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ON SIZE MULTIPARTITE RAMSEY NUMBERS OF $mK_{1,n}$ VERSUS P_3 OR $K_{1,3}$

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Abstract. For given two simple graphs G and H, the size multipartite Ramsey number $m_j(G,H)$, is the smallest natural number t such that every 2−edge coloring on the edges of the complete balanced multipartite graph $K_{j\times t}$ has a monochromatic copy of G in the first color or H in the second color. Hattingh and Henning (1998) gave the results for the size bipartite Ramsey numbers of stars versus paths, $m_2(K_{1,m},P_n)$, for $m,n \geq 2$. In 2016, we have derived the size tripartite Ramsey numbers $m_3(mK_{1,n},P_3)$, for $m \geq 1, n \geq 2$, where $mK_{1,n}$ is a disjoint union of m copies of a star $K_{1,n}$ and P_3 is a path of order 3. In this paper, we determine the size multipartite Ramsey numbers $m_j(mK_{1,n},P_3)$ and $m_j(mK_{1,n},K_{1,3})$, for all integers $m,n \geq 2$ and $j \geq 3$. We also provide an exact value of $m_2(mK_{1,n},P_3)$, for $m,n \geq 2$ and $m_3(mK_{1,n},K_{1,3})$, for $(m=1,n\geq 1)$ or $(n=1,m\geq 1)$.

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1. Introduction

For two simple graphs G and H, we say that $F \longrightarrow (G,H)$, if for any red-blue coloring of the edges of a graph F, we always have a red subgraph G or a blue subgraph H. The classical Ramsey number $r(m,n)$ is defined as the smallest natural number p such that $K_p \longrightarrow (K_m,K_n)$. Burger and Vuuren (2004) have generalised this definition as follows. Let j,l,n,s and t be natural numbers with $n,s \geq 2$. Then, the size multipartite Ramsey number $m_j(K_{n\times l},K_{s\times t})$ is the smallest natural number ζ such that $K_{j\times\zeta} \longrightarrow (K_{n\times l},K_{s\times t})$ [1]. They also determined the exact values of $m_j(K_{2\times 2},K_{3\times 1})$, for $j \geq 2$. For the bounds of the size multipartite Ramsey numbers, they gave a general lower bound, a probabilistic lower bound, and a diagonal bipartite upper bound for the size multipartite Ramsey numbers. Burger and Vuuren (2004) have established the following growth property and the asymptotic limit for the size multipartite Ramsey numbers, that will be used in this paper.

Proposition 1.1. [1] *Let $a,c \geq 2$ and j,k,b,d be natural numbers. Then*

$$m_j(K_{a\times b},K_{c\times d}) \leq m_k(K_{a\times b},K_{c\times d}) \text{ if } k \leq j.$$

Theorem 1.2. [1] *$m_j(K_{n\times l},K_{s\times t}) \longrightarrow 1$ as $j \longrightarrow \infty$ for any $n,s \geq 2$ and $l,t \geq 1$.*

Syafrizal et al. [8] generalized this concept by removing the completeness requirement. So, the size multipartite Ramsey number, $m_j(G,H)$, is defined as the smallest positive

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integers ζ such that $K_{j\times\zeta} \longrightarrow (G,H)$. If $j = 2$ then we have the bipartite Ramsey number $m_2(G,H)$. In 1998, Hattingh and Henning [4] gave the results for the size bipartite Ramsey numbers of stars versus paths, $m_2(K_{1,m}, P_n)$, for $m, n \geq 2$. For $j \geq 3$, Syafrizal et al. determined the size multipartite Ramsey numbers for P_3 versus stars [9]. Then, Surahmat et al. [7] gave the size tripartite Ramsey numbers for paths P_n versus stars, where $3 \leq n \leq 6$. Furthermore, we gave the size multipartite Ramsey numbers for stars versus cycles [5] and the size tripartite Ramsey numbers for a disjoint union of m copies of a star $K_{1,n}$ versus P_3 [6], which can be seen in Theorem 1.3.

Theorem 1.3. [6] *For positive integers $m \geq 2$ and $n \geq 3$, we have that* $m_3(mK_{1,n}, P_3) = \lceil \frac{m(n+1)}{3} \rceil$.

