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Topological Indices and Bounds for the Energy of Fibonacci Graph

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Abstract

Fibonacci sequence of numbers plays a significant role in communication networks, coding theory, encryption and many such areas. A Fibonacci graph $F_{d,2n}$ is a d regular graph where d is such that F_d is the largest Fibonacci number less than or equal to n. In this paper we present few lower and upper bounds for the energy of the Fibonacci graph in terms of n, d and the determinant of the adjacency matrix, $detA$. We have also investigated some topological indices for the Fibonacci graph and the obtained results are tabulated.

Keywords: Fibonacci Graph, energy of a graph, bounds for energy of a graph, Ramanujan graph, Topological Indices.

2000 Mathematical subject classifications: 05C50, 05C35.

1 Introduction

In this paper we consider only finite, connected, simple gaphs. Let $G(V,E)$ be a connected graph with n vertices and m edges. Let $\lambda_1 \geq \lambda_2 \geq, \ldots, \geq \lambda_n$ be the eigenvalues of the $(0,1)$-adjacency matrix $A = A(G)$ of G. These eigenvalues are called the eigenvalues of G and they form its spectrum. The energy of a graph G is introduced by Gutman [7] and was defined as

$$\varepsilon(G) = \sum_{i=1}^{n} |\lambda_i|.$$

The largest eigenvalue, namely λ_1 is called the spectral radius of G and is well known that

$$detA = \prod_{i=1}^{n} \lambda_i$$

and

$$\sum_{i=1}^{n} \lambda_i = 0.$$

A graph in which all the vertices are of same degree d is called a d-regular graph.

In this article we mainly focus on an interesting family of graphs called Fibonacci graphs. These graphs have been used for the purpose of performing effective communications in networks.

A Fibonacci graph $F_{d,2n}$ is a graph with vertex set $V = V_1 \cup V_2$, where

$$V_1 = \{v_1, v_2, \dots, v_n\},$$

$$V_2 = \{v_{n+1}, v_{n+2}, \dots, v_{2n}\}$$

and v_iv_j for $1 \le i \le n$ and $n+1 \le j \le 2n$ is an edge if $j-i+1$ or $j-i+1-n$ is a member of the set

$$S = \{F_1, F_2, \dots, F_d\}.$$

Here d is such that F_d is the largest Fibonacci number less than or equal to n and the Fibonacci numbers are defined as follows:

$$F_0 = 1, F_1 = 1 \quad \text{and} \quad F_n = F_{n-1} + F_{n-2} \quad \text{for} \quad n \ge 2.$$

The below graph is a Fibonacci graph of $F_{5,20}$ with degree of each vertex being 5 and with 20 vertices.

For the various energies of the Fibonacci graph $F_{d,2n}$ and related results one may refer the paper by Adiga et al. [1].

Numerous upper and lower bounds for graph energy are known [15]. Among the pioneering results of the theory of graph energy are the lower and upper bound of $\varepsilon(G)$, discovered by McClelland [16]. The upperbound $\sqrt{2mn}$ has been much studied in the chemical literature. McClelland's lower bound for energy depends on the parameter n, m and $detA$ given by,

$$\varepsilon(G) \ge \sqrt{2m + n(n-1)|detA|^{2/n}}.$$

For bipartite graphs,

$$\varepsilon(G) \ge \sqrt{4m + n(n-2)|detA|^{2/n}}.$$

Caprossi et al.[4] discovered the following simple lower bound:

$$\varepsilon(G) \ge 2\sqrt{m}.$$

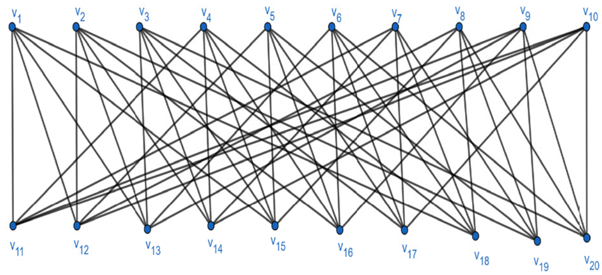

Figure 1: Fibonacci Graph $F_{5,20}$

Recently Das et al.[5] have given the following lower bound

$$\varepsilon(G) \geq \frac{2m}{n} + n - 1 + ln\left(\frac{n|detA|}{2m}\right).$$

