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Reliability analysis of acyclic transmission network based on minimal cuts using copula in repair

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Abstract: In the present paper, we have considered acyclic transmission network in which number of nodes are capable of receiving or sending a signal to the target nodes. To model the proposed acyclic transmission network, the present study combined the concepts of Markov process and minimal cuts incorporating copula to find the various reliability measures. The considered network can have four possible states namely operable, partial failure, critical failure and complete failure. The considered network can be repaired in two different ways. When the network is in critical state it is repaired with general repair whereas in complete failure state repaired with the help of two different repair rates namely general and exponential which has been incorporated with the application of Gumbel-Hougaard family of copula. Various reliability characteristics such as transition state probabilities, asymptotic behavior, reliability, mean time to failure and sensitivity of the proposed network have been evaluated with the help of minimal cuts coupling with Markov process using Gumbel-Hougaard copula, supplementary variable technique and Laplace transforms.

Keywords: Network reliability, Acyclic transmission network, Mean time to failure, Sensitivity, Supplementary variable techniques, Gumbel-Hougaard family of copula.

1. Introduction

Networks are the prominent part in many real-world systems such as computer architecture, data communications, software engineering, voice communications, transportations, oil and gas production and electrical power systems (Gertsbakh and Shpungin 2016). A network is a combination of nodes and links which is also known as vertices and edges respectively. A network model is defined by G= (V, E) in which 'V' and 'E' show nodes and edges set respectively. Examples of nodes (vertices) are railway stations, road intersections, mobile routers in mobile communication etc. and links (edges) are railways, roads, wireless paths etc. (Colbourn 1987).

Network Reliability is the probability of transferring the information/flows/messages from a source node to a sink node. Network reliability is an important concept in the planning, designing, manufacturing, and maintenance of controlling of networks. The network reliability evaluation problem occurs in a wide range of situations including telecommunications, interconnection networks, parallel processing networks and many others (Gertsbakh and Shpungin 2016). On the basis of connectivity, when all terminals are connected to each other i.e. if all nodes are terminal then it is called all-terminal connectivity. If there are only two terminals then the possibility to reach signal from one node to another one, that probably lead to source-terminal or *s-t* connectivity (Levitin 2005).Traditionally, network reliability considers binary state network in which both the components and system can possibly be in two states: completely working or totally failed. However, network reliability analysis in the context of multi-state network is based on performance level not on the connectivity level.

Basically, the network reliability problems are classified into two parts with respect to of flow viz. Binary state flow network (BFN)/multi-state flow network (MFN) and Multi-state node network (MNN). In BFN, the capacity of each arc is either 0 or 1 whereas, in MFN, the capacity of each arc can be a non-negative integer. The BFN/MFN satisfies the flow conservation law whereas MNN does not. These networks have their own utilization in many real-life problems, like electrical power distribution, transportation networks, cellular telephones and computer networks here it is suggested that refer to (Levitin 2005, Yeh 2012).

The considered acyclic transmission network (ATN) consists of number of nodes which are capable of receiving and sending a signal via different edges. The proposed network consists of a root node where the signal source is placed, a number of leaf nodes that can only receive a signal and a number of intermediate nodes (neither source node nor sink node) which are capable of transmitting the received signal to some other nodes. The whole network is in working condition if a signal from the root node is transmitted to the leaf nodes, otherwise, the network fails. An example of the acyclic transmission network is a radio relay station, where a transmitter located at the root node (position) and receivers are located in the terminal nodes (positions). The aim of this network is to propagate the signals from transmitters to the receivers (Gertsbakh and Shpungin 2016, Yeh 2006). (Levitin 2003) evaluated the reliability for acyclic transmission networks of multi-state elements with time delays from an algorithm based on extended universal generating function method. In this work it is assumed that the network fails if the signal generated at the source node cannot reach the terminal nodes within a specified time. (Levitin 2001) calculated the reliability of acyclic transmission networks with its multi-state elements from an algorithm based on universal generating function technique. A Multistate–node acyclic network (MNAN) was first investigated by Malinowski and Preuss in 1996. In this study researchers evaluated the reliability of multistate node of acyclic networks using minimal cuts (MC) based on an algorithm and some simple concepts. (Levitin 2001 and Malinowski and Preuss 1996) also discussed reliability evaluations for MNAN by one of the best-known algorithm and these algorithms are based on universal generating function (UGF) technique and branch and bound method. (Levitin 2005) discussed different algorithms and applied UGF method to find the reliability of binary and multi state systems.

