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AN APPROXIMATION SCHEME FOR THE NUMERICAL SOLUTION OF HIV INFECTION OF CD4$^+$ T-CELLS USING CHEBYSHEV WAVELETS

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Abstract. In this paper, the Chebyshev wavelets method for solving a model for HIV infection of CD4$^+$ T-cells is studied. The properties of Chebyshev wavelets and their operational matrices are first presented and then are used to convert into algebraic equations. Also the convergence and error analysis for the proposed technique is discussed. Illustrative examples are given to demonstrate the validity and applicability of the technique. The efficiency of the proposed method is compared with other traditional methods and it is observed that the Chebyshev wavelet method is more convenient than the other methods in terms of applicability, efficiency, accuracy, error and computational effort.

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Keywords. Chebyshev wavelets; HIV Infection; CD4$^+$T- cells; Convergence analysis; Operational matrix.

1. Introduction

Many nonlinear mathematical models have been developed to describe infection by the human immunodeficiency virus (HIV). In 1989, a model for the infection of the human immune system by HIV was developed by Perelson [19]. This model of virus spread has three variables: the population sizes of uninfected cells, infected cells, and free virus particles. Perelson et al. [20] extended the model described in [19] and developed a new model by considering four variables: cells that are uninfected, cells that are latently infected, cells that are actively infected, free virus particles. Their model is described by a system of four ordinary differential equations. It was noted that the model can replicate many of the symptoms of AIDS observed clinically. Culshaw and Ruan [4] reduced the model described in [20] to a system of three ordinary differential equations by assuming that all the infected cells are capable of producing the virus.

In this paper, Chebyshev wavelets method (CWM) is applied to the dynamics of a model for HIV infection of CD4$^+$T cells [4]. This model is characterized by a system of nonlinear differential equations. The components of the basic three component model are the concentration of susceptible CD4$^+$T cells infected by the HIV viruses and free HIV virus particles in the blood are denoted, respectively, by T(t),I(t)and V(t). CD4$^+$T cells are also named as leukocytes or T helper cells. These with order cells in human immunity systems fight against diseases. HIV use cells in order to propagate. The number of CD4$^+$T cells in a healthy person is 800/1200 mm^3. These quantities satisfy [29]

$$\frac{dT}{dt} = q - \alpha T + rT\left(1 - \frac{T+1}{T_{max}}\right) - kVT \tag{1.1}$$

$$\frac{dI}{dt} = kVT - \beta I \tag{1.2}$$

$$\frac{dV}{dt} = N\beta I - rV \tag{1.3}$$

with the initial conditions : T(0) = r_1, I(0)=r_2 and V(0)=r_3.

Throughout this paper, we take $q = 0.1, \alpha = 0.02$, $\beta = 0.3$, $r = 3$, $\nu = 2.4$, k = 0.0027, N=10, $T_{max} = 1500$. The logistic growth of the healthy $CD4^+T$ cells are now described by $\left(1 - \frac{T+1}{T_{max}}\right)$, and proliferation of infected $CD4^+T$ cells is neglected. The term kVT describes the incidence of HIV infection of healthy $CD4^+T$ cells, where k >0 is the infection rate. Each infected $CD4^+T$ cells is assumed to produce N virus particles during its lifetime, including any of its daughter cells. The body is believed to produce $CD4^+T$ cells from precursors in the bone marrow and thymus at a constant rate q. When stimulated by antigen or mitogen, T cells multiply through mitosis with a rate r. T_{max} is the maximum level of $CD4^+T$ cell concentration in a body [16, 20, 21, 27], α is the natural turnover rate of T cells and β is the per capita rate of disappearance of infected cells. $N\beta$ is the rate of production of virions by infected cells, where N is the average number of virus particles produced by an infected T cell and ν is the death rate of virus particles [2,13]. In a normal human body, the level of $CD4^+T$ cells in the peripheral blood is regulated at a level between 800 and 1200 mm^{-3}. $CD4^+T$ cells are most abundant white blood cells of the human immune system, which fight against diseases. HIV wreaks most havocilly these cells causing their decline and destruction, thus decreasing the resistance of the human immune system. The dynamic model has proved valuable in understanding the dynamics of HIV-1 infection. This type of model is available in [1, 5, 17, 25].