Koolen and Moulton [14] gave the following upper bound for $\varepsilon(G)$:

$$\text{If} \quad 2m \geq n, \quad \text{then} \quad \varepsilon(G) \leq \frac{2m}{n} + \sqrt{(n-1)[2m - (\frac{2m}{n})^2]}.$$

Adiga and Rakshith [2] gave upper bound for the extended energy of graphs and extended equienergetic graphs as

$$\varepsilon_{ex}(G) \leq \frac{\Delta}{\delta}\epsilon(G)$$

and

$$\varepsilon_{ex}(G) \leq \sqrt{\frac{n}{2}\left(\frac{F(G)}{\delta^2} + \delta^2 r(G)\right)},$$

where Δ and δ are the maximum and minimum degree of the graph G respectively, $F(G)$ is the forgotten topological index and $r(G)$ is the inverse degree sum of the graph G.

In Section 2 of this paper we establish upper and lower bounds for the energy of Fibonacci graph $F_{d,2n}$ under certain conditions.

In this era of unexpected pandemic, there is always a need for a rapid development of chemical and pharmaceutical techniques which involves new nanomaterials, crystalline materials and new drugs. To determine the chemical properties of these materials and drugs it involves large number of experiments and herculean tasks for the pharmaceutical researchers. Fortunately, the strong connection between topology of molecular structures and their physical behaviors, chemical characteristics and biological features, such as meting point, boiling point, and toxicity of drugs ease this task.

The study of the topological indices on chemical structure of chemical materials and drugs can provide a theoretical basis for the manufacturing of drugs and chemical materials.

Topological indices are designed on the basis of transformation of a molecular graph into a number that characterizes the topology of the molecular graph. Some major types of topological indices of graphs are degree-based, distance-based and counting-related topological indices. Topological indices have many applications in theoretical chemistry, especially in QSPR/QSAR research.

The oldest topological index is the Weiner index. Harary Weiner [20, 21, 22, 23] in his series of papers showed that the sum of distances between all pairs of vertices correlate well with various properties of alkanes. Later it was introduced in Graph theory as distance of graph and transmission of graph. Now a days it is used in quantitative structure-activity relationship studies. Other well studied topological indices include Hosoya Z index [12], the fist and second Zagreb index [10], the Randić index [18] and others. Now a days vast number of degree based topological indices are in the focus of interest of mathematicians and Mathematical chemists.

In the concluding section of this paper we compute some topological indices of the Fibonacci graph.

2 Upper and Lower Bounds for the Energy of $F_{d,2n}$

In recent times, a lot of interest has been shown on Ramanujan graphs by mathematicians in various fields like Number Theory and Communication Theory. For more details one may refer the paper by Ram Murthy [17] and the reference cited therein.

In Spectral graph theory, a "Ramanujan Graph" is a regular graph whose spectral gap is almost as large as possible and they are excellent spectral expanders. In other words if $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \geq \lambda_n$ are the eigenvalues of a connected d-regular graph and if

$$\lambda(G) = max_{i\neq 1}|\lambda_i| = max(|\lambda_2||\lambda_3|\ldots|\lambda_n|),$$

then G is Ramanujan if

$$\lambda(G) \leq 2\sqrt{(d-1)}.$$

Ramanujan graphs resolve extremal problems in communication network theory and they are also helpful in cryptography.

In this section, we give upper and lower bounds for the energy $\varepsilon(F_{d,2n})$ in terms of n, d and the determinant of the adjacency matrix.