Note that, if we consider in [6], for $n \geq 3$ and $m = 3k + p$ where $p \in \{0, 1, 2\}$ and positive integers k, it is proved that $m_3(mK_{1,n}, P_3) = k(n + 1) + \lceil \frac{p(n+1)}{3} \rceil$. By easy calculation, we can have that $k(n + 1) + \lceil \frac{p(n+1)}{3} \rceil = \lceil \frac{m(n+1)}{3} \rceil$.

Now, we consider $j \geq 2$. In this paper, we use the generalizing concept of the size multipartite Ramsey number of G and $H, m_j(G,H)$, for $j \geq 2$. We determine the size multipartite Ramsey numbers $m_j(mK_{1,n}, H)$, where H is P_3 or $K_{1,3}$, for all integers $m, n \geq 2, j \geq 2$. P_3 is a path of order 3, $K_{1,3}$ is a star of order 4 and $mK_{1,n}$ is a disjoint union of m copies of a star $K_{1,n}$. For additions, we determine the size tripartite Ramsey numbers $m_3(mK_{1,n}, K_{1,n})$, for $m = 1, n \geq 1$ and $n = 1, m \geq 1$.

We call some basic definitions will be used to show the results, as follows. Let G be a finite and simple graph. Let vertex and edge sets of graph G are denoted by $V(G)$ and $E(G)$, respectively. A *matching* of a graph G is defined as a set of edges without a common vertex. Let $e = uv$ be an edge in G, then u is called *adjacent* to v. The *neighborhood* $N(v)$ of a vertex v is the set of vertices adjacent to v in G. The degree $d(v)$ of a vertex v is $|N(v)|$. The *maximum degree* of G is denoted by $\Delta(G)$, where $\Delta(G) = max\{d(v)|v \in V(G)\}$. The *minimum degree* of G is denoted by $\delta(G)$, where $\delta(G) = min\{d(v)|v \in V(G)\}$. A star $K_{1,n}$ is the graph on $n + 1$ vertices with one vertex of degree n, called the *center* of this star, and n vertices of degree 1, called the *leaves*. A factor of a graph G is a spanning subgraph of G. A graph G is said to be factorable into the factors $F_1, F_2, ..., F_k$ if these factors are (pairwise) edge-disjoint and $\bigcup_{i=1}^{k} E(F_i) = E(G)$. If G is factored into $F_1, F_2, ..., F_k$, then $\mathcal{F} = \{F_1, F_2, ..., F_k\}$ is called *a factorization* of G [2]. Any red-blue coloring of graph $K_{j\times t}$ is represented by any factorization $\{F_1, F_2\}$ of graph $K_{j\times t}$, where F_1 is the red graph and F_2 is the blue graph. A path in G that contains every vertex of G is called a *Hamiltonian path* of G, while a cycle in G that contains every vertex of G is called a *Hamiltonian cycle* of G. A graph that contains a Hamiltonian cycle is itself called *Hamiltonian*. In 1952, Dirac gave the following sufficient condition for a graph to be Hamiltonian.

Theorem 1.4. [3]
If G is a graph of order n and the minimum degree of $G, \delta(G) \geq \frac{n}{2}$, then G is a Hamiltonian.

2. Results

In this section, we determine for any graphs G and H the necessary and sufficient conditions for $m_j(G,H)=1$ as in Theorem 2.1 and the exact value of size multipartite Ramsey numbers of $mK_{1,n}$ versus P_3, for all integers $j,m,n\geq 2$ as in Theorem 2.2.

Theorem 2.1. *For any simple graph G and H, the size multipartite Ramsey number $m_j(G,H)=1$ if and only if $j\geq r(G,H)$.*

Proof. Let $m_j(G,H)=1$, there is the smallest number j such that $K_{j\times 1}\longrightarrow (G,H)$. We consider that $K_{j\times 1}\cong K_j$, then $j\geq r(G,H)$ since $K_j\longrightarrow (G,H)$. By Proposition 1.1, if $k\leq j$, then $m_j(G,H)\leq m_k(G,H)$. Then, the sequence $m_j(G,H)$ is non-increasing for increasing j. We will show that there is a natural number k such that $m_k(G,H)=1$. It is clear that $k=r(G,H)$, since any red-blue coloring of the edges of graph $K_k\cong K_{k\times 1}$ contains red G or blue H as a subgraph. □