Many researchers applied different techniques for solving the modeling for HIV infection of $CD4^+T$ cells. Homotopy Perturbation Method (HPM) [14], Homotopy Perturbation Transform Method (HPTM) [12], Homotopy Analysis Method (HAM) [7], The Laplace Adomian decomposition method [18], Bessel collocation method [25], Multistep Laplace Adomian decomposition method [5], Variational Iteration method (VIM) based on pade approximants [15], Differential Transform Method (DTM) [24], Multistage Homotopy Perturbation Method [8] are the methods available in literature for solving a model for HIV infection of $CD4^+T$ cells. Especially, fractional order model for HIV infection of $CD4^+T$ cell was carried out by Gokdokgan et al. [9].The purpose of this work is to solve a model for HIV infection of $CD4^+T$ cells using Chebyshev wavelets using operational matrix of integration. Chebyshev wavelets are capable of converting the given system of differential equations into system of algebraic equations. Many researchers have employed the Chebyshev wavelets using operational matrix of integration and the details are available in [3,6,11,23,28].

The remaining part of the paper is organized as follows: In section 2, we describe the basic formulation of wavelets and Chebyshev wavelets required for our subsequent development. Section 3 is devoted to the Chebyshev wavelet solution for a model for HIV infection of $CD4^{+}$T-cells. Convergence analysis and the error bound for the proposed method have been discussed in section 4. In section 5, we demonstrate the accuracy of the proposed scheme by considering numerical examples. Concluding remarks are given in the final section.

2. Properties of Chebyshev Wavelets

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets [10] as:

$\psi_{a,b}(x) = |a|^{-\frac{1}{2}} \psi\left(\frac{x-b}{a}\right), a, b \in R, a \neq 0$

If we restrict the parameters 'a' and 'b' to discrete values as $a = a_0^{-k}$, $b = nb_0a_0^{-k}$, $a_0 > 1$, $b_0 > 0$ and n, k are positive integers, we have the following family of discrete wavelets: $\psi_{k,n}(x) = |a|^{-\frac{1}{2}} \psi\left((a_0)^k x - nb_0\right)$ where $\psi_{k,n}(t)$ forms an orthonormal basis.

Chebyshev wavelets $\psi_{n,m}(t) = \psi(k, \hat{n}, m, x)$ have four arguments: $\hat{n} = 2n-1, n = 1, 2, 3, ..., 2^{k-1}$, k can assume any positive integer, m is the order of Chebyshev polynomials of the first kind. They are defined on the interval $[0, 1)$ as

$$\psi_{n,m}(x) = \left\{ \begin{array}{cc} 2^{\frac{k}{2}}\tilde{T}_m\left(2^kx - 2n + 1\right) & , for \frac{n-1}{2^{k-1}} \leq x \leq \frac{n}{2^{k-1}} \\ 0 & , otherwise \end{array} \right\} \tag{2.1}$$

where $m = 0, 1, 2, ..., M-1, n = 1, 2, 3, ..., 2^{k-1}$. and

$$\tilde{T}_m(x) = \left\{ \begin{array}{c} \frac{1}{\sqrt{\pi}} \quad , m = 0 \\ \sqrt{\frac{2}{\pi}}T_m(x), m > 0 \end{array} \right\}.$$

$T_m(x)$ are the famous Chebyshev polynomials of the first kind of degree m, which are orthogonal with respect to the weight function $W(x) = \frac{1}{\sqrt{1-x^2}}$, on the interval [-1,1], and satisfy the following recursive formula:

$T_0(x) = 1, T_1(x) = x, T_{m+1}(x) = 2xT_m(x) - T_{m-1}(x)$, m=1,2,3, The set of Chebyshev wavelets is an orthogonal set with respect to the weight function

$W_n(x) = W(2^kx - 2n + 1)$.

