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THE CONNECTED SIZE RAMSEY NUMBER FOR PAIR OF MATCHINGS AND SMALL PATHS

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Abstract. The connected size Ramsey number of a graph G and H, denoted by $\hat{r}_c(G,H)$ is the smallest integer $\hat{r}_c$ such that there is a connected graph F of $\hat{r}_c$ edges with the property that any red-blue coloring of the edges of F yields either a red subgraph G or a blue subgraph H. In this paper, we consider the connected size Ramsey number for a pair of a matching and a path. In particular, we derive an upper bound of $\hat{r}_c(tK_2,P_n)$ for $n,t \geq 2$. We also determine the exact values of $\hat{r}_c(tK_2,P_n)$, for some fixed n and t.

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1. Introduction

Given graphs G and H, the notation $F \rightarrow (G,H)$ means that in any red-blue coloring of the edges of F there exists a red copy of G or a blue copy of H in F. We denote $F \nrightarrow (G,H)$ to mean that there is some red-blue coloring of the edges of F such that F contains neither a red G nor a blue H. This coloring is called as the (G,H)-*coloring* of F. A *matching*, denoted by $mK_2, m \geq 2$, is a graph consisting $2m$ vertices and m independent edges.

The size Ramsey number of graph G and H denoted by $\hat{r}(G,H)$ is the smallest integer k such that there is a graph F with k edges satisfying $F \rightarrow (G,H)$. The size Ramsey number of a graph was introduced by Erdös, Faudree, and Rousseau in 1978. They proved that the size Ramsey number $\hat{r}(K_n,K_m) = \binom{r(K_n,K_m)}{2}$ and determined the size Ramsey numbers for a pair of stars and for the products of two graphs [6].

Since its introduction in [6], there are only a few results. The results on size Ramsey number for paths, trees, cycles, regular graphs, and directed paths can be found in [1], [2], [3], and [9]. Additionally, the size Ramsey number for graph pairs with one is either a matching or a star, respectively can be found in [4], [5], [8], and [10]. A survey of results concerning the size Ramsey number for many pairs of graphs can be obtained in [7].

The study of the size Ramsey number involving matching was proposed by Burr *et.al* [4] in 1977. They showed that for positive integers k, l, m, and n, the size Ramsey number $\hat{r}(mK_{1,s}, nK_{1,t}) = (m+n-1)(s+t-1)$. Later, in 1981, Erdös and Faudree have determined the size Ramsey numbers for a matching versus a path. Some of their results are as follows.

Theorem 1.1. [5] $\hat{r}(2K_2,P_n) = n+1$.

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Theorem 1.2. [5] *For* $t \geq 1$, $\hat{r}(tK_2, P_4) = \lceil \frac{5t}{2} \rceil$ *and*
$\hat{r}(tK_2, P_5) = \begin{cases} 3t, & \text{for even } t, \\ 3t+1, & \text{for odd } t. \end{cases}$

Note that up to now the exact values of $\hat{r}(mK_2, P_n)$ are known only for $n \leq 5$.

In general, the smallest graph F satisfying $F \to (G, H)$ can be connected or disconnected. For instance, the smallest graph F satisfying $F \to (mK_{1,s}, nK_{1,t})$ is the graph $(m+n-1)K_{1,s+t-1}$ and the smallest graph F satisfying $F \to (2K_2, P_n)$ is a C_{n+1}. We also know that the size Ramsey number $\hat{r}(tK_2, P_n)$ for $n = 4, 5$ is satisfied by $\lfloor \frac{t}{2} \rfloor C_{t+1}$ for even t and $\lfloor \frac{t}{2} \rfloor C_{t+1} \cup P_n$ for odd t.

In various purposes, connectedness is very important. So, in this paper, we will consider finding the smallest connected graph F satisfying $F \to (G, H)$ for some fixed G and H. In that case, we define the *connected size Ramsey number* of a pair of graphs G and H, denoted by $\hat{r}_c(G, H)$, as the smallest integer k such that there is a connected graph F with k edges satisfying $F \to (G, H)$. Of course, for any G and H, we have that $\hat{r}(G, H) \leq \hat{r}_c(G, H)$. Since $\hat{r}(2K_2, P_n)$ is satisfied by a C_{n+1} and a C_{n+1} is connected, then $\hat{r}(2K_2, P_n) = \hat{r}_c(2K_2, P_n)$.

