Advanced Studies in Contemporary Mathematics
28 (2018), No. 2, pp. 231 – 244

www.jangjeon.or.kr
http://dx.doi.org/10.17777/ascm2018.28.2.231

COMPUTATION OF ADRIATIC INDICES OF CERTAIN OPERATORS OF REGULAR AND COMPLETE BIPARTITE GRAPHS

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Abstract. A topological index of a graph G is a numerical parameter related to G which characterizes its molecular topology and used for quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR). Adriatic indices are bond-additive topological indices. They are analyzed on the testing sets provided by the Inernational Academy of Mathematical Chemistry (IAMC) and it has been shown that they have good predictive properties in many cases. In this paper, we study the certain adriatic indices of regular and complete bipartite graphs using some graph operators.

1. Introduction

Let $G = (V, E)$ be a graph. We denote the number of vertices and edges of G by n and m, respectively. Thus $|V(G)| = n$ and $|E(G)| = m$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge e connecting the vertices u and v will be denoted by $e = uv$.

In chemical graph theory, molecular topology and mathematical chemistry, a topological graph theoretical index sometimes also known as the connectivity index is a type of a molecular descriptor which is calculated by means of the molecular graph, a graph in which the vertices correspond to the atoms and the edges to the bonds of a molecule, of a chemical compound. That is, a topological index is a function defined on a (molecular) graph regardless of the labeling of its vertices. Till now, many of different topological indices have been employed in QSAR/QSPR studies, some of which have been proved to be successful [18]. Recently, the set of 148 discrete Adriatic indices has been proposed (see [19] and for further studies of the discrete adriatic indices (discrete adriatic indices are bond-additive topological indices) see [20, 21]). They have shown good predictive properties in one-parametric linear models and outperform benchmark descriptors proposed by IAMC in several cases. Among those successful topological indices, there are four bond-additive discrete adriatic indices, called the sum lordeg index, inverse sum lordeg index,

Date: Received: November, ?? 2017, Accepted:
2010 *Mathematics Subject Classification.* Primary 05C05; Secondary 05C07, 05C35.
Key words and phrases. Adriatic indices, regular and complete bipartite graphs, graph operators.
I. N. Cangul is the corresponding author.
This paper is in final form and no version of it will be submitted for publication elsewhere.

misbalance lodeg index and misbalance losdeg index, which are respectively defined to be

$$\Upsilon(G) = \sum_{uv\in E(G)} \left[\sqrt{lnd_u} + \sqrt{lnd_v}\right]$$

$$\Phi(G) = \sum_{uv\in E(G)} \left[\frac{1}{\sqrt{lnd_u} + \sqrt{lnd_v}}\right]$$

$$\Psi(G) = \sum_{uv\in E(G)} |lnd_u - lnd_v|$$

and

$$\phi(G) = \sum_{uv\in E(G)} |ln^2 d_u - ln^2 d_v|.$$

In forthcoming sections, we will obtain some results on the four of the bond-additive discrete adriatic indices for line, subdivision, vertex-semitotal, edge-semitotal, total, jump and para-line graphs of some standard graphs.

2. Line graphs

The line graph $L(G)$ is the graph with vertex set $V(L(G)) = E(G)$ whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common. For more details, see e.g. [1].

Here the discrete adriatic indices of the line graph of r-regular and complete bipartite graphs are discussed.

Theorem 1. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\Upsilon[L(G)] = nr(r-1)\sqrt{ln(2r-2)}$$

and

$$\Phi[L(G)] = \frac{nr(r-1)}{4\sqrt{ln(2r-2)}}.$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the line graph of G is also a $2r-2$-regular graph with $\frac{nr}{2}$ vertices and $\frac{nr}{2}(r-1)$ edges. Then

$$\begin{aligned}
\Upsilon[L(G)] &= \tfrac{nr(r-1)}{2}\left[\sqrt{ln(2r-2)} + \sqrt{ln(2r-2)}\right] \\
&= nr(r-1)\sqrt{ln(2r-2)} \\
\Phi[L(G)] &= \tfrac{nr(r-1)}{2}\left[\tfrac{1}{\sqrt{ln(2r-2)}+\sqrt{ln(2r-2)}}\right] \\
&= \tfrac{nr(r-1)}{4\sqrt{ln(2r-2)}}.
\end{aligned}$$