Reliability analysis of the systems is traditionally done with the help of probability distributions. Usually a single distribution is used in failure/repair analysis. But if two different distributions are to be applied simultaneously in repair /failure, one can take help of copula. The copula is a function which joins or couples a multivariate distribution function to its one-dimensional marginal distribution functions. Copulas are multivariate distributions functions whose one-dimensional margins are uniform on the interval [0, 1].The copula approach is very natural when any system is repaired/failed by a couple of ways (Nelsen 2006). There are some important families of copulas with their different characteristics. The family of Archimedean copulas has been studied by numbers of authors. (Ram and Singh 2008) applied Gumbel-Hougaard family of copula and determined the availability, M.T.T.F and cost analysis of complex system under preemptive repeat repair discipline. (Nailwal and Singh 2016) calculated the reliability of cold standby redundant systems with preventive maintenance using Gumbel-Hougaard family of the copula. (Munjal and Singh 2014) considered the complex repairable system consisting of 2-out-of-3: G subsystems connected in parallel for finding the reliability characteristics and using Gumbel-Hougaard family of copula. (Nailwal and Singh 2011), the researchers evaluated the performance and reliability analysis of a complex system having three types of repairs with the application of copula. (Srinivasan and Subramanian 2006) the researchers considered standby systems with more than two units and these systems are studied only when either the lifetime or the repair time is exponentially distributed. (Kumar and Singh 2013) discussed the reliability analysis of a complex system having two repairable subsystems viz. A and B connected in series. This study also included a special type of delay i.e. reboot delay and used Gumbel-Hougaard family of copula to obtain various transition state probabilities, reliability, availability, MTTF, cost analysis and sensitivity analysis. (Nailwal and Singh 2012) investigated the reliability characteristics of a complex system having nine subsystems arranged in the form of a matrix in which each row contains three subsystems. The considered system analyzed the different types of power failures which also lead to failure of the system.

From above discussion, it is clear that many researchers analyzed the reliability of different networks by incorporating the probabilistic evaluation based on inclusion-exclusion, the sum of disjoint products methods and universal generating method (UGF). Researchers also analyzed reliability of acyclic transmission network by using UGF method and also discussed many algorithms based on different concepts like minimal cut, Dijkstra's and Kruskal's algorithm. Further, it is also clear from above discussion that reliability analysis of the acyclic transmission network using both Markov process and minimal cut yet to be studied.

Keeping above facts in view, here in the proposed work we tried to combine the concepts of Markov process and minimal cut to evaluate the reliability characteristics of the acyclic transmission network. In the considered network there are four different possible states, namely operable, partial failure, critical failure and complete failure. The network is said to be in partial failure state, if one of the edges fails but the signal is transmitted to both the sinks. But if any further failure occurs in the network and the signal flows to only one of the sinks then it is said to be in critical state. If there is no flow to any of the sinks then the network is in complete failure state. When the network is in the critical states, it is repaired by general distribution whereas when it is in complete failure state then the network is repaired by two different types of distributions namely general and exponential. The proposed acyclic transmission network has been studied to evaluate the reliability characteristics with the application of minimal cuts and Markov process incorporating Gumbel-Hougaard family of copula, supplementary variable technique and Laplace transforms. The reliability measures such as transition state probabilities, asymptotic behavior, MTTF and sensitivity of the network has been obtained. The considered acyclic transmission network and the transition diagram of the network are shown in Figures 1 and 2 respectively.