In this paper, we investigate the connected size Ramsey number $\hat{r}_c(mK_2, H)$ for H is either a P_3 or a P_5.

2. Main Results

In this section, we are going to present our results. We start by deriving an upper bound of $\hat{r}_c(nK_2, P_3)$. Then, we determine the exact values of $\hat{r}_c(nK_2, P_3)$ for some values of n. Moreover, we also derive an upper bound of $\hat{r}_c(nK_2, P_5)$ and present the exact values of $\hat{r}_c(nK_2, P_5)$ for some values of n.

Lemma 2.1. *For* $n \geq 3$, $\hat{r}_c(nK_2, P_3) \leq 3n - 1 - \lfloor \frac{n}{2} \rfloor$.

Proof. We will define a connected graph G satisfies $G \to (nK_2, P_3)$. We start with even n. Consider the graph G as in Figure 1. The graph G is formed from a path with $\frac{n}{2}$ vertices and $\frac{n}{2}$ cycles of length 4 by identifying each vertex of the path with a fixed vertex of a cycle C_4. The number of edges of G is $|E(G)| = 4\frac{n}{2} + \frac{n}{2} - 1 = \frac{5n}{2} - 1 = 3n - 1 - \lfloor \frac{n}{2} \rfloor$. Let μ be any red-blue coloring of the edges of G. Suppose G contains no blue P_3. Then there are at least two independent red edges in each C_4. Since there exists $\frac{n}{2}$ C_4 in G , we have at least n independent red edges in G. Thus, G contains a red nK_2. So, $G \longrightarrow (nK_2, P_3)$.

Next, if n is odd, consider the graph G depicted in Figure 2. The graph G is formed from a path with $\lfloor \frac{n}{2} \rfloor + 1$ vertices by attaching a vertex of C_4 to $\lfloor \frac{n}{2} \rfloor$ first vertices of path and two edges pendant to the last vertex of the path. The number of edges of G is $|E(G)| = 4\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor - 1 + 3 = 5\lfloor \frac{n}{2} \rfloor + 2 = 3n - 1 - \lfloor \frac{n}{2} \rfloor$.
Let μ be any red-blue coloring of the edges of G. Suppose G contains no blue P_3. By similar argument as in the previous case, we have at least two independet red edges in each C_4 and one red pendant edge. Therefore, we have at least $2\left\lfloor \frac{n}{2} \right\rfloor + 1$ red edges in G. So, $G \longrightarrow (nK_2, P_3)$. □

Theorem 2.2. $\hat{r}_c(3K_2, P_3) = 7$.

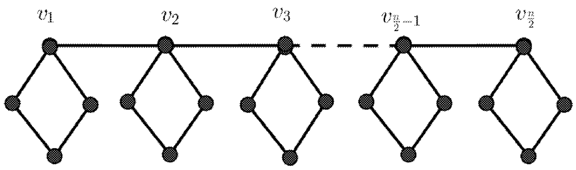

FIGURE 1. The graph $G \to (nK_2, P_3)$, n is even.

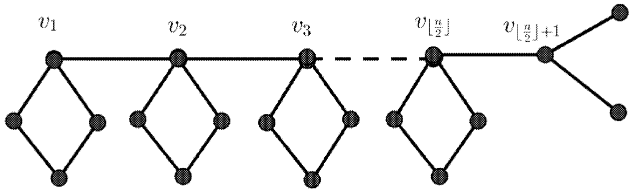

FIGURE 2. The graph $G \to (nK_2, P_3)$, n is odd.