□

Theorem 2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then*

$$\Upsilon[L(K_{r,s})] = rs(r+s-2)\sqrt{ln(r+s-2)}$$

and

$$\Phi[L(K_{r,s})] = \frac{rs}{4}(r+s-2)\left[\frac{1}{\sqrt{ln(r+s-2)}}\right].$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then the line graph of $K_{r,s}$ is also an $r+s-2$-regular graph with rs vertices and $\frac{1}{2}rs(r+s-2)$ edges. Then

$$\begin{aligned}
\Upsilon[L(K_{r,s})] &= \tfrac{rs}{2}(r+s-2)\left[\sqrt{ln(r+s-2)} + \sqrt{ln(r+s-2)}\right] \\
&= rs(r+s-2)\sqrt{ln(r+s-2)} \\
\Phi[L(K_{r,s})] &= \tfrac{rs}{2}(r+s-2)\left[\frac{1}{\sqrt{ln(r+s-2)}+\sqrt{ln(r+s-2)}}\right] \\
&= \tfrac{rs}{4}(r+s-2)\left[\frac{1}{\sqrt{ln(r+s-2)}}\right].
\end{aligned}$$

□

By the above result with $r = s$, we have completed the study of the regular bipartite graphs $K_{r,r}$ with $r > 1$.

3. Subdivision graphs

The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two, or equivalently, by inserting an additional vertex into each edge of G. For more details see [2].

Here the discrete adriatic indices of the subdivision graphs of r-regular and complete bipartite graphs are discussed.

Theorem 3. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\Upsilon[S(G)] = nr[\sqrt{ln(2)} + \sqrt{ln(r)}],$$

$$\Phi[S(G)] = \left[\frac{nr}{\sqrt{ln2} + \sqrt{lnr}}\right],$$

$$\Psi[S(G)] = nr\left|ln\frac{2}{r}\right|,$$

and

$$\phi[S(G)] = nr|ln^2(2) - ln^2(r)|.$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the subdivision graph of G has $n + \frac{nr}{2}$ vertices and nr edges. The edge partition of $S(G)$ is as follows:

(d_u, d_v) where $uv \in E(G)$	$(2, r)$
Number of edges	nr

Then

$$\begin{aligned}
\Upsilon[S(G)] &= nr\left[\sqrt{ln2}+\sqrt{lnr}\right],\\
\Phi[S(G)] &= nr\tfrac{1}{\sqrt{ln2}+\sqrt{lnr}}\\
&= \left[\tfrac{nr}{\sqrt{ln2}+\sqrt{lnr}}\right],\\
\Psi[S(G)] &= nr\left|ln2-lnr\right|\\
&= nr\left|ln\tfrac{2}{r}\right|,\\
\phi[S(G)] &= nr|ln^2(2)-ln^2(r)|.
\end{aligned}$$

□

Theorem 4. *Let $K_{r,s}$ be a complete bipartite graph with $1 \le r \le s$. Then*

$$\Upsilon[S(K_{r,s})] = rs[2\sqrt{ln2}+\sqrt{lnr}+\sqrt{lns}].$$

$$\Phi[S(K_{r,s})] = rs\left[\frac{1}{\sqrt{ln2}+\sqrt{lnr}}+\frac{1}{\sqrt{ln2}+\sqrt{lns}}\right].$$

$$\Psi[S(K_{r,s})] = rs\left[|ln\frac{2}{r}|+|ln\frac{2}{s}|\right].$$

$$\phi[S(K_{r,s})] = rs[|ln^2 2-ln^2 r|+|ln^2 2-ln^2 s|].$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \le r \le s$. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. Then the subdivision graph of $K_{r,s}$ has $r+s+rs$ vertices and $2rs$ edges. The edge partition of $S(K_{r,s})$ would be as follows:

(d_u, d_v) where $uv \in E(G)$	$(2, r)$	$(2, s)$
Number of edges	rs	rs

Then

$$\begin{aligned}
\Upsilon[S(K_{r,s})] &= rs[\sqrt{ln2}+\sqrt{lnr}]+rs[\sqrt{ln2}+\sqrt{lns}]\\
&= rs[2\sqrt{ln2}+\sqrt{lnr}+\sqrt{lns}],\\
\Phi[S(K_{r,s})] &= rs\left[\tfrac{1}{\sqrt{ln2}+\sqrt{lnr}}]+rs[\tfrac{1}{\sqrt{ln2}+\sqrt{lns}}\right]\\
&= rs\left[\tfrac{1}{\sqrt{ln2}+\sqrt{lnr}}+\tfrac{1}{\sqrt{ln2}+\sqrt{lns}}\right],\\
\Psi[S(K_{r,s})] &= rs|ln2-lnr|+rs|ln2-lns|\\
&= rs\left[|ln\tfrac{2}{r}|+|ln\tfrac{2}{s}|\right],\\
\phi[S(K_{r,s})] &= rs|ln^2 2-ln^2 r|+rs|ln^2 2-ln^2 s|\\
&= rs[|ln^2 2-ln^2 r|+|ln^2 2-ln^2 s|].
\end{aligned}$$

□

By the above result with $r = s$, we completed the study of the regular bipartite graphs $K_{r,r}$ with $r > 1$.

4. Vertex-semitotal graphs

The vertex-semitotal graph $T_1(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S(G)) \cup E(G)$ is the graph obtained from G by adding a new vertex for each edge of G and by joining each new vertex to the end vertices of the corresponding edge. For more details see [16].

Here the discrete adriatic indices of vertex-semitotal graphs of r-regular and complete bipartite graphs are studied.

Theorem 5. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\begin{aligned} \Upsilon[T_1(G)] &= nr[\sqrt{ln2} + 2\sqrt{ln2r}], \\ \Phi[T_1(G)] &= nr\left[\frac{1}{\sqrt{ln2}+\sqrt{ln2r}} + \frac{1}{4\sqrt{ln2r}}\right], \\ \Psi[T_1(G)] &= nr\left|ln\frac{1}{r}\right|, \\ \phi[T_1(G)] &= nr|ln^2 2 - ln^2 2r|. \end{aligned}$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the vertex-semitotal graph of G has $\frac{nr}{2} + n$ vertices and $\frac{3nr}{2}$ edges. The edge partition of $T_1(G)$ is then as follows:

(d_u, d_v) where $uv \in E(G)$	$(2, 2r)$	$(2r, 2r)$
Number of edges	nr	$nr/2$

Then we have

$$\begin{aligned} \Upsilon[T_1(G)] &= nr[\sqrt{ln2} + \sqrt{ln2r}] + \tfrac{nr}{2}[\sqrt{ln2r} + \sqrt{ln2r}] \\ &= nr[\sqrt{ln2} + 2\sqrt{ln2r}], \\ \Phi[T_1(G)] &= nr\left[\frac{1}{\sqrt{ln2}+\sqrt{ln2r}}\right] + \frac{nr}{2}\left[\frac{1}{\sqrt{ln2r}+\sqrt{ln2r}}\right] \\ &= nr\left[\frac{1}{\sqrt{ln2}+\sqrt{ln2r}} + \frac{1}{4\sqrt{ln2r}}\right], \\ \Psi[T_1(G)] &= nr|ln2 - ln2r| + \tfrac{nr}{2}|ln2r - ln2r| \\ &= nr\left|ln\frac{1}{r}\right|, \\ \phi[T_1(G)] &= nr|ln^2 2 - ln^2 2r| + \tfrac{nr}{2}|ln^2 2r - ln^2 2r| \\ &= nr|ln^2 2 - ln^2 2r|. \end{aligned}$$

□

Theorem 6. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then*

$$\Upsilon[T_1(K_{r,s})] = 2rs[\sqrt{ln2r} + \sqrt{ln2s} + \sqrt{ln2}],$$

$$\Phi[T_1(K_{r,s})] = rs\left[\frac{1}{\sqrt{ln2r}+\sqrt{ln2}} + \frac{1}{\sqrt{2s}+\sqrt{ln2}} + \frac{1}{\sqrt{2r}+\sqrt{2s}}\right],$$

$$\Psi[T_1(K_{r,s})] = rs\left[\left|lnr\right| + \left|lns\right| + \left|ln(\frac{r}{s})\right|\right]$$

and

$$\phi[T_1(K_{r,s})] = rs[|ln^2 2r - ln^2 2| + |ln^2 2s - ln^2 2| + |ln^2 2r - ln^2 2s|].$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and rs edges, where $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. Then the vertex-semitotal graph of $K_{r,s}$ has $r+s+rs$ vertices and $3rs$ edges. The edge partition of $T_1(K_{r,s})$ is as follows:

(d_u, d_v) where $uv \in E(G)$	$(2, 2r)$	$(2, 2s)$	$(2r, 2s)$
Number of edges	rs	rs	rs

Then

$$\begin{aligned}
\Upsilon[T_1(K_{r,s})] &= rs[\sqrt{ln2r} + \sqrt{ln2}] + rs[\sqrt{ln2s} + \sqrt{ln2}] + rs[\sqrt{ln2r} + \sqrt{ln2s}] \\
&= 2rs[\sqrt{ln2r} + \sqrt{ln2s} + \sqrt{ln2}], \\
\Phi[T_1(K_{r,s})] &= rs\left[\frac{1}{\sqrt{ln2r}+\sqrt{ln2}}\right] + rs\left[\frac{1}{\sqrt{ln2s}+\sqrt{ln2}}\right] + rs\left[\frac{1}{\sqrt{2r}+\sqrt{2s}}\right] \\
&= rs\left[\frac{1}{\sqrt{ln2r}+\sqrt{ln2}} + \frac{1}{\sqrt{2s}+\sqrt{ln2}} + \frac{1}{\sqrt{2r}+\sqrt{2s}}\right], \\
\Psi[T_1(K_{r,s})] &= rs|ln2r - ln2| + rs|ln2s - ln2| + rs|ln2r - ln2s| \\
&= rs\left[|ln2r - ln2| + |ln2s - ln2| + |ln2r - ln2s|\right], \\
\phi[T_1(K_{r,s})] &= rs|ln^2 2r - ln^2 2| + rs|ln^2 2s - ln^2 2| + rs|ln^2 2r - ln^2 2s| \\
&= rs\left[|ln^2 2r - ln^2 2| + |ln^2 2s - ln^2 2| + |ln^2 2r - ln^2 2s|\right].
\end{aligned}$$

□

By the above result with $r = s$, we have completed the study of regular bipartite graphs $K_{r,r}$ with $r > 2$.

5. Edge-semitotal graphs

An edge-semitotal graph $T_2(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S(G)) \cup E(L(G))$ is the graph obtained from G by inserting a new vertex into each edge of G and by joining with edges those pairs of these new vertices which lie on adjacent edges of G. For more details [16].

Here the discrete adriatic indices of the edge-semitotal graph of the r-regular and complete bipartite graphs are discussed.

Theorem 7. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\Upsilon[T_2(G)] = rn[\sqrt{lnr} + r\sqrt{ln2r}],$$

$$\Phi[T_2(G)] = rn\left[\frac{1}{\sqrt{lnr}+\sqrt{ln2r}} + \frac{r-1}{4\sqrt{ln2r}}\right],$$

$$\Psi[T_2(G)] = rn|ln(\frac{1}{2})|$$

and

$$\phi[T_2(G)] = rn|ln^2 r - ln^2 2r|.$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the edge-semitotal graph of G have $\frac{nr}{2} + n$ vertices and $\frac{nr}{2}(r+1)$ edges. The edge partition of $T_2(G)$ is as follows:

(d_u, d_v) where $uv \in E(G)$	$(r, 2r)$	$(2r, 2r)$
Number of edges	rn	$rn(r-1)/2$

Then

$$\Upsilon[T_2(G)] = rn[\sqrt{lnr} + \sqrt{ln2r}] + \frac{rn(r-1)}{2}[\sqrt{ln2r} + \sqrt{ln2r}].$$

$$= rn[\sqrt{lnr} + r\sqrt{ln2r}].$$

$$\Phi[T_2(G)] = rn\left[\frac{1}{\sqrt{lnr} + \sqrt{ln2r}}\right] + \frac{rn(r-1)}{2}\left[\frac{1}{\sqrt{ln2r} + \sqrt{ln2r}}\right].$$

$$= rn\left[\frac{1}{\sqrt{lnr} + \sqrt{ln2r}} + \frac{r-1}{4\sqrt{ln2r}}\right].$$