2.1. **Function Approximation.**

A function $f(x)$ defined over [0,1) may be expanded as

$$f(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{nm}\psi_{nm}(x) \tag{2.2}$$

where $c_{nm} = \langle f(x), \psi_{nm}(x) \rangle$, in which $\langle ., . \rangle$ denotes the inner product.

If the infinite series in Eq.(2.2) is truncated, then Eq.(2.2) can be written as

$$f(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm}\psi_{nm}(x) = C^T\psi(x) \tag{2.3}$$

where C and $\psi(t)$ are $2^{k-1}M \times 1$ matrices given by

$$C = \left[c_{10}, c_{11}, \cdots, c_{1M-1}, c_{20}, c_{21}, \cdots, c_{2M-1}, \cdots, c_{2^{K-1}0}, \cdots, c_{2^{K-1}M-1}\right]^T$$

$$\psi(x) = \left[\psi_{10}(x), \cdots, \psi_{1M-1}(xt), \psi_{20}(x), \cdots, \psi_{2^{K-1}M-1}(x)\right]^T$$

The integration of the product of two Chebyshev wavelets vector functions with respect to the weight function $W_n(x)$, is derived as

$\int_0^1 W_n(x)\psi(x)\psi^T(x)\,dx = I$ where I is an identity matrix.

A function f(x,y) defined on [0,1] x [0,1] can be approximated as the following $f(x,y) \approx \psi^T(x)K\psi(y)$.

Here the entries of the matrix $K = [k_{i,j}]_{(2^{k-1}MX2^{k-1}M)}$ are obtained by

$k_{i,j} = \left(\psi_i(x), (f(x,y), \psi_j(y))_{W_n(y)}\right)_{W_n(x)}, i, j = 1, 2, \ldots, 2^{k-1}M.$

The integration of vector $\psi(x)$ is given by

$\int_0^T \psi(t')dt' = P\psi(x)$ where P is $\left(2^{k-1}\right)M \times 2^{k-1}M$ operational matrix of integration and is given in [3] as

$$P = \frac{1}{2^k}\begin{bmatrix} L & F & F & \cdots & F \\ O & L & F & \cdots & F \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ O & O & \cdots & O & L \end{bmatrix} \tag{2.4}$$

In Eq.(2.4) F and L are M x M matrices given by

$F = \begin{bmatrix} 2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$ and

$$L = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \ldots & 0 \\ \frac{\sqrt{2}}{\sqrt{4}} & 0 & \frac{1}{4} & 0 & 0 & \ldots & 0 \\ -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 & \frac{1}{6} & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\sqrt{2}}{2}(-1)^r\left(\frac{1}{r-2} - \frac{1}{r}\right) & \cdots & -\frac{1}{2(r-2)} & 0 & \frac{1}{2r} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \frac{\sqrt{2}}{2}(-1)^M\left(\frac{1}{M-2} - \frac{1}{M}\right) & 0 & 0 & 0 & -\frac{1}{2(M-2)} & \cdots & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \frac{-2\sqrt{2}}{3} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\sqrt{2}}{2}\left(\frac{1-(-1)^r}{r} - \frac{1-(-1)^{r-2}}{r-2}\right) & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\sqrt{2}}{2}\left(\frac{1-(-1)^M}{M} - \frac{1-(-1)^{M-2}}{M-2}\right) & 0 & \dots & 0 \end{bmatrix}$$

$$O = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

The property of the product of two Chebyshev wavelet vector functions are obtained by using
$\psi(x)\psi^T(x) \approx \bar{Y}\psi(x)$ where $\bar{Y}$ is a $\left(2^{k-1}\right)M \times 2^{k-1}M$ matrix, called the operational matrix of product.