Assumptions:
(i) Initially, the acyclic transmission network is in the good state.
(ii) In the considered network there are four possible minimal cuts:
(a){1, 2}{1, 3}
(b){3, 5}{2, 4}
(c) {1, 2} {2, 3} {3, 5}
(d){1, 3}{2, 3}{2, 4}
(iii) The network has four possible states: good, degraded, critical and completely failed.
(iv) Network has three types of failure: partial, critical and complete failure.
(v) The network is repaired when it is in the critical and complete failure states.
(vi) Transitions from critical states S_6 and S_9 to initial state S_0 follow the general distribution.
(vii) Transitions from the completely failed states S_2, S_4, S_7 and S_{10} to initial state S_0 follow two different distributions incorporating Gumbel-Hougaard family of the copula.

Table 1: Descriptions of notations used in Transition Diagram

States	Descriptions
S_0	The state when all the edges of the network are in working condition.
S_1	The state when the edge {1, 3} fails and the network is in degraded state.
S_2	The state when the edges {1, 2} and {1, 3} in the network fail and network is in completely failed state.
S_3	The state when the edge {3, 5} fails and the network is in the degraded state.
S_4	The state when the edges {3, 5} and {2, 4} in the network fail and the network is in the completely failed state.
S_5	The state when the edge {1, 2} fails and the network is in the degraded state.
S_6	The state when the edges {1, 2} and {2, 3} in the network fail and the network is in the critical state.
S_7	The state when the edges {1, 2}, {2, 3} and {3, 5} in the network fail and the network is in the completely failed state.
S_8	The state when the edge {1, 3} fails then the network is in the degraded state.
S_9	The state when the edges {1, 3} and {2, 3} in the network fail and the network is in the critical state.
S_{10}	The state when the edges {1, 3}, {2, 3} and {2, 4} in the network fail and the network is in the completely failed state.
$\lambda_{12} / \lambda_{13} / \lambda_{24} / \lambda_{35} / \lambda_{23}$	The different failure rates corresponding to their edges in the network using possible minimal cuts.

$P_i(t)$	The probability that the network is in S_i state at instant time t, i=1 to 10.
$\overline{P}_i(s)$	Laplace transform of $P_i(t)$.
$\phi_1(x)$	Repair rate in states S_6 and S_9 in the network with the elapsed repair time x .
$\phi_2(x)$	Repair rate in states S_2, S_4, S_7 and S_{10} in the network with the elapsed repair time x .
$\varphi(x)$	Coupled repair rate

Letting $u_1 = e^x$ and $u_2 = \phi_2(x)$ the expression for joint probability (completely failed state S_2, S_4, S_7 and S_{10} to good state S_0) according to Gumbel-Hougaard family of copula is given by

$$\varphi(x) = \exp\left[(x)^\theta + (\log\phi_2(x))^\theta\right]^{1/\theta}$$

Figure 1: Acyclic Transmission Network

Figure 2: Transition diagram for the acyclic network

2. Solution of the Model

Taking Laplace transformation of equations (A.1.1)-(A.1.15) and using equation (A.1.16), we obtain the transition state probabilities of the proposed acyclic network as

$$\overline{P}_0(s) = \frac{1}{B(s)} \tag{2.1}$$

$$\overline{P}_1(s) = \frac{\lambda_{13}}{(s+\lambda_{12})} \frac{1}{B(s)} \tag{2.2}$$

$$\overline{P}_2(s) = \frac{\lambda_{13}\lambda_{12}}{(s+\lambda_{12})}\left[\frac{1-s\overline{\varphi(s)}}{s}\right].\frac{1}{B(s)} \tag{2.3}$$