$$\Psi[T_2(G)] = rn|lnr - ln2r| + \frac{rn(r-1)}{2}|ln2r - ln2r|.$$

$$= rn|ln\frac{1}{2}|.$$

$$\phi[T_2(G)] = rn|ln^2r - ln^2 2r| + \frac{rn(r-1)}{2}|ln^2 2r - ln^2 2r|.$$

$$= rn|ln^2r - ln^2 2r|.$$

□

Theorem 8. *Let $K_{r,s}$ be a complete bipartite graph with $1 \le r \le s$. Then*

$$\Upsilon[T_2(K_{r,s})] = rs\left[\sqrt{lnr} + \sqrt{lns} + 2\sqrt{ln(r+s)} + (r+s-2)\sqrt{ln(r+s)}\right],$$

$$\Phi[T_2(K_{r,s})] = rs\left[\frac{1}{\sqrt{lnr} + \sqrt{ln(r+s)}} + \frac{1}{\sqrt{lns} + \sqrt{ln(r+s)}} + \frac{rs(r+s-2)}{4\sqrt{r+s}}\right],$$

$$\Psi[T_2(K_{r,s})] = rs\left[|ln(\frac{r}{r+s})| + |ln(\frac{s}{r+s})|\right]$$

and

$$\phi[T_2(K_{r,s})] = rs\left[|ln^2r - ln^2(r+s)| + |ln^2s - ln^2(r+s)|\right].$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \le r \le s$. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. Then the edge-semitotal graph of $K_{r,s}$ would have $r+s+rs$ vertices and $sr[1+\frac{1}{2}(r+s)]$ edges. The edge partition of $T_2(K_{r,s})$ is as follows:

(d_u, d_v) where $uv \in E(G)$	$(r, r+s)$	$(s, r+s)$	$(r+s, r+s)$
Number of edges	rs	rs	$rs(r+s-2)/2$

Then

$$\begin{aligned}
\Upsilon[T_2(K_{r,s})] &= rs\left[\sqrt{lnr}+\sqrt{ln(r+s)}\right]+rs\left[\sqrt{ln(s)}+\sqrt{ln(r+s)}\right]\\
&\quad +\tfrac{1}{2}rs(r+s-2)\left[\sqrt{ln(r+s)}+\sqrt{ln(r+s)}\right]\\
&= rs\left[\sqrt{lnr}+\sqrt{lns}+2\sqrt{ln(r+s)}+(r+s-2)\sqrt{ln(r+s)}\right],\\
\Phi[T_2(K_{r,s})] &= rs\left[\tfrac{1}{\sqrt{lnr}+\sqrt{ln(r+s)}}\right]+rs\left[\tfrac{1}{\sqrt{lns}+\sqrt{ln(r+s)}}\right]\\
&\quad +\tfrac{rs(r+s-2)}{2}\left[\tfrac{1}{\sqrt{r+s}+\sqrt{r+s}}\right]\\
&= rs\left[\tfrac{1}{\sqrt{lnr}+\sqrt{ln(r+s)}}+\tfrac{1}{\sqrt{lns}+\sqrt{ln(r+s)}}+\tfrac{rs(r+s-2)}{4\sqrt{r+s}}\right],\\
\Psi[T_2(K_{r,s})] &= rs|lnr-ln(r+s)|+rs|lns-ln(r+s)|\\
&\quad +\tfrac{rs(r+s-2)}{2}|ln(r+s)-ln(r+s)|\\
&= rs\left[|ln(\tfrac{r}{r+s})|+|ln(\tfrac{s}{r+s})|\right],\\
\phi[T_2(K_{r,s})] &= rs|ln^2r-ln^2(r+s)|+rs|ln^2s-ln^2(r+s)|\\
&\quad +\tfrac{rs(r+s-2)}{2}|ln^2(r+s)-ln^2(r+s)|\\
&= rs\left[|ln^2r-ln^2(r+s)|+|ln^2s-ln^2(r+s)|\right].
\end{aligned}$$

□

By the above result with $r = s$, we have completed the regular bipartite graphs $K_{r,r}$ with $r > 1$.