3. Chebyshev wavelet solution for a model for HIV infection of $CD4^{+}T$-cells

The equations given in Eq. (1.1), Eq.(1.2) and Eq.(1.3) can be written in the following form

$$E(t)\dot{x}(t) = f(t, x(t), u(t)), x(0) = x_0 \tag{3.1}$$

where

$$f(t, x(t), u(t)) = H(t)x(t) + U(t) \tag{3.2}$$

Here $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\dot{x}(t) = \begin{bmatrix} \frac{dT}{dt} \\ \frac{dI}{dt} \\ \frac{dV}{dt} \end{bmatrix}$

$O = \begin{bmatrix} -\alpha - kV + \nu\left(1 - \frac{T+1}{T_{max}}\right) & 0 & 0 \\ kv & -\beta & 0 \\ 0 & N\beta & -\nu \end{bmatrix}$; $x(t) = \begin{bmatrix} T \\ I \\ V \end{bmatrix}$; $U(t) = \begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix}$.

Let $\dot{x}(t) = F\psi(t); E(t) = e\psi(t); H(t) = h\psi(t); U(t) = g\psi(t)$

This implies $x(t) = \int_0^t F\psi(t)dt$
$x(t) = FP\psi(t) + x_0$ [26]
Eq.(3.1) takes the form $E(t)\dot{x}(t) = H(t)x(t) + U(t)$, becomes
e $\psi(t)F\psi(t) = h\psi(t)(FP\psi(t) + x_0) + g\psi(t)$
$[e\psi(t)F - h\psi(t)FP] = [[hx_0, 0, 0, \cdots, 0] + g]$
$[e - hP]\psi F = G_1$ where $G_1 = [[hx_0, 0, 0, \cdots, 0] + g]$
The unknown matrix can be solved using Kronecker product as
$(\psi F)^T = \left[e \otimes I - h \otimes P^T\right]^{-1} G_1{}^T$
i.e $F^T\psi^T = \left[e \otimes I - h \otimes P^T\right]^{-1} G_1{}^T$

$$F^T = \left[e \otimes I - h \otimes P^T\right]^{-1} G_1{}^T (\psi^{-1})^T$$

where $F^T = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{2^{k-1}(M-1)} \end{bmatrix}$ and

$$G_1{}^T = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{2^{k-1}(M-1)} \end{bmatrix}$$

The Kronecker product $A \otimes P^T$ is defined as in [22].

4. Convergence Analysis

Theorem 4.1. [23]

A function f(x) defined on [0,1), is with bounded second derivative, say $|f''(x)| \leq B$ *can be expanded as an infinite sum of Chebyshev wavelets, and the series converges uniformly to the function* $f(x)$*, that is* $f(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{nm}\psi_{nm}(x)$ *where* $c_{nm} = \langle f(x), \psi_{nm}(x)\rangle$*, in which* $\langle ., . \rangle$ *denotes the inner product in* $L^2[0,1]$.

Theorem 4.2. *(Accuracy estimation)* [23]

Let f(x) be a continuous function defined on [0,1), with bounded second derivative $|f''(x)|$ *bounded by B, then we have the following accuracy estimation :*

$$\sigma < \frac{\sqrt{\pi}B}{8}\left(\sum_{n=2^k}^{\infty}\sum_{m=M}^{\infty}\frac{1}{(m-1)^4}\right)^{\frac{1}{2}}$$

where $\sigma = \left(\int_0^1 \left[f(x) - \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{nm}\psi_{nm}(x)\right]^2 w_n(x)\, dx\right)^{\frac{1}{2}}$

5. Numerical Application

In this section, the numerical simulation is performed. The initial conditions are given as $T(0) = r_1 = 0.1, I(0) = r_2 = 0$ and $V(0) = r_3 = 0.1; q = 0.1, \alpha = 0.02, \beta = 0.3, r = 3; \nu = 2.4, k = 0.0027, N = 10, T_{max} = 1500$.

Applying the Chebyshev wavelet method discussed in the previous section, we obtain the solutions for $T(t), I(t)$ and $V(t)$ which are as shown in figure 1. The results obtained are then compared with the other traditional methods which are available in literature.

In figure1, CWM solutions of $T(t), I(t)$ and $V(t)$ have been explored. Comparison of solutions for $T(t), I(t)$ and $V(t)$ using LADM, VIM, MVIM, DTM have been revealed in figures 2, 3 and 4 respectively. In figure 5, error for the solutions $T(t), I(t)$ and $V(t)$ has been elucidated.