$$\overline{P}_3(s) = \frac{\lambda_{35}}{(s+\lambda_{24})}.\frac{1}{B(s)} \tag{2.4}$$

$$\overline{P}_4(s) = \frac{\lambda_{24}\lambda_{35}}{(s+\lambda_{24})}\left[\frac{1-s\overline{\varphi(s)}}{s}\right].\frac{1}{B(s)} \tag{2.5}$$

$$\overline{P}_5(s) = \frac{\lambda_{12}}{(s+\lambda_{23})}.\frac{1}{B(s)} \tag{2.6}$$

$$\overline{P}_6(s) = \frac{\lambda_{25}}{(s+\lambda_{35})}.\frac{1}{B(s)} \tag{2.7}$$

$$\overline{P}_7(s) = \frac{\lambda_{35}\lambda_{12}\lambda_{25}}{(s+\lambda_{23})(s+\lambda_{35})}\left[\frac{1-\overline{s\varphi(s)}}{s}\right].\frac{1}{B(s)} \tag{2.8}$$

$$\overline{P}_8(s) = \frac{\lambda_{13}}{(s+\lambda_{23})}.\frac{1}{B(s)} \tag{2.9}$$

$$\overline{P}_9(s) = \frac{\lambda_{23}}{(s+\lambda_{24})}.\frac{1}{B(s)} \tag{2.10}$$

$$\overline{P}_{10}(s) = \frac{\lambda_{23}\lambda_{13}\lambda_{24}}{(s+\lambda_{24})(s+\lambda_{23})}\left[\frac{1-\overline{s\varphi(s)}}{s}\right].\frac{1}{B(s)} \tag{2.11}$$

where

$$B(s) = \left[(s+c_1) - \frac{\lambda_{12}\lambda_{13}\overline{s\varphi(s)}}{(s+\lambda_{12})} - \frac{\lambda_{24}\lambda_{35}\overline{s\varphi(s)}}{(s+\lambda_{24})} - \frac{\lambda_{25}\lambda_{35}\lambda_{12}\overline{s\varphi(s)}}{(s+\lambda_{23})(s+\lambda_{35})} - \frac{\lambda_{24}\lambda_{23}\lambda_{13}\overline{s\varphi(s)}}{(s+\lambda_{24})(s+\lambda_{23})}\right]$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any instance are as follows:

$$\overline{P}_{\text{up}}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_3(s) + \overline{P}_5(s) + \overline{P}_6(s) + \overline{P}_8(s) + \overline{P}_9(s) \tag{2.12}$$

$$\overline{P}_{\text{up}}(s) = \left[1 + \frac{\lambda_{13}}{(s+\lambda_{12})} + \frac{\lambda_{35}}{(s+\lambda_{24})} + \frac{\lambda_{12}}{(s+\lambda_{23})} + \frac{\lambda_{25}}{(s+\lambda_{35})} + \frac{\lambda_{13}}{(s+\lambda_{23})} + \frac{\lambda_{23}}{(s+\lambda_{24})}\right]\frac{1}{B(s)} \tag{2.13}$$

$$\overline{P}_{\text{down}}(s) = \overline{P}_2(s) + \overline{P}_4(s) + \overline{P}_7(s) + \overline{P}_{10}(s)$$

$$\overline{P}_{\text{down}}(s) = \frac{\lambda_{13}\lambda_{12}}{(s+\lambda_{12})}\left[\frac{1-\overline{s\varphi(s)}}{s}\right].\frac{1}{B(s)} + \frac{\lambda_{24}\lambda_{35}}{(s+\lambda_{24})}\left[\frac{1-\overline{s\varphi(s)}}{s}\right].\frac{1}{B(s)} + \frac{\lambda_{12}\lambda_{35}\lambda_{25}}{(s+\lambda_{23})(s+\lambda_{35})}$$
$$\left[\frac{1-\overline{s\varphi(s)}}{s}\right].\frac{1}{B(s)} + \frac{\lambda_{24}\lambda_{13}\lambda_{23}}{(s+\lambda_{23})(s+\lambda_{24})}.\left[\frac{1-\overline{s\varphi(s)}}{s}\right].\frac{1}{B(s)}$$