Theorem 2.2. *For positive integers $m,n\geq 2$, we have*

$$m_j(mK_{1,n},P_3)=\begin{cases}\lceil\frac{m}{2}\rceil n+\lfloor\frac{m}{2}\rfloor; & \text{for } j=2,\\ \lceil\frac{m(n+1)}{j}\rceil; & \text{for } 3\leq j<r(mK_{1,n},P_3),\\ 1; & \text{for } j\geq r(mK_{1,n},P_3),\end{cases}$$

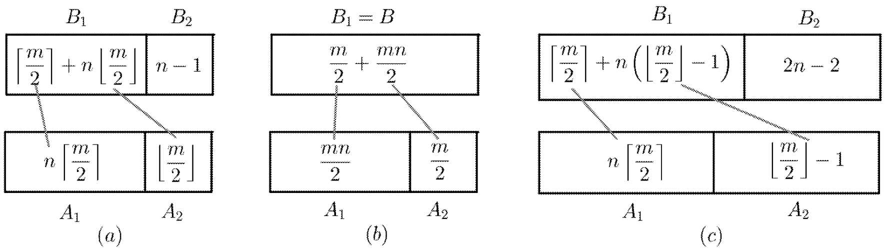


Figure 1. (a). Graph $K_{2\times t}$, if m is odd. (b). Graph $K_{2\times t}$, if m is even. (c). Graph $K_{2\times(t-1)}$, if m is odd.

Proof.
Case 1. For $j=2$. Let $t=\lceil\frac{m}{2}\rceil n+\lfloor\frac{m}{2}\rfloor$.

We consider any factorization $\{G_1,G_2\}$ of graph $K_{2\times t}$, such that $G_2\not\supseteq P_3$. So, $\Delta(G_2)\leq 1$ or G_2 is a matching. This implies that $t-1\leq\delta(G_1)\leq t$. Let A and B be two partite sets of graph $K_{2\times t}$. We will show that G_1 contains $mK_{1,n}$. Let $A_1=\{a\in A|a$ is a leaf of $mK_{1,n}\}$, $A_2=\{a\in A|a$ is a center of $mK_{1,n}\}$, $B_1=\{b\in B|b\in V(mK_{1,n})\}$ and $B_2=\{b\in B|b\notin V(mK_{1,n})\}$. We distinguish two cases as follows.

(1) m is odd.
Without lost of generality, there are $\lfloor\frac{m}{2}\rfloor$ centers of $mK_{1,n}$ in A and $\lceil\frac{m}{2}\rceil$ centers of $mK_{1,n}$ in B. Therefore, $|A_1|=n\lceil\frac{m}{2}\rceil$ and $|A_2|=\lfloor\frac{m}{2}\rfloor$. Note that $|A_1\cup A_2|=|A_1|+|A_2|=n\lceil\frac{m}{2}\rceil+\lfloor\frac{m}{2}\rfloor=t$. Since $|A_2|=\lfloor\frac{m}{2}\rfloor$, there are $n\lfloor\frac{m}{2}\rfloor$ leaves of $mK_{1,n}$ in B. See Figure 1.(a). So, $|B_1|=\lceil\frac{m}{2}\rceil+n\lfloor\frac{m}{2}\rfloor$. Since $t>|B_1|$, so we have G_1 contains $mK_{1,n}$.

(2) m is even.
There are $\frac{m}{2}$ centers of $mK_{1,n}$ in each set A and B. Therefore, $|A_1| = \frac{mn}{2}$ and $|A_2| = \frac{m}{2}$. Note that $|A_1 \cup A_2| = |A_1| + |A_2| = \frac{mn}{2} + \frac{m}{2} = t$. Since $|A_2| = \frac{m}{2}$, there are $\frac{mn}{2}$ leaves of $mK_{1,n}$ in B. See Figure 1.(b). So, $|B_1| = \frac{m}{2} + \frac{mn}{2}$ and $B_2 = \emptyset$. Since $t = |B_1| = |B|$, so we have G_1 contains $mK_{1,n}$.

Therefore, $m_2(mK_{1,n}, P_3) \leq t$.