In tables 1-3, the results of LADM, VIM, MVIM, DTM and CWM are shown. The CWM solutions are in good agreement with the other methods mentioned in the literature.

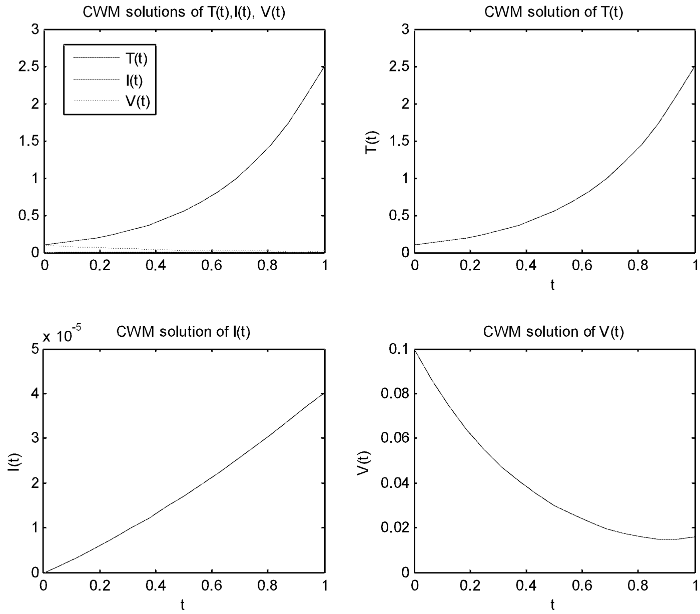

FIGURE 1. CWM solutions of T(t), I(t), V(t).

TABLE 1. Numerical comparison for T (t)

t	LADM [7]	VIM[18]	MVIM[29]	DTM[24]	CWM
0	0.1	0.1	0.1	0.1	0.1
0.2	0.2088072731	0.2088073214	0.2088080868	0.211648	0.2088073215
0.4	0.4061052625	0.4061346587	0.4062407949	0.422685	0.4061245634
0.6	0.7611467713	0.7624530350	0.764487245	0.817940	0.7641476415
0.8	1.3773198590	1.3978805880	1.4140941730	1.546211	1.3977746217
1.0	2.3291697610	2.5067466690	2.5919210760	2.854053	2.5571462314

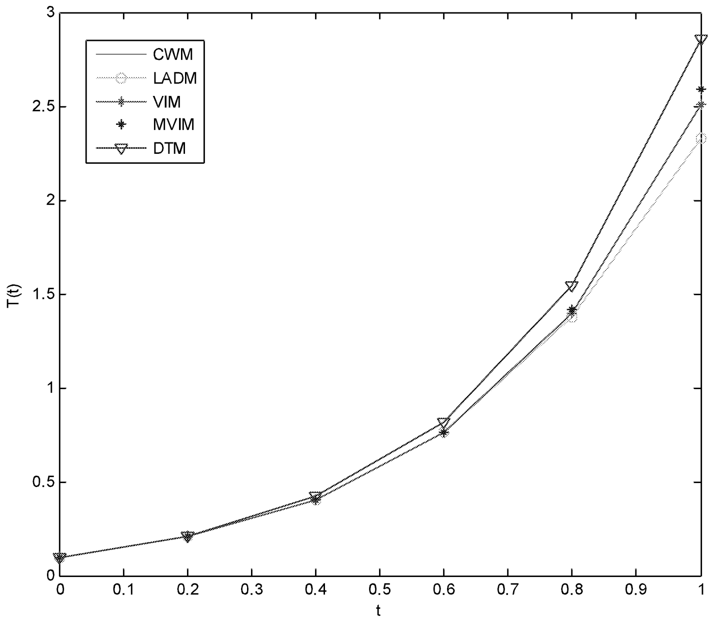

Figure 2. Comparison of solutions using LADM, VIM, MVIM, DTM for T(t)

