and I.N. Cangul, On the Zagreb indices of the line graphs of the subdivision graphs, *Appl. Math. Compt.* 218 (2011), 699-702.

- [15] V.S. Shegehalli and R. Kanabur, Computation of new degree-based topological indices of graphene, *J. Math.* 5 (2016), Article Id 4341919, 6 pages.
- [16] G.H. Shirdel, H. Rezapour and A.M Sayadi, The Hyper Zagreb index of graph operations, *Iran. J. Math. Chem.*, 4(2) (2013), 213-220.
- [17] B. Shwetha Shetty, V. Lokesha, P.S. Ranjini and K.C. Das, Computing some topological indices of smart polymer, *Digest J. Nanomaterials and Biostructures*, 7(3) (2012), 1097-1102.
- [18] M. Togan, A. Yurttas, A.S. Cevik, I.N. Cangul, Zagreb indices and multiplicative Zagreb indices of double graphs of subdivision graphs, *TWMS J. App. Eng. Math.* Vol 9(2) (2019), 404-412.
- [19] M. Veylaki, M.J. Nikmehr and H.A. Tavallaee, The third and Hyper-Zagreb coindices of some graph operations, *J. Appl. Math. Compt.* 50(1-2) (2016), 315-325.
- [20] D. Vukicevic and B. Furtula, Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, *J. Math. Chem.* 4 (2009), 1369-1376.
- [21] D. Vukicevic, M. Gasperov, Bond Additive Modeling 1. Adriatic Indices, *Croat. Chem. Acta*, 83(3) (2010), 243-260.