6. Total graphs

The total graph of a graph G is denoted by $T(G)$ with vertex set $V(G) \cup E(G)$ and any two vertices of $T(G)$ are adjacent if and only if they are either incident or adjacent in G. For more details, see [1].

Here the discrete adriatic indices of the total graph of the r-regular and complete bipartite graphs are discussed.

Theorem 9. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\Upsilon[T(G)] = nr(r+2)\sqrt{ln2r}$$

and

$$\Phi[T(G)] = \frac{nr(r+2)}{4\sqrt{ln2r}}.$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the total graph of G is also a $2r$-regular graph with $\frac{nr}{2} + nr$ vertices and $\frac{nr^2}{2} + nr$ edges. Then

$$\begin{aligned}
\Upsilon[T(G)] &= \left(\tfrac{nr^2}{2}+nr\right)[\sqrt{ln2r}+\sqrt{ln2r}]\\
&= nr(r+2)\sqrt{ln2r},\\
\Phi[T(G)] &= \left(\tfrac{nr^2}{2}+nr\right)\tfrac{1}{\sqrt{ln2r}+\sqrt{ln2r}}\\
&= \tfrac{nr(r+2)}{4\sqrt{ln2r}}.
\end{aligned}$$

□

Theorem 10. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then*

$$\Upsilon[T(K_{r,s})] = 2rs\left[\sqrt{ln2s} + \sqrt{ln2r} + \sqrt{ln(r+s)}\right] + rs(r+s-2)\sqrt{ln(r+s)},$$

$$\Phi[T(K_{r,s})] = rs\left[\frac{1}{\sqrt{ln2s} + \sqrt{2r}} + \frac{1}{\sqrt{ln2s} + \sqrt{ln(r+s)}} + \frac{1}{\sqrt{ln2r} + \sqrt{ln(r+s)}}\right]$$

$$+\frac{rs(r+s-2)}{4\sqrt{ln(r+s)}},$$

$$\Psi[T(K_{r,s})] = rs\left[\left|ln\frac{s}{r}\right| + \left|ln\frac{2s}{r+s}\right| + \left|ln\frac{2r}{r+s}\right|\right]$$

and

$$\phi[T(K_{r,s})] = rs\left[\left|ln^2 2s - ln^2 2r\right| + \left|ln^2 2s - ln^2(r+s)\right| + \left|ln^2 2r - ln^2(r+s)\right|\right].$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. Then the total graph of $K_{r,s}$ has $r+s+rs$ vertices and $\frac{1}{2}rs(r+s-2)+3rs$ edges. The edge partition of $T(K_{r,s})$ is as follows:

(d_u, d_v) where $uv \in E(G)$	$(2s, 2r)$	$(2s, r+s)$	$(2r, r+s)$	$(r+s, r+s)$
Number of edges	rs	rs	rs	$rs(r+s-2)/2$