Table 2. Numerical comparison for I (t)

t	LADM [7]	VIM[18]	MVIM[29]	DTM[24]	CWM
0	0.0	0.0	0.1e-13	0.0	0.0
0.2	0.603270728e-5	0.6032634661e-5	0.60327016510e-5	6.36664e-06	0.60327046634e-5
0.4	0.131591617e-4	0.1314878543e-4	0.13158301670e-4	1.39924e-05	0.1316784536e-4
0.6	0.212683688e-4	0.2101417193e-4	0.21223310013e-4	2.26514e-05	0.2112628765e-4
0.8	0.300691867e-4	0.2795130456e-4	0.30174509323e-4	3.32836e-05	0.2998139732e-4
1.0	0.398736542e-4	0.2431562317e-4	0.40025404050e-4	4.85399e-05	0.328765432e-4

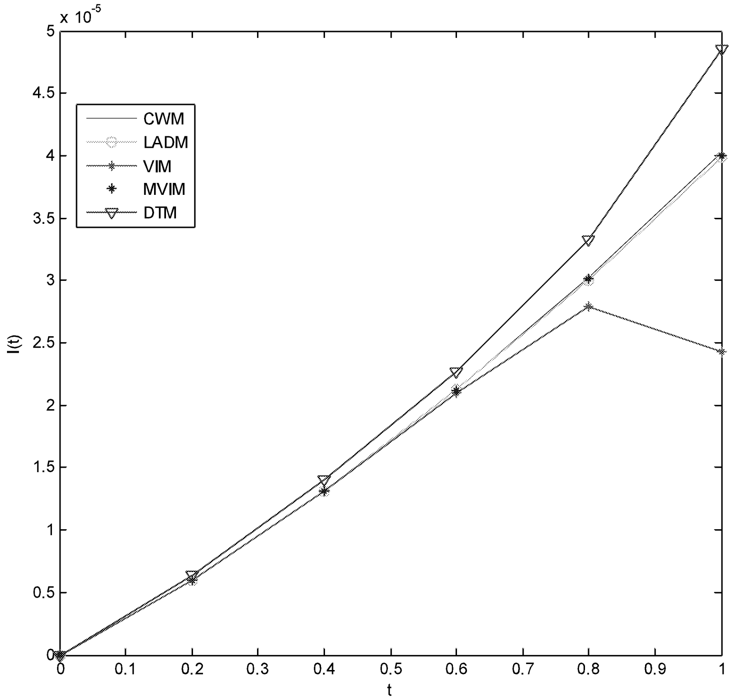

Figure 3. Comparison of solutions using LADM, VIM, MVIM, DTM for I(t)

Table 3. Numerical comparison for V(t)

t	LADM [7]	VIM[18]	MVIM[29]	DTM[24]	CWM
0	0.1	0.1	0.1	0.1	0.1
0.2	0.06187996025	0.06187995314	0.06187990876	0.061880	0.06187990765
0.4	0.03831324883	0.03830820126	0.03829595768	0.038309	0.03832341574
0.6	0.02439174349	0.02392029257	0.02371029480	0.023920	0.02381098734
0.8	0.009967218934	0.01621704553	0.01470041902	0.016212	0.01621389765

6. Conclusion

In this paper, Chebyshev wavelet method for the model for HIV infection of $CD4^{+}T$-cells has been studied. The properties of Chebyshev wavelets and its operational matrices are first presented and then are used to convert into algebraic equations. Also the convergence and error analysis for the proposed technique have been discussed. Illustrative examples have been given to demonstrate the validity and applicability of the technique. The efficiency of the proposed method has been