3. Asymptotic behavior of the Network

Using Abel's lemma in Laplace transformation

$\underset{s\to 0}{Lim}\{s\overline{F}(s)\} = \underset{t\to\infty}{Lim}F(t) = F$, provided the limit on right hand exists the following time independent up and down states probabilities are obtained:

$$P_{up} = \left[1 + \frac{\lambda_{13}}{\lambda_{12}} + \frac{\lambda_{35}}{\lambda_{24}} + \frac{\lambda_{12}}{\lambda_{23}} + \frac{\lambda_{25}}{\lambda_{35}} + \frac{\lambda_{13}}{\lambda_{23}} + \frac{\lambda_{23}}{\lambda_{24}}\right].\frac{1}{B(0)} \tag{3.1}$$

$$P_{down} = \lambda_{13}.\frac{1}{\varphi} + \lambda_{35}.\frac{1}{\varphi} + \frac{\lambda_{12}\lambda_{25}}{\lambda_{23}}.\frac{1}{\varphi} + \lambda_{13}.\frac{1}{\varphi} \tag{3.2}$$

where $B(0) = \lim_{s \to 0} B(s)$

Particular case: When repair follows exponential distribution. In this case the result can be

$$\overline{S_1}(s) = \frac{\phi_1(x)}{s + \phi_1(x)}\ ,\ \bar{S}_2(s) = \frac{\exp[x^\theta + \{\log\phi_2(x)\}^\theta]^{\frac{1}{\theta}}}{s + \exp[x^\theta + \{\log\phi_2(x)\}^\theta]^{\frac{1}{\theta}}} \tag{3.3}$$

4. Reliability analysis of the network

Let the failure rate of the network with different edges be $\lambda_{13} = 0.3, \lambda_{12} = 0.2, \lambda_{35} = 0.4,$ $\lambda_{24} = 0.1, \lambda_{23} = 0.15, \lambda_{25} = 0.5$, repair rates $\phi = 0, \theta = 1$ and $x = 1$. Putting these values in equation (2.13) and using equation (3.3) one can obtain Table 2. Also, if repair follows exponential distribution, then

$$\text{Reliability } (R) = P_{up}(t) = e^{1.2t} + 0.3e^{-0.2t} - 0.3e^{-1.2t} + 0.363636e^{-0.1t} - 0.363636e^{-1.2t} + 0.19047619e^{-0.15t}$$
$$- 0.19047619e^{-1.2t} + 0.2857142e^{-0.15t} - 0.2857142e^{-1.2t}$$

Table 2: Time vs. Reliability

Time(t)	Pup(t)
0	1
1	0.9424
2	0.8389
3	0.7338
4	0.6367
5	0.5555
6	0.4834
7	0.4212
8	0.3674
9	0.3209
10	0.2806

Figure 3: Time vs. Reliability

5. Mean time to failure of the network

Mean time to failure (MTTF) of the network is given by

$$M.T.T.F = \lim_{s \to 0} \overline{P_{up}}(s) \tag{5.1}$$

Assuming that the repair follows exponential distribution, i.e.

$$\overline{S_1}(s)=\frac{\phi_1(x)}{s+\phi_1(x)},\ \overline{S}_2(s)=\frac{\exp[x^\theta+\{\log\phi_2(x)\}^\theta]^{\frac{1}{\theta}}}{s+\exp[x^\theta+\{\log\phi_2(x)\}^\theta]^{\frac{1}{\theta}}}$$

Now, let the repair rate $\phi=0,\theta=1$ and $x=1$, then

(i). Setting $\lambda_{13}=0.3,\lambda_{35}=0.4,\lambda_{24}=0.1,\lambda_{23}=0.15,\lambda_{25}=0.5$, and varying λ_{12} as 0.01, 0.02, 0.03, 0.04, 0.05, one can obtain variation of MTTF with respect to λ_{12} from equation (5.1), values are given in Table 3.