Lemma 2.1. *[24] If G is a bipartite graph with n vertices and if $k = \lfloor \frac{n}{4} \rfloor$, then*

$$\lambda_2(G) \leq \begin{cases} k & \text{if } n = 4k \text{ or } 4k+1 \\ \sqrt{k(k+1)} & \text{if } n = 4k+2 \text{ or } 4k+3. \end{cases}$$

In particular if $k = \lfloor \frac{n}{2} \rfloor$, then

$$\lambda_2(F_{d,2n}) \leq \begin{cases} k & \text{if } n \text{ is even} \\ \sqrt{k(k+1)} & \text{if } n \text{ is odd.} \end{cases} \tag{2.1}$$

Lemma 2.2. *[24] If G is a connected graph, then*

$$\lambda_2(G) \leq \frac{\sqrt{(n^2-4)}}{2} - 1.$$

In particular

$$\lambda_2(F_{d,2n}) \leq \sqrt{n^2-1} - 1, \tag{2.2}$$

where λ_2 is the second largest eigenvalue of the graph $F_{d,2n}$.

Theorem 2.3. *We have*

$$\varepsilon(F_{d,2n}) \leq d + (\sqrt{n^2-1} - 1) + 2nd - d^2 - log|detA| + logd + log(\sqrt{(n^2-1)} - 1),$$

where A is the adjacency matrix of $F_{d,2n}$.

Proof. Since $F_{d,2n}$ is non singular, we have $|\lambda_i| > 0$ for $i = 1, 2, \ldots, 2n$. Thus

$$|detA| = \prod_{i=1}^{2n} |\lambda_i| > 0.$$

Also, we have

$$\sum_{i=1}^{2n} \lambda_i^2 = \sum_{i=1}^{2n} |\lambda_i|^2 = 2nd.$$

Now, Consider the function

$$f(x) = x^2 - x - log(x), \quad x > 0.$$

Since $f'(x) = 2x - 1 - \frac{1}{x}$, $f'(x)$ increases for $x \geq 1$ and decreases for $x < 1$.

Thus $f(x) \geq f(1) = 0$ for $x > 0$.

This implies that $x \leq x^2 - log(x)$ for $x > 0$, with equality holds if and only if $x = 1$.

By virtue of this result and Lemma 2.2 we obtain

$$\begin{aligned}
\varepsilon(F_{d,2n}) &= \lambda_1 + \lambda_2 + \sum_{i=3}^{2n} |\lambda_i| \\
&\leq d + \sqrt{n^2-1} - 1 + \sum_{i=3}^{2n} |\lambda_i| \\
&\leq d + (\sqrt{n^2-1} - 1) + \sum_{i=3}^{2n} |\lambda_i|^2 - \sum_{i=3}^{2n} log(|\lambda_i|) \\
&= d + \sqrt{n^2-1} - 1 + 2nd - d^2 - \lambda_2^2 - log(|detA|) + logd + log|\lambda_2|
\end{aligned}$$

which implies

$$\varepsilon(F_{d,2n}) \leq d + \sqrt{n^2-1} - 1 + 2nd - d^2 - log|detA| + logd + log(\sqrt{n^2-1} - 1).$$

This completes the proof.

Remark 2.4. *If we use Lemma 2.1, then for odd n, we have*

$$\varepsilon(F_{d,2n}) \leq d + \sqrt{\lfloor \frac{n}{2} \rfloor (\lfloor \frac{n}{2} \rfloor + 1)} + 2nd - d^2 - log|detA| + logd + log\sqrt{\lfloor \frac{n}{2} \rfloor (\lfloor \frac{n}{2} \rfloor + 1)}.$$

Remark 2.5. *If we use Lemma 2.1, then for even n, we have*

$$\varepsilon(F_{d,2n}) \leq d + \lfloor \frac{n}{2} \rfloor + 2nd - d^2 - log|detA| + logd + log\lfloor \frac{n}{2} \rfloor.$$

Similar to the above result in the next theorem we give another upper bound for $\varepsilon(F_{d,2n})$.