Proof. By using Lemma 2.1, we have $\hat{r}_c(3K_2, P_3) \leq 7$. Next, we will show that $\hat{r}_c(3K_2, P_3) \geq 7$. To do this, we assume that F is a connected graph with $|E(F)| \leq 6$. We will show that $F \nrightarrow (3K_2, P_3)$. Consider a connected graph F with $|E(F)| \leq 6$.
Consider the following two cases.
Case 1. Suppose $\Delta(F) = 2$.
In this case, graph F is a path or a cycle of length at most 6. It is easy to find a 2-coloring of F so that $F \nrightarrow (3K_2, P_3)$. So, we obtain $(3K_2, P_3)$-coloring.
Case 2. $\Delta(F) \geq 3$.
Let v be a vertex in F with $d(v) \geq 3$ and set $F^* = F - \{v\}$. Observe that $|E(F^*)| \leq 6 - 3 = 3$. If F^* is connected, then by Theorem 1.1, there is a $(2K_2, P_3)$-coloring in F^*. By coloring all edges incident to v with red, we obtain $F \nrightarrow (3K_2, P_3)$.
Now, assume that F^* is disconnected. Since $|E(F^*)| \leq 3$, the subgraph F^* have at most one component containing P_3. If there is no component of F^* containing P_3, then we color all edges of F^* with blue. Therefore, we color all edges incident to v with red. By this coloring, we obtain $F \nrightarrow (3K_2, P_3)$. If F^* contains exactly one component G_1 containing P_3, then by Theorem 1.1, there is a $(2K_2, P_3)$-coloring in G_1. Therefore, we color all edges incident to v with red and all edges in the other components with blue. By this coloring, we obtain $F \nrightarrow (3K_2, P_3)$. In all cases, we obtains $F \nrightarrow (3K_2, P_3)$. Thus, $\hat{r}_c(3K_2, P_3) \geq 7$. □

Theorem 2.3. $\hat{r}_c(4K_2, P_3) = 9$.

Proof. By using Lemma 2.1, we have $\hat{r}_c(4K_2, P_3) \leq 9$. Next, we will show that $\hat{r}_c(4K_2, P_3) \geq 9$. To do this, we assume that F is a connected graph with $|E(F)| \leq 8$. We will show that $F \nrightarrow (4K_2, P_3)$. Consider a connected graph F with $|E(F)| \leq 8$. Decompose the graph F into two connected subgraph F_1 and F_2 with $|E(F_1)| = 2$ and $|E(F_2)| \leq 6$. By Theorem 2.2, there is a $(3K_2, P_3)$-coloring in F_2. Therefore, we color all edges of F_1 with red. Together with the coloring on F_2, we obtain a $(4K_2, P_3)$-coloring in F. So, $F \nrightarrow (4K_2, P_3)$. □

Theorem 2.4. $\hat{r}_c(5K_2, P_3) = 12$.

Proof. By using Lemma 2.1, we have $\hat{r}_c(5K_2, P_3) \leq 12$. Next, we will show that $\hat{r}_c(5K_2, P_3) \geq 12$. Suppose F be a connected graph with $|E(F)| \leq 11$. We will show that $F \nrightarrow (5K_2, P_3)$. Consider the following two cases.

Case 1. Suppose $\Delta(F) = 2$.

In this case, graph F is a path or a cycle with length 11. It is easy to find a 2-coloring of F so that $F \nrightarrow (5K_2, P_3)$. So, we obtain $(5K_2, P_3)$-coloring.

Case 2. $\Delta(F) \geq 3$.

Let v be a vertex in F with $d(v) \geq 3$ and $F^* = F - v$. Observe that $|E(F^*)| \leq 11 - 3 = 8$. If F^* is connected, then by Theorem 2.3, there is a $(4K_2, P_3)$-coloring in F^*. By coloring all edges incident to v with red, we obtain $F \nrightarrow (5K_2, P_3)$.

Now, assume that F^* is not connected. Since $|E(F^*)| \leq 8$, the subgraph F^* have at most 3 components containing P_3.

First, suppose F^* contains exactly one component G_1 containing P_3. In this case, G_1 have at most 8 edges. By Theorem 2.3, there is a $(4K_2, P_3)$-coloring in G_1. Therefore, we color all edges incident to v with red and all edges in the other components with blue. By this coloring, we obtain $F \nrightarrow (5K_2, P_3)$.

Now, suppose F^* contains exactly two components containing P_3, namely G_1 and G_2. Then the combination of the number of edges in each component is $(6,2)$, $(5,3)$, $(4,4)$, $(4,3)$, $(4,2)$, $(3,3)$, $(3,2)$, or $(2,2)$. By coloring the remaining component (if any) with blue, in all combinations, except $(4,4)$, there exists a $(4K_2, P_3)$-coloring of F^*. Furthermore, by coloring all edges incident to v with red, we show that $F \nrightarrow (5K_2, P_3)$. For the case $(4,4)$, we consider the subgraph induced by four edges in each component. If there is at most one component isomorphic to C_4, then we can color all edges of each component of F^* with red and blue so that F^* contains at most three red K_2 and no blue P_3. By coloring all edges incident to v with red, there exists a $(5K_2, P_3)$-coloring for F. Now, suppose the both components isomorphic to C_4. Choose one vertex x_i of $G_i, i = 1, 2$ which is adjacent to v. Color all edges incident with x_i by red and the remaining edges of $F - x_i$ are colored by red and blue so that $F - \{x_1, x_2\}$ contains two red K_2 and no blue P_3. By this coloring, there exists a $(5K_2, P_3)$-coloring for F. So, we obtain $F \nrightarrow (5K_2, P_3)$.