(ii). Assuming $\lambda_{12}=0.2,\lambda_{35}=0.4,\lambda_{24}=0.1,\lambda_{23}=0.15,\lambda_{25}=0.5$, and changing λ_{13} as 0.01, 0.02, 0.03, 0.04, 0.05 can see variation of MTTF with respect to λ_{13} from equation (5.1), values are given in Table 3.

(iii). Letting $\lambda_{13}=0.3,\lambda_{12}=0.2,\lambda_{35}=0.4,\lambda_{23}=0.15,\lambda_{25}=0.5$, and altering λ_{24}, as 0.01, 0.02, 0.03, 0.04, 0.05 one can determine variation of MTTF with respect to λ_{24} from equation (5.1), values are given in Table 3.

(iv). Assuming $\lambda_{13}=0.3,\lambda_{12}=0.2,\lambda_{24}=0.1,\lambda_{23}=0.15,\lambda_{25}=0.5$, and changing λ_{35}, as 0.01, 0.02, 0.03, 0.04, 0.05, the variation of MTTF with respect to λ_{35} can be obtained from equation (5.1), values are given in Table 3.

(v). Setting $\lambda_{13}=0.3,\lambda_{12}=0.2,\lambda_{35}=0.4,\lambda_{24}=0.1,\lambda_{25}=0.5$, and varying λ_{23}, as 0.01, 0.02, 0.03, 0.04, 0.05 one can obtain variation of MTTF with respect to λ_{23} from equation (5.1), values are given in Table 3.

The variation obtained in MTTF with respect to different failure rates is shown in Figure 4.

Table 3: Variation in MTTF w.r.t. Failure rates

Failure rate	**MTTF w.r.t λ_{12}**	**MTTF w.r.t λ_{13}**	**MTTF w.r.t λ_{24}**	**MTTF w.r.t λ_{35}**	**MTTF w.r.t λ_{23}**
0.01	8.4887	10.4032	38.1943	7.3246	47.0814
0.02	8.46404	10.2604	21.5277	7.3576	26.249
0.03	8.4466	10.1262	15.9721	7.3856	23.1666
0.04	8.42948	9.9999	13.1941	7.4202	15.8327
0.05	8.41269	9.8809	11.5278	7.4509	13.7495

Figure 4: Failure rate vs. MTTF

6. Sensitivity of the network

Sensitivity of the network reliability with respect to parameters ξ can be obtained by

$$S_{\xi} = \frac{\partial R}{\partial \xi}\text{, where } \xi = \lambda_{12}, \lambda_{13}, \lambda_{24}, \lambda_{35} \text{ and } \lambda_{23} \tag{6.1}$$

The computed sensitivities with respect to different parameters are listed in Table 4. The same is also shown in Figure 5.

Table 4: Sensitivities w.r.t. different parameters

Time	$S_{\lambda_{12}}$	$S_{\lambda_{13}}$	$S_{\lambda_{35}}$	$S_{\lambda_{23}}$	$S_{\lambda_{24}}$
0	0	0	0	0	0
1	-5.41994	2.946685	-1.82551	-0.33854	-0.29901
2	-23.5437	11.96128	-10.6537	-0.30977	-0.23265
3	-78.4001	39.73755	-38.5026	-0.17368	-0.03419
4	-235.319	119.7443	-118.641	-0.03886	0.188641
5	-667.298	340.4482	-339.479	0.05726	0.393291
6	-1824.47	932.0885	-931.24	0.106631	0.5662
7	-4862.35	2485.818	-2485.08	0.111935	0.705562
8	-12715.2	6502.973	-6502.32	0.078946	0.813897
9	-32767.9	16762.53	-16762	0.013617	0.895022
10	-83469.5	42705.8	-42705.3	-0.0789	0.952881

Figure 5: Sensitivity vs. Time

7. Conclusion

In the present study, various reliability measures have been computed for the acyclic transmission network with the help of minimal cuts and Markov process incorporating different types of failure. In this model various reliability measures like transition probabilities, asymptotic behavior, reliability, MTTF and sensitivities with respect to different parameters have been obtained with the help of the proposed method unlike done in the past.