Then

$$\begin{aligned}
\Upsilon[T(K_{r,s})] &= rs\Big[\sqrt{ln2s}+\sqrt{ln2r}\Big]+rs\Big[\sqrt{ln2s}+\sqrt{ln(r+s)}\Big] \\
&\quad +rs\Big[\sqrt{ln2r}+\sqrt{ln(r+s)}\Big]+\tfrac{rs(r+s-2)}{2}\Big[\sqrt{ln(r+s)}+\sqrt{ln(r+s)}\Big] \\
&= 2rs\Big[\sqrt{ln2s}+\sqrt{ln2r}+\sqrt{ln(r+s)}\Big]+rs(r+s-2)\sqrt{ln(r+s)}, \\
\Phi[T(K_{r,s})] &= rs\Big[\tfrac{1}{\sqrt{ln2s}+\sqrt{2r}}\Big]+rs\Big[\tfrac{1}{\sqrt{ln2s}+\sqrt{ln(r+s)}}\Big]+rs\Big[\tfrac{1}{\sqrt{ln2r}+\sqrt{ln(r+s)}}\Big] \\
&\quad +\tfrac{rs(r+s-2)}{2}\Big[\tfrac{1}{\sqrt{ln(r+s)}+\sqrt{ln(r+s)}}\Big] \\
&= rs\Big[\tfrac{1}{\sqrt{ln2s}+\sqrt{2r}}+\tfrac{1}{\sqrt{ln2s}+\sqrt{ln(r+s)}}+\tfrac{1}{\sqrt{ln2r}+\sqrt{ln(r+s)}}\Big] \\
&\quad +\tfrac{rs(r+s-2)}{4\sqrt{ln(r+s)}}, \\
\Psi[T(K_{r,s})] &= rs\Big|ln2s-ln2r\Big|+rs\Big|ln2s-ln(r+s)\Big|+rs\Big|ln2r-ln(r+s)\Big| \\
&\quad +\tfrac{rs(r+s-2)}{2}\Big|ln(r+s)-ln(r+s)\Big| \\
&= rs\Big[\Big|ln\tfrac{s}{r}\Big|+\Big|ln\tfrac{2s}{r+s}\Big|+\Big|ln\tfrac{2r}{r+s}\Big|\Big], \\
\phi[T(K_{r,s})] &= rs\Big|ln^2 2s-ln^2 2r\Big|+rs\Big|ln^2 2s-ln^2(r+s)\Big|+rs\Big|ln^2 2r-ln^2(r+s)\Big| \\
&\quad +\tfrac{rs(r+s-2)}{2}\Big|ln^2(r+s)-ln^2(r+s)\Big| \\
&= rs\Big[\Big|ln^2 2s-ln^2 2r\Big|+\Big|ln^2 2s-ln^2(r+s)\Big|+\Big|ln^2 2r-ln^2(r+s)\Big|\Big].
\end{aligned}$$

□

By the above result with $r=s$, we have completed the study of regular bipartite graphs $K_{r,r}$ with $r>2$.

7. Jump-Graphs

The jump-graph $J(G)$ of a graph G is the graph defined on $E(G)$ where two vertices are adjacent if and only if they are not adjacent in G. For more details, see e.g. [8].

Theorem 11. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\Upsilon[J(G)] = \frac{nr}{2}(n-2r+1)\sqrt{ln(n-2r+1)}$$

and

$$\Phi[J(G)] = \frac{nr(n-2r+1)}{8\sqrt{ln(n-2r+1)}}.$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the jump-graph of G is also $n-2r+1$-regular with $\frac{nr}{2}$ vertices and $\frac{nr}{4}(n-2r+1)$ edges. Then

$$\begin{aligned}
\Upsilon[J(G)] &= \tfrac{nr}{4}(n-2r+1)[\sqrt{ln(n-2r+1)}+\sqrt{ln(n-2r+1)}] \\
&= \tfrac{nr}{2}(n-2r+1)\sqrt{ln(n-2r+1)}, \\
\Phi[J(G)] &= \tfrac{nr}{4}(n-2r+1)\left[\frac{1}{\sqrt{ln(n-2r+1)}+\sqrt{ln(n-2r+1)}}\right] \\
&= \frac{nr(n-2r+1)}{8\sqrt{ln(n-2r+1)}}.
\end{aligned}$$

□

Theorem 12. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then*

$$\Upsilon[J(K_{r,s})] = rs(r-1)(s-1)[\sqrt{ln(r-1)+ln(s-1)}]$$

and

$$\Phi[J(K_{r,s})] = \frac{rs(r-1)(s-1)}{4\sqrt{ln(r-1)+ln(s-1)}}.$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then the jump-graph of $K_{r,s}$ is also an $(r-1)(s-1)$-regular graph with rs vertices and $\frac{rs}{2}(r-1)(s-1)$ edges. Then

$$\begin{aligned}
\Upsilon[J(K_{r,s})] &= \tfrac{rs}{2}(r-1)(s-1)[\sqrt{ln(r-1)(s-1)}+\sqrt{ln(r-1)(s-1)}] \\
&= rs(r-1)(s-1)[\sqrt{ln(r-1)+ln(s-1)}], \\
\Phi[J(K_{r,s})] &= \tfrac{rs}{2}(r-1)(s-1)\left[\frac{1}{\sqrt{ln(r-1)(s-1)}+\sqrt{ln(r-1)(s-1)}}\right. \\
&= \frac{rs(r-1)(s-1)}{4\sqrt{ln(r-1)+ln(s-1)}}.
\end{aligned}$$

□

8. Para-line graphs

We now define a new concept called the para-line graph $P(G)$ of a graph G. Given a graph G, insert two vertices to each edge xy of G. These two vertices will be denoted by $(x,y),(y,x)$ where (x,y) (resp. (y,x)) is the one incident to x (resp. y). We define the vertex set and the edge set as follows:

$$V(P(G)) = \{(x,y) \in V(G) \times V(G) : \ xy \in E(G)\}$$

and

$$E(P(G)) = \{((x,w),(x,z)) : \ (x,w),(x,z) \in V(P(G)), w \neq z\} \cup \{((x,y),(y,x)) : \ xy \in E(G)\}.$$

The resultant graph is called the para-line graph of G. For more details, see [17].