Theorem 2.6. *We have*

$$\varepsilon(F_{d,2n}) \leq d + \sqrt{d(2n-d)(2n-1)}.$$

Proof. Consider the function

$$f(x) = x^2 - kx + 1, \quad k > 0$$

which is increasing for $x \geq \frac{k}{2}$ and decreasing for $x < k/2$. This implies

$$f(x) \geq f(\frac{k}{2}).$$

Thus

$$x^2 - kx + 1 \geq \frac{k^2}{4} - \frac{k^2}{2} + 1.$$

Or equivalently,

$$x \leq \frac{x^2}{k} + \frac{k}{4}, \quad k > 0. \tag{2.3}$$

Since $F_{d,2n}$ is a d-regular graph, we have $\lambda_1 = d$. Therefore, on using (2.3), we have

$$\begin{aligned}\varepsilon(F_{d,2n}) &= \lambda_1 + \sum_{i=2}^{2n} |\lambda_i| \\ &\leq \lambda_1 + \sum_{i=2}^{2n} |\frac{\lambda_i^2}{k}| + \sum_{i=2}^{2n} |\frac{k}{4}| \\ &= d - \frac{d^2}{k} + \frac{2nd}{k} + \frac{2nk}{4} - \frac{k}{4}.\end{aligned}$$

Let

$$g(x) = d - \frac{d^2}{x} + \frac{2nd}{x} + \frac{2nx}{4} - \frac{x}{4}.$$

We can easily verify that the function $g(x)$ is minimum at $x = 2\sqrt{\frac{2nd-d^2}{2n-1}}$ and the minimum value is

$$d + \sqrt{d(2n-d)(2n-1)}.$$

Hence,

$$\varepsilon(F_{d,2n}) \quad \leq \quad d + \sqrt{d(2n-d)(2n-1)}.$$

□

Remark 2.7. *One can also prove the above theorem on applying Cauchy's-Schwarz's inequality. For more details see [3].*

Theorem 2.8. *Suppose $F_{d,2n}$ $(n \geq 3)$ is not a Ramanujan graph. Then*

$$\varepsilon(F_{d,2n}) \geq 4\sqrt{d-1} + (2n-2)\left(\frac{|detA|}{4(d-1)}\right)^{\dfrac{1}{2n-2}},$$

where A is the adjacency matrix of $F_{d,2n}$.

Proof. We know that $F_{d,2n}(n \geq 3)$ is a d-regular, bipartite graph and its eigenvalues are of the form

$$\lambda_1 = -\lambda_{2n}, \lambda_2 = -\lambda_{2n-1}, \ldots, \lambda_n = -\lambda_{n+1}.$$

By the arithmetic-geometric mean inequality, we have

$$\begin{aligned}
\varepsilon(F_{d,2n}) &= 2\lambda_2 + \sum_{\substack{i=1 \\ i\neq 2,2n-1}}^{2n} |\lambda_i| \\
&\geq 2\lambda_2 + (2n-2)\left(\prod_{\substack{i=1 \\ i\neq 2,2n-1}}^{2n-2} |\lambda_i|\right)^{\dfrac{1}{2n-2}} \\
&= 2\lambda_2 + (2n-2)\left(\frac{|detA|}{{\lambda_2}^2}\right)^{\dfrac{1}{2n-2}}.
\end{aligned}$$

□

Now we consider the function

$$f(x) = 2x + (2n-2)\left(\frac{|detA|}{x^2}\right)^{\dfrac{1}{2n-2}}.$$

Observe that f is an increasing function for $x \geq (|detA|)^{\dfrac{1}{2n}}$. Moreover, we have

$$\varepsilon(F_{d,2n}) \leq \sqrt{4n^2 d} = 2n\sqrt{d}. \tag{2.4}$$

Since $F_{d,2n}$ is not a Ramanujan graph, from (2.4) we have

$$\lambda_2 > 2\sqrt{d-1} > \sqrt{d} \geq \frac{\varepsilon(F_{d,2n})}{2n} = \frac{\sum_{i=1}^{2n}|\lambda_i|}{2n} \geq |detA|^{\dfrac{1}{2n}}.$$