To show that $m_2(mK_{1,n}, P_3) \geq t$, we consider a factorization $\{F_1, F_2\}$ of graph $K_{2\times(t-1)}$, such that F_2 is a maximal matching. If m is even, then $|V(K_{2\times(t-1)})| = 2(t-1) = 2(\lceil \frac{m}{2} \rceil n + \lfloor \frac{m}{2} \rfloor - 1) = mn + m - 2 = m(n+1) - 2 < |V(mK_{1,n})|$. Then, $F_1 \not\supseteq mK_{1,n}$. If m is odd, then without lost of generality, there are $\lfloor \frac{m}{2} \rfloor - 1$ centers of $mK_{1,n}$ in A and $\lceil \frac{m}{2} \rceil$ centers of $mK_{1,n}$ in B. Therefore, $|A_1| = n\lceil \frac{m}{2} \rceil$ and $|A_2| = \lfloor \frac{m}{2} \rfloor - 1$. Note that $|A_1 \cup A_2| = |A_1| + |A_2| = t-1$. Since $|A_2| = \lfloor \frac{m}{2} \rfloor - 1$, there are $n(\lfloor \frac{m}{2} \rfloor - 1)$ leaves of $mK_{1,n}$ in B. So, $|B_1| = \lceil \frac{m}{2} \rceil + n(\lfloor \frac{m}{2} \rfloor - 1) = (n+1)(\lfloor \frac{m}{2} \rfloor - 1) + 2$ and $|B_2| = 2n - 2$. See Figure 1.(c). We consider that the vertices in $A \cup B_1$ only form $(m-1)K_{1,n}$. So, F_1 does not contain $mK_{1,n}$. Therefore, $m_2(mK_{1,n}, P_3) \geq t$.

Case 2. For $3 \leq j < r(mK_{1,n}, P_3)$. Let $t = \lceil \frac{m(n+1)}{j} \rceil$.

For $j = 3, n \geq 3$ see Theorem 1.3. Otherwise, we consider a factorization $\{F_1, F_2\}$ of graph $K_{j\times(t-1)}$, such that F_2 is an empty graph. So, $F_1 = K_{j\times(t-1)}$. Since $|V(F_1)| = j(t-1) = j\lceil \frac{m(n+1)}{j} \rceil - j < m(n+1) = |V(mK_{1,n})|$. Then, $F_1 \not\supseteq mK_{1,n}$. Therefore, $K_{j\times(t-1)}$ doest not contain red $mK_{1,n}$ and blue P_3. Hence, $m_j(mK_{1,n}, P_3) \geq t$.

Now, we consider any factorization $\{G_1, G_2\}$ of graph $K_{j\times t}$, such that $G_2 \not\supseteq P_3$. So, G_2 is a matching. We will construct a disjoint union of m copies of star $K_{1,n}$ in G_1. We consider two following cases.

(1) For $|E(G_2)| \geq \lceil \frac{m}{2} \rceil$, we define a vertex set W that contains the end points of $\lceil \frac{m}{2} \rceil$ edges in G_2. We consider that if m is even, then $|W| = m$, otherwise $|W| = m+1$. For some $w \in W$, we define a vertex set A as follows:

$$A = \begin{cases} W; & \text{if } m \text{ is even,} \\ W - \{w\}; & \text{if } m \text{ is odd.} \end{cases}$$

(2) For $|E(G_2)| < \lceil \frac{m}{2} \rceil$, we define a vertex set A, where $|A| = m$ for any $A \subset V(G_2)$.

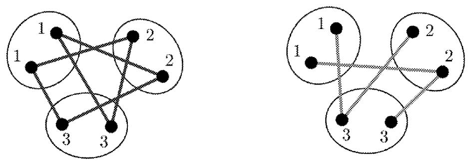
Based on these two cases, we have $|A| = m$. Every vertex in A must be connected with n vertices outside A by red edges. Therefore, we need mn vertices outside A to be leaves of m center points in A. The number of vertices outside A is $j\lceil \frac{m(n+1)}{j} \rceil + k \geq mn$, where $k = -2\lceil \frac{m}{2} \rceil + 1$, for m is odd and $k = -2\lceil \frac{m}{2} \rceil$, for m is even. Hence, there are $mK_{1,n}$ in G_1, where the center points are all of vertices in A. Therefore, $m_j(mK_{1,n}, P_3) \leq t$.

Case 3. For $j \geq r(mK_{1,n}, P_3)$.
This case is a direct consequence of Theorem 2.1. □

Theorem 2.3 is the size tripartite Ramsey numbers of combination graph $mK_{1,n}$ versus $K_{1,3}$, for $m = 1, n \geq 1$ and $n = 1, m \geq 1$.