Next, suppose F^* contains exactly 3 components containing P_3, namely G_1, G_2, and G_3. Then the combination of the number of edges in each component is $(4,2,2)$, $(3,3,2)$, $(3,2,2)$, or $(2,2,2)$. Except for the case $(4,2,2)$, in all combinations, there exists a $(4K_2, P_3)$-coloring of F^*. Furthermore, by coloring all edges incident to v with red, we show that $F \nrightarrow (5K_2, P_3)$. For the case $(4,2,2)$, denote $N(v^*) = \{x, y, z\}$. Now, color all edges incident to vertices x, y and z by red. The remaining edges are colored by red and blue so that there is one red K_2 and no blue P_3. By this coloring, there exists a $(5K_2, P_3)$-coloring for F. So, we obtain $F \nrightarrow (5K_2, P_3)$.

In all cases, we obtain $(5K_2, P_3)$-coloring in F. So, $F \nrightarrow (5K_2, P_3)$. Thus, the proof is complete. □

Theorem 2.5. $\hat{r}_c(6K_2, P_3) = 14$.

Proof. By using Lemma 2.1, we have $\hat{r}_c(6K_2, P_3) \leq 14$. Next, we will show that $\hat{r}_c(6K_2, P_3) \geq 14$. To do this, we assume that F be a connected graph with $|E(F)| \leq 13$. We will show that $F \nrightarrow (6K_2, P_3)$. Consider a connected graph F with $|E(F)| \leq 13$. Decompose the graph F into two connected subgraph F_1 and F_2 with $|E(F_1)| = 2$ and $|E(F_2)| \leq 11$. By Theorem 2.4, there is a $(5K_2, P_3)$-coloring in F_2. Therefore, we color all edges of F_1 with red. Together with the coloring on F_2, we obtain a $(6K_2, P_3)$-coloring in F. So, $F \nrightarrow (6K_2, P_3)$. □

In the following, we are going to derive the upper bound of the connected size Ramsey $\hat{r}_c(nK_2, P_5)$. Then, we will calculate the exact values of$\hat{r}_c(nK_2, P_5)$ for some fixed n.

Lemma 2.6. *For $n \geq 3$, $\hat{r}_c(nK_2, P_5) \leq \begin{cases} \frac{7n}{2} - 1, & \text{if } n \text{ is even,} \\ 7\lfloor \frac{n}{2} \rfloor + 4, & \text{if } n \text{ is odd.} \end{cases}$*

Proof. We start by showing that $\hat{r}_c(nK_2, P_5) \leq \frac{7n}{2} - 1$ if n is even. Consider the graph F depicted in Figure 3. We will show that $F \longrightarrow (nK_2, P_5)$. The graph F is a graph

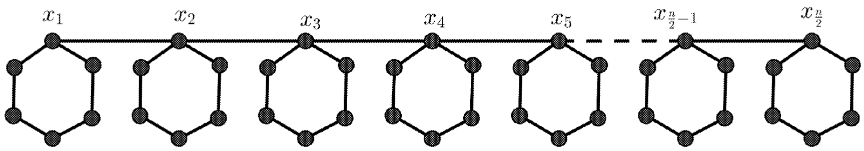

FIGURE 3. The graph $F \rightarrow (nK_2, P_5)$, n is even.

obtained from union a path with $\frac{n}{2}$ vertices and $\frac{n}{2}$ cycles with length 6 by identifying each vertex of the path with a fixed vertex of a cycle C_6. The number of edges of F is $|E(F)| = 6\frac{n}{2} + \frac{n}{2} - 1 = \frac{7n}{2} - 1$.
Let μ be any red-blue coloring of the edges of F. Suppose F contains no blue P_5. Then there are at least two red independent edges in each C_6. Since there exists $\frac{n}{2}$ C_6 in F , we have at least n red edges in F. Thus, F contains a red nK_2. So, $F \longrightarrow (nK_2, P_5)$.