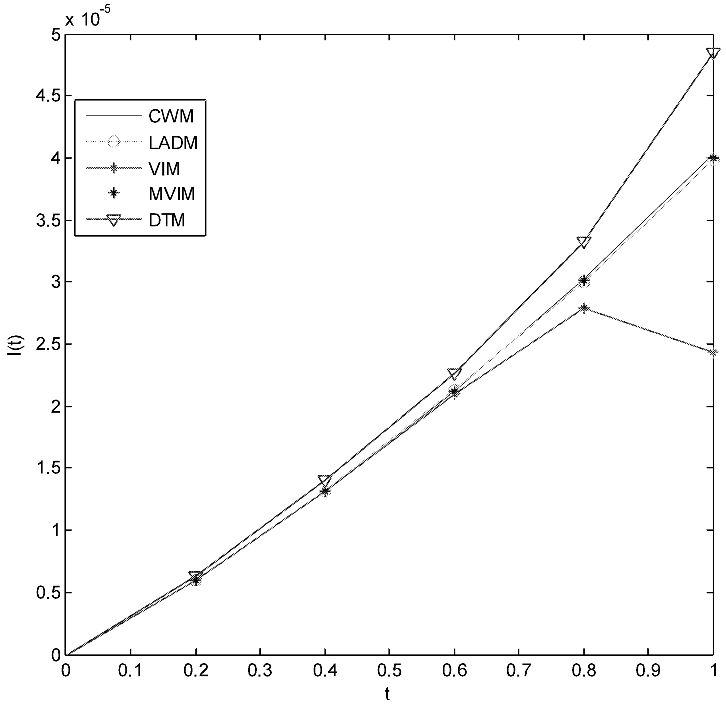

FIGURE 4. Comparison of solutions using LADM, VIM, MVIM, DTM for V(t)

compared with other methods given in literature and it is observed that the Chebyshev wavelet method is more convenient than the mentioned methods in terms of applicability, efficiency, accuracy, error, and computational effort.

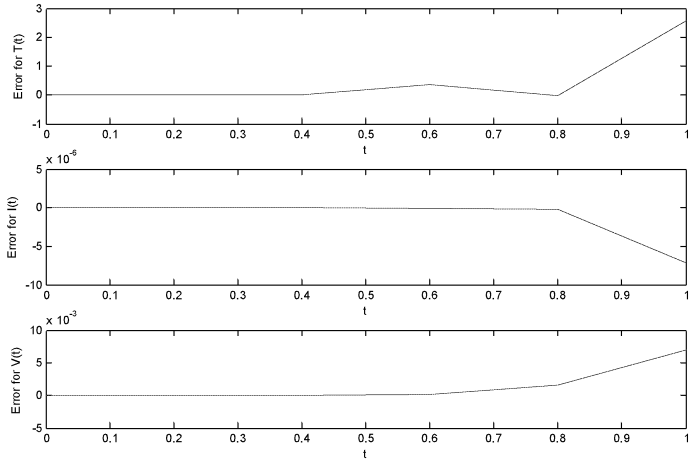

FIGURE 5. Error for T(t), I(t) and V(t)

REFERENCES

[1] A.A.M. Arafa, S.Z Rida, and M.Khalil, Fractional modeling dynamics of HIV and CD4+ T-cells during primary infection, Nonlinear Biomed Phys. 36 (2012).

[2] B.Asquith, and C.R.M. Bangham, The dynamics of T-cell fratricide: application of a robust approach to mathematical modeling in immunology. J Theoret Biol. 222 (2003) 53-69.

[3] J. Biazar, and H. Ebrahimi, Chebyshev wavelets approach for nonlinear systems of Volterra integral equations, Comput. Math. Appl. 63 (2012) 608-616.

[4] R.V. Culshaw, and S. Ruan, A delay-differential equation model of HIV infection of CD4+ T-cells, Math. Biosci. 165 (2000) 27-39.

[5] N. Dogan, Numerical treatment of the model for HIV infection of CD+4T cells by using multistep Laplace Adomian Decomposition Method, Discrete Dynamics in Nature and Society (2012), Article ID 976352.

[6] . M. Ghasemia, and M. Tavassoli Kajani, Numerical solution of time-varying delay systems by Chebyshev wavelets, App Math Model. 35 (2011) 5235-5244.

[7] M. Ghoreishi , A.I.B.Md. Ismail , and A.K. Alomari, Application of the Homotopy analysis method for solving a model for HIV infection of CD4+ T- cells, Math Comput. Model. 54 (2011) 3007-3015.