Theorem 2.3. $m_3(mK_{1,n}, K_{1,3}) = \begin{cases} \lceil \frac{n}{2} \rceil + 1; & \textit{for } m = 1 \textit{ and } n \geq 1, \\ \lceil \frac{2m}{3} \rceil + 1; & \textit{for } m = 3 \textit{ and } n = 1, \\ \lceil \frac{2m}{3} \rceil; & \textit{for } m \geq 2, m \neq 3, \textit{ and } n = 1. \end{cases}$

Proof.
Case 1. For $m = 1$ and $n \geq 1$.
To show that $m_3(K_{1,n}, K_{1,3}) \geq \lceil \frac{n}{2} \rceil + 1$, consider a factorization $\{F_R, F_B\}$ of $K_{3\times\lceil \frac{n}{2} \rceil}$ such that F_B is a cycle with order $3\lceil \frac{n}{2} \rceil$. Then, F_B does not contain $K_{1,3}$. Since F_R is the complement of graph F_B relative to graph $K_{3\times\lceil \frac{n}{2} \rceil}$, then $d(x) = 2\lceil \frac{n}{2} \rceil - 2 < n$, for all $x \in V(F_R)$. This implies that $F_R \not\supseteq K_{1,n}$. See Figure 2, for $n = 3$.

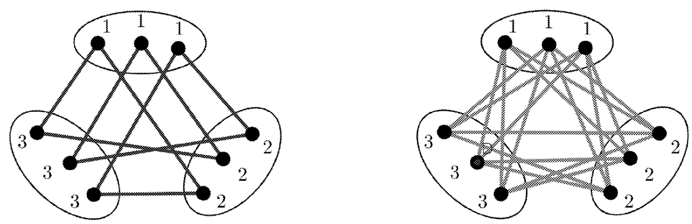


$F_B = C_6 = 1231231$ $F_R \not\supseteq K_{1,3}$ $F_R \not\supseteq 3K_{1,1}$

FIGURE 2. Two examples for $m_3(K_{1,3}, K_{1,3}) \geq 2$ and $m_3(3K_{1,1}, K_{1,3}) \geq 2$.

Now, we show that $m_3(K_{1,n}, K_{1,3}) \leq \lceil \frac{n}{2} \rceil + 1$. We consider any factorization $\{G_R, G_B\}$ of $K_{3\times(\lceil \frac{n}{2} \rceil+1)}$ such that G_B does not contain a blue $K_{1,3}$. So, $\Delta(G_B) \leq 2$. Since graph G_R is the complement of graph G_B relative to graph $K_{3\times(\lceil \frac{n}{2} \rceil+1)}$, then $\delta(G_R) \geq 2(\lceil \frac{n}{2} \rceil + 1) - \Delta(G_B) = 2\lceil \frac{n}{2} \rceil$. This implies that $G_R \supseteq K_{1,n}$.

Case 2. For $m = 3$ and $n = 1$.
To show that $m_3(3K_{1,1}, K_{1,3}) \geq 3$, consider a factorization $\{F_R, F_B\}$ of $K_{3\times 2}$ such that $F_B = C_6 = 1231231$. See Figrue 2. Then, F_B does not contain a blue $K_{1,3}$. This implies that $F_R = 2K_{1,2}$. Then, $F_R \not\supseteq 3K_{1,1}$. Now, we show that $m_3(3K_{1,1}, K_{1,3}) \leq 3$. We



$G_2 \not\supseteq K_{1,3}$ $G_1 \supseteq 3K_{1,1}$

FIGURE 3. $G_2 \not\supseteq K_{1,3}$ but $G_1 \supseteq 3K_{1,1}$.

consider any factorization $\{G_R, G_B\}$ of $K_{3\times 3}$ such that G_B does not contain a blue $K_{1,3}$. So, $\Delta(G_B) \leq 2$. Since graph G_R is the complement of graph G_B relative to graph $K_{3\times 3}$, then $\delta(G_R) \geq 4$. This implies that $G_R \supseteq 3K_{1,1}$. See Figure 3.