Now, we will show $\hat{r}_c(nK_2, P_5) \leq 7\lfloor \frac{n}{2} \rfloor + 4$. if n is odd.
Consider the graph G depicted in Figure 4. The graph G is formed from a path with $\lfloor \frac{n}{2} \rfloor + 5$ vertices and $\lfloor \frac{n}{2} \rfloor$ cycles with length 6 by identifying a fixed vertex of a cycle C_6 with a vertex v_i, $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, of the path. The number of edges of G is $|E(G)| = 6\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor - 1 + 5 = 7\lfloor \frac{n}{2} \rfloor + 4$.
Let μ be any red-blue coloring of the edges of F. Suppose F contains no blue P_5. By similar argument as in the previous case, we have at least two red independet edges in each C_6 and one red edge in the subpath induced by $\{v_i | i = \lfloor \frac{n}{2} \rfloor + 1, \cdots, \lfloor \frac{n}{2} \rfloor + 5\}$. Therefore, we have at least $2\lfloor \frac{n}{2} \rfloor + 1$ red edges in F. So, $G \longrightarrow (nK_2, P_5)$. □

Theorem 2.7. $\hat{r}_c(3K_2, P_5) = 11$.

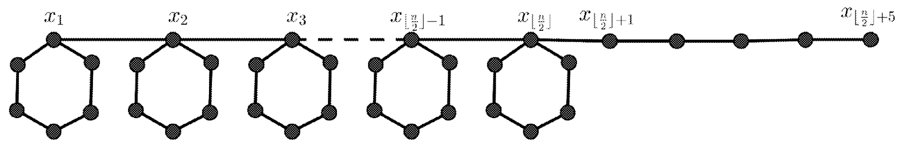

FIGURE 4. The graph $G \rightarrow (nK_2, P_5)$, n is odd.

Proof. By Lemma 2.6, we show that $\hat{r}_c(3K_2, P_5) \leq 11$.
Now, we will show that $\hat{r}_c(3K_2, P_5) \geq 11$. To do this, we will show that any connected graph F, with $|E(F)| \leq 10$ satisfies $F \nrightarrow (3K_2, P_5)$. Let F be a connected graph with $|E(F)| \leq 10$. We consider the two following cases.
Case 1. $\Delta(F) = 2$.
In this case, graph F is a path or a cycle of length at most 10. It is easy to color all edges in F with red and blue so that there is a $(3K_2, P_5)$-coloring in F. So, $F \nrightarrow (3K_2, P_5)$.
Case 2. Suppose that $\Delta(F) \geq 3$.
In this case we consider the longest path P in F.
Suppose F contains cycle $C_i, i \leq 5$. Let P be the longest path in F. The length of P is at most 9. First, choose a vertex x of P with the largest degree. Color all edges incident with x by red. Therefore, we consider the subgraph $F^* = F - \{x\}$. The number of edges of F^* at most 7. If F^* contains no P_5, then we color all edges in F^* with blue. Otherwise, we can choose one vertex y of F^* so that $F^* - \{y\}$ no contains P_5. Color all edges incident with y by red and all edges in $F^* - \{y\}$ with blue. In this coloring, there is a $(3K_2, P_5)$-coloring in F.
Now, Suppose F contains cycle $C_i, i \geq 6$. We consider the number of cycles C_i in F.

First, suppose F contains exactly one cycle $C_i, i \geq 6$. In this case, we will use the same method as the case F contains cycle $C_i, i \leq 5$. We choose a vertex x of P with the largest degree. Color all edges incident with x by red. Therefore, we consider the subgraph $F^* = F - \{x\}$. The number of edges of F^* at most 7. If F^* contains no P_5, then we color all edges in F^* with blue. Otherwise, we choose one vertex y of F^* so that $F^* - \{y\}$ no contains P_5. Color all edges incident with y by red and all edges in $F^* - \{y\}$ with blue. In this coloring, there is a $(3K_2, P_5)$-coloring in F.