Theorem 13. *Let G be an r-regular graph with $n \geq 2$ vertices. Then*

$$\Upsilon[P(G)] = 2mr\sqrt{lnr}$$

and

$$\Phi[P(G)] = \frac{mr}{2\sqrt{lnr}}.$$

Proof. Let G be an r-regular graph with $n \geq 2$ vertices. Then the para-line graph of G is also an r-regular graph with $2m$ vertices and mr edges. Also

$$\begin{aligned}\Upsilon[P(G)] &= mr[\sqrt{lnr} + \sqrt{lnr}] \\ &= 2mr\sqrt{lnr}, \\ \Phi[P(G)] &= mr\left[\frac{1}{\sqrt{lnr}+\sqrt{lnr}}\right] \\ &= \frac{mr}{2\sqrt{lnr}}.\end{aligned}$$

□

Theorem 14. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then*

$$\Upsilon[P(K_{r,s})] = rs[r\sqrt{lnr} + s\sqrt{lns}],$$

$$\Phi[P(K_{r,s})] = rs\left[\frac{1}{\sqrt{lnr} + \sqrt{lns}} + \frac{r-1}{4\sqrt{lnr}} + \frac{s-1}{4\sqrt{lns}}\right],$$

$$\Psi[P(K_{r,s})] = rs\left|ln\frac{r}{s}\right|$$

and

$$\phi[P(K_{r,s})] = rs|ln^2 r - ln^2 s|.$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$. Then the para-graph of $K_{r,s}$ has $2rs$ vertices and $\frac{rs(r+s)}{2}$ edges. The edge partition of $P(K_{r,s})$ is as follows:

(d_u, d_v) where $uv \in E(G)$	(r, s)	(r, r)	(s, s)
Number of edges	rs	$rs(r-1)/2$	$rs(s-1)/2$

Then

$$\begin{aligned}\Upsilon[P(K_{r,s})] &= rs[\sqrt{lnr} + \sqrt{lns}] + \tfrac{rs(r-1)}{2}[\sqrt{lnr} + \sqrt{lnr}] + \tfrac{rs(s-1)}{2}[\sqrt{lns} + \sqrt{lns}] \\ &= rs[r\sqrt{lnr} + s\sqrt{lns}], \\ \Phi[P(K_{r,s})] &= rs\left[\tfrac{1}{\sqrt{lnr}+\sqrt{lns}}\right] + \tfrac{rs(r-1)}{2}\left[\tfrac{1}{\sqrt{lnr}+\sqrt{lnr}}\right] + \tfrac{rs(s-1)}{2}\left[\tfrac{1}{\sqrt{lns}+\sqrt{lns}}\right] \\ &= rs\left[\tfrac{1}{\sqrt{lnr}+\sqrt{lns}} + \tfrac{r-1}{4\sqrt{lnr}} + \tfrac{s-1}{4\sqrt{lns}}\right], \\ \Psi[P(K_{r,s})] &= rs|lnr - lns| + \tfrac{rs(r-1)}{2}|lnr - lnr| + \tfrac{rs(s-1)}{2}|lns - lns| \\ &= rs\left|ln\tfrac{r}{s}\right|, \\ \phi[P(K_{r,s})] &= rs|ln^2 r - ln^2 s| + \tfrac{rs(r-1)}{2}|ln^2 r - ln^2 r| + \tfrac{rs(s-1)}{2}|ln^2 s - ln^2 s| \\ &= rs|ln^2 r - ln^2 s|.\end{aligned}$$

□

Acknowledgment 1. *The second author is thankful to the authorities of V. S. K. University, Ballari for Scholarship.*

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