Figure 3 provides the variation of reliability with respect to time. By observing the figure one can visualize that it decreases from its initial stage with respect to time.

From Figure 4, we can easily visualize that MTTF with respect to λ_{35}, λ_{13} and λ_{12} are continuously and slowly decreasing though, the variation is found to be decreasing linearly. Further, it is observed that the MTTF with respect to λ_{24} is decreasing exponentially whereas it is decreasing continuously with the different rates. The highest and the lowest value of MTTF have been found with respect to failure rates of edges λ_{23} and λ_{35} respectively.

Figure 5 shows the sensitivity of the proposed network with respect to different parameters. Critical examination of the figure reveals that sensitivity of system corresponding to the parameters λ_{12} and λ_{35} is decreasing with respect to time whereas it is increasing corresponding to parameter λ_{13}. Moreover, it is observed that sensitivity of the system corresponding to the parameters λ_{23} and λ_{24} firstly decreases then slowly increases with respect to time. The system is found to be most sensitive with respect to parameter λ_{13}.

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Appendix A.1

Formulation of Mathematical model

By probability considerations and continuity arguments we can obtain the following set of difference-differential equations governing the present mathematical model

$$\left[\frac{d}{dt}+\lambda_{13}+\lambda_{35}+\lambda_{12}+\lambda_{13}\right]P_0(t)=\int_0^\infty \varphi(x)P_2(x,t)dx+\int_0^\infty \varphi(x)P_4(x,t)dx+\int_0^\infty \varphi(x)P_7(x,t)dx$$
$$+\int_0^\infty \varphi(x)P_{10}(x,t)dx \tag{A.1.1}$$

$$\left[\frac{d}{dt}+\lambda_{12}\right]P_1(t)=\lambda_{13}P_0(t) \tag{A.1.2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)\right]P_2(x,t)=0 \tag{A.1.3}$$

$$\left[\frac{d}{dt}+\lambda_{24}\right]P_3(t)=\lambda_{35}P_0(t) \tag{A.1.4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)\right]P_4(x,t)=0 \tag{A.1.5}$$

$$\left[\frac{d}{dt}+\lambda_{23}\right]P_5(t)=\lambda_{12}P_0(t) \tag{A.1.6}$$

$$\left[\frac{d}{dt}+\lambda_{35}\right]P_6(t)=\lambda_{23}P_0(t) \tag{A.1.7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)\right]P_7(x,t)=0 \tag{A.1.8}$$

$$\left[\frac{d}{dt}+\lambda_{23}\right]P_8(t)=\lambda_{13}P_0(t) \tag{A.1.9}$$

$$\left[\frac{d}{dt}+\lambda_{24}\right]P_9(t)=\lambda_{23}P_0(t) \tag{A.1.10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)\right]P_{10}(x,t)=0 \tag{A.1.11}$$

Boundary conditions

$$P_2(0,t)=\lambda_{12}P_1(t) \tag{A.1.12}$$

$$P_4(0,t)=\lambda_{24}P_3(t) \tag{A.1.13}$$

$$P_7(0,t)=\lambda_{35}P_6(t) \tag{A.1.14}$$

$$P_{10}(0,t)=\lambda_{24}P_9(t) \tag{A.1.15}$$

Initial conditions

$$P_i(0)=\begin{cases}1 & if\ i=0\\ 0 & i>1\end{cases} \tag{A.1.16}$$