Next, suppose there is more than one cycle $C_i, i \geq 6$ in F. Let P be the longest path in F. Select a vertex v of P with the largest degree that is contained in the most cycles C_i of F. Color all edges incident with v by red. Next, choose a vertex x of $F - \{v\}$ so that $F - \{v, x\}$ contains no P_5. Color all edges incident with x by red and all edges in $F - \{v, x\}$ by blue. In this coloring, there is a $(3K_2, P_5)$-coloring in F.

□

Theorem 2.8. $\hat{r}_c(5K_2, P_5) = 18$.

Proof. According Lemma 2.6, we show that $\hat{r}_c(5K_2, P_5) \leq 18$. Now, we will prove that $\hat{r}_c(5K_2, P_5) \geq 18$.
Let F be a connected graph with $|E(F)| \leq 17$. We will show that $F \nrightarrow (5K_2, P_5)$. Let

consider the two following cases.

Case 1. $\Delta(F) = 2$.
In this case, graph F is a path or a cycle of length at most 17. It is easy to select four vertices u, v, w, and x in F such that $F - \{u, v, w, x\}$ contains no P_5. Then, color all edges incident with u, v, w, and x by red and the edges in $F - \{u, v, w, x\}$ with blue. By this coloring, there is a $(5K_2, P_5)$-coloring in F. So, $F \nrightarrow (5K_2, P_5)$.

Case 2. Suppose that $\Delta(F) \geq 3$.
In this case we consider the longest path P in F.
Suppose F contains cycle $C_i, i \leq 5$. Let P be the longest path in F. The length of P is at most 16. First, choose a vertex p_1 of P with the largest degree. Observe that $|E(F - \{p_1\})|$ is at most 14. Then inductively, we can choose p_2, p_3, and p_4 of P so that $F - \{p_1, p_2, p_3, p_4\}$ contain no P_5. Color all edges incident with p_i by red and the remaining edges of $F - \{p_1, p_2, p_3, p_4\}$ by blue. In this coloring, there is a $(5K_2, P_5)$-coloring in F.

Now, Suppose F contains cycle $C_i, i \geq 6$. Suppose F contains exactly one or two cycles $C_i, i \geq 6$. In this case, we use the same method as the case F contains cycle $C_i, i \leq 5$. Let P be a longest path in F. First, choose a vertex p_1 of P with the largest degree. Then inductively, we can choose p_2, p_3, and p_4 of P so that $F - \{p_1, p_2, p_3, p_4\}$ contain no P_5. Color all edges incident with p_i by red and the remaining edges of $F - \{p_1, p_2, p_3, p_4\}$ by blue. In this coloring, there is a $(5K_2, P_5)$-coloring in F.

Next, suppose F contains more than two cycles $C_i, i \geq 6$. First, choose a vertex p_1 of P with the largest degree that is contained in the most cycles C_i in F. Then inductively, we can choose p_2, p_3, and p_4 of P so that $F - \{p_1, p_2, p_3, p_4\}$ contain no P_5. Color all edges incident with p_i by red and the remaining edges of $F - \{p_1, p_2, p_3, p_4\}$ by blue. In this coloring, there is a $(5K_2, P_5)$-coloring in F.

□

Theorem 2.9. *Let t be positive integer with $t \geq 2$. If $\hat{r}_c((2t-1)K_2, P_5) = 7t - 3$, then $\hat{r}_c(2tK_2, P_5) = 7t - 1$.*

Proof. Letting $n = 2t$ in Lemma 2.6, we obtain that $\hat{r}_c(2tK_2, P_5) \leq 7t - 1$. Now, we will prove that $\hat{r}_c(2tK_2, P_5) \geq 7t - 1$. Let F be a connected graph with $|E(F)| \leq 7t - 2$. We will show that $F \nrightarrow (2tK_2, P_5)$. Decompose graph F into two connected subgraph F_1 and F_2 with $|E(F_1)| \leq 7t - 4$ and $|E(F_2)| = 2$. Observe that, there exists a $((2t-1)K_2, P_5)$-coloring in F_1. By giving the red color to all edges in F_2, we show that F contains neither a red $2tK_2$ nor a blue P_5. So, $F \nrightarrow (2tK_2, P_5)$. Thus, the proof is complete. □

The Theorem 2.9 has an immediate consequence.

Corollary 2.10. $\hat{r}_c(4K_2, P_5) = 13$.

Corollary 2.11. $\hat{r}_c(6K_2, P_5) = 20$.

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