[8] A. Gokdogan, and M. Merdan. A Multistage Homotopy perturbation method for solving Human T- cell Lymphotropic Virus I (HTLV-I) Infection of CD4+ T-cells model, Middle East Jornal of Scientific Research, 9(4) (2011) 503-509.

[9] A. Gokdogan, A. Yildirimb, and M. Merdan, Solving a fractional order model of HIV infection of CD4+ T cells, Math Comput. Model. 54 (2011) 21322138.

[10] J.S.Gu, and W.S.Jiang, The Haar wavelets operational matrix of integration, Int.J.Syst.Sci. 27 (1996) 623-628.

[11] S. Gh. Hosseini, and F. Mohammadi, A New Operational Matrix of Derivative for Chebyshev wavelets and its applications in solving ordinary differential equations with non analytic solution, Applied Mathematical Sciences 5 (2011) 2537 - 2548.

[12] Y. Khan , Hector Vazquez-Leal, and Q. Wu. An efcient iterated method for mathematical biology model, Neural Comput and Applic. 23 (2013) 677-682.
[13] W. Liancheng, and Y.L. Michael, Mathematical analysis of the global dynamics of a model for HIV infection of CD4+ T cells. Math Biosci. 200 (2006) 44-57.
[14] M. Merdan. Homotopy perturbation method for solving a Model for HIV infection of CD+4T cells, Istanb. Commerce Uni. J. Sci. 6 (2007) 39-52.
[15] M. Merdan, A. Gokdogan, and A. Yildirim, On the numerical solution of the model for HIV infection of CD4+ T cells, Comput. Math. Appl., 62 (2011) 118-123.
[16] M. Nowak, and R. May, Mathematical biology of HIV infections: Antigenic variation and diversity threshold, Math. Biosci. 106 (1991) 1-21.
[17] M.Y.Ongun, The Laplace Adomian Decomposition Method for solving a model for HIV infection of CD4+T cells, Math Comput. Model. 53 (2011) 597-603.
[18] M.Y. Ongun, The Laplace Adomian Decomposition Method for solving a model for HIV infection of CD4+T cells, Math Comput. Model., 53 (2011) 597-603.
[19] A.S. Perelson,Modelling the interaction of the immune system with HIV, in: C. Castillo-Chavez (Ed.), Mathematical and Statistical Approaches to AIDS Epidemiology, Springer, Berlin, (1989) 350-370.
[20] A.S. Perelson, D.E. Kirschner, R. De Boer, Dynamics of HIV infection of CD4+ T-cells, Mathematical Biosciences, 114 (1993) 81-125.
[21] A.S. Perelson, and P.W. Nelson, Mathematical analysis of HIV-I Dynamics in Vivo, SIAM Rev. 41 (1999) 3-44.
[22] P.A.Regalia , and S.K. Mitra, Kronecker product, Unitary matrices and signal Processing applications, SIAM , 31(1989) 586-613.
[23] S.Sohrabi, Comparison Chebyshev wavelets method with BPFs method for solving Abels integral equation, Ain Shams Eng J 2 (2011) 249-254.
[24] V. K. Srivastava , M.K. Awasthi, Sunil Kumar. Numerical approximation for HIV infection of CD4+ T cells mathematical model, Ain Shams Eng J 5(2) (2014) 625-629.
[25] Y.Suayip, A numerical approach to solve the model for HIV infection of CD4+T cells, App Math Model. 36(12) (2012) 5876-5890.
[26] S.G.Venkatesh, S.K.Ayyaswamy, S.Raja Balachandar, and K.Kannan , Legendre wavelet method for the singular system of transistor circuits, International journal of applied engineering research, 9 (2014) 213-221.
[27] L. Wang, and M.Y. Li, Mathematical analysis of the global dynamics of a model for HIV infection of CD4+ T cells, Math. Biosci. 200 (2006) 44-57.
[28] C. Yang, Chebyshev polynomial solution of nonlinear integral equations, Journal of the Franklin Institute 349 (2012) 947-956.
[29] S.Yzbas, A numerical approach to solve the model for HIV infection of CD4+ T cells, App Math Model. 36 (2012) 5876-5890.

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