Case 3. For $m \geq 2, m \neq 3$ and $n = 1$.
To show that $m_3(mK_{1,1}, K_{1,3}) \geq \lceil \frac{2m}{3} \rceil$, we color the edges of graph $K_{3\times(\lceil \frac{2m}{3} \rceil-1)}$ by red. It is clear that $K_{3\times(\lceil \frac{2m}{3} \rceil-1)} \not\supseteq mK_{1,1}$, since $|V(K_{3\times(\lceil \frac{2m}{3} \rceil-1)})| = 3\lceil \frac{2m}{3} \rceil - 3 < 2m = |V(mK_{1,1})|$. Therefore, $K_{3\times(\lceil \frac{2m}{3} \rceil-1)}$ contains neither the red $mK_{1,1}$ nor a blue $K_{1,3}$.

Now, we show that $m_3(mK_{1,1}, K_{1,3}) \leq \lceil \frac{2m}{3} \rceil$. We consider any factorization $\{G_R, G_B\}$ of $K_{3\times\lceil \frac{2m}{3} \rceil}$ such that G_B does not contain a blue $K_{1,3}$. So, $\Delta(G_B) \leq 2$. Since graph G_R is the complement of graph G_B relative to graph $K_{3\times\lceil \frac{2m}{3} \rceil}$, then $\delta(G_R) \geq 2\lceil \frac{2m}{3} \rceil - 2 \geq m$, for $m \geq 2, m \neq 3$. We know that $|V(G_R)| = 3\lceil \frac{2m}{3} \rceil \geq 2m, \forall m \geq 2, m \neq 3$. Let graph G'_R be any subgraph of G_R, where $|V(G'_R)| = 2m$. Then, $\delta(G'_R) \geq m = \frac{|V(G'_R)|}{2}$. By Theorem 1.4, G'_R is a Hamiltonin graph. So, $G_R \supseteq G'_R \supseteq C_{2m}$. Therefore, $G_R \supseteq mK_{1,1}$. □

Theorem 2.4. *For positif integers $m, n \geq 2$ and $j \geq 3$, we have*
$$m_j(mK_{1,n}, K_{1,3}) = \begin{cases} \lceil \frac{m(1+n)}{j} \rceil; & \text{for } 3 \leq j < r(mK_{1,n}, K_{1,3}), \\ 1; & \text{for } j \geq r(mK_{1,n}, K_{1,3}). \end{cases}$$

Proof.
Case 1. For $3 \leq j < r(mK_{1,n}, K_{1,3})$.
To show that $m_j(mK_{1,n}, K_{1,3}) \geq \lceil \frac{m(1+n)}{j} \rceil$, let $t = \lceil \frac{m(1+n)}{j} \rceil$. We color the edges of graph $K_{j\times(t-1)}$ by red. Since $|V(K_{j\times(t-1)})| = jt - j = j\lceil \frac{m(1+n)}{j} \rceil - j < m(1+n) = |V(mK_{1,n})|$, then $K_{j\times(t-1)}$ contains neither the red $mK_{1,n}$ nor a blue $K_{1,3}$.

Now, we show that $m_j(mK_{1,n}, K_{1,3}) \leq t$. We consider any factorization $\{G_R, G_B\}$ of $K_{j\times t}$ such that G_B does not contain a blue $K_{1,3}$. So, $\Delta(G_B) \leq 2$. Since graph G_R is the complement of graph G_B relative to graph $K_{j\times t}$, then $\delta(G_R) \geq (j-1)t - 2 = (j-1)\lceil \frac{m(1+n)}{j} \rceil - 2 \geq n$, for $m, n \geq 2$ and $j \geq 3$. Thus, $|V(K_{j\times t})| = j\lceil \frac{m(1+n)}{j} \rceil \geq m(1+n)$. Note that, G_B is a graph (may be disconnected) whose every component is isomorphic to a cycle, a path or singleton. We divide the vertices in G_B into m partitions, each containing $n+1$ vertices. Every vertex in A adjacent to at most one vertex in B, for every different partition A and B in G_B. Thus, we construct $mK_{1,n}$ in G_R as follows. Let $a \in A$. If a is adjacent to b in G_B, for any $b \in B$, then induced subgraph of G_R by $\{a\} \cup B - \{b\}$ contains a star $K_{1,n}$. Otherwise, induced subgraph of G_R by $\{a\} \cup B$ contains a star $K_{1,n}$. Since there are m partitions, so $G_R \supseteq mK_{1,n}$.

Case 2. For $j \geq r(mK_{1,n}, K_{1,3})$.
This case is a direct consequence of Theorem 2.1.

□

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