This implies

$$f(\lambda_2) \geq f(2\sqrt{d-1}),$$

which implies

$$2\lambda_2 + (2n-2)\left(\frac{|detA|}{{\lambda_2}^2}\right)^{\dfrac{1}{2n-2}} \geq 4\sqrt{d-1} + (2n-2)\left(\frac{|detA|}{4(d-1)}\right)^{\dfrac{1}{2n-2}}.$$

Hence

$$\varepsilon(F_{d,2n}) \geq 4\sqrt{d-1} + (2n-2)\left(\frac{|detA|}{4(d-1)}\right)^{\dfrac{1}{2n-2}}.$$

More generally we have the following theorem.

Theorem 2.9. *Suppose $F_{d,2n}(n \geq 3)$ is not a Ramanujan graph and if*

$$\lambda_2 \geq x_0 > 2\sqrt{d-1},$$

then

$$\varepsilon(F_{d,2n}) \geq 2x_0 + (2n-2)\left(\frac{|detA|}{x_0^2}\right)^{\dfrac{1}{2n-2}}.$$

3 Topological Indices for the Fibonacci graph $F_{d,2n}$

In this section we compute few prominent distance based, degree based, degree-distance based and eccentricity based topological indices for the Fibonacci graph.

3.1 Distance based Topological indices

Wiener index is a distance-based topological descriptor which is widely studied as it is readily computed and it correlates with many physio-chemical properties of organic compounds. Weiner index has found applications in fields such as crystallography, communication theory, facility location, ornithology and so on.

Since the advent of Wiener index several other topological indices have been formulated such as Weiner polarity index, the reciprocal complementary Wiener index, old Harary index and Harary index.

3.1.1 Weiner index

The Weiner index $W(G)$ is defined as the sum of the distance between all pairs of vertices in G, that is

$$W(G) = \sum_{u,v \epsilon V(G)} d(u,v).$$

Lemma 3.1. *The Fibonacci graph $F_{d,2n}$ is a graph with vertex set $V = V_1 \cup V_2$ where,*

$$V_1 = \{v_1, v_2, \ldots, v_n\},$$

$$V_2 = \{v_{n+1}, v_{n+2}, \ldots, v_{2n}\}.$$

Then

$$d_{F_{d,2n}}(v_i, v_{n+j}) = \begin{cases} 1, & \text{if} \quad v_i v_{n+j} \in E(F_{d,2n}) \\ 3, & \text{if} \quad v_i v_{n+j} \notin E(F_{d,2n}), \end{cases} \quad \text{where} \quad 1 \leq i \leq j \leq n,$$

$$d_{F_{d,2n}}(v_i, v_k) = 2 \quad \text{for} \quad 1 \leq i < k \leq n,$$

and

$$d_{F_{d,2n}}(v_j, v_l) = 2 \quad \text{for} \quad n+1 \leq j < l \leq 2n.$$

Theorem 3.2. *The Weiner index of $F_{d,2n}$ is given by*

$$W(F_{d,2n}) = 5n^2 - 2n - 2nd.$$

Proof. We know that

$$W(F_{d,2n}) = \sum d_{F_{d,2n}}(v_i, v_j) = A_1 + A_2 \tag{3.1}$$

where

$$A_1 = \sum_{1 \leq i \leq j \leq n} d_{F_{d,2n}}(v_i, v_{n+j})$$

and

$$A_2 = \sum_{v_i v_k \notin E(F_{d,2n})} d_{F_{d,2n}}(v_i, v_k) + \sum_{v_j v_l \notin E(F_{d,2n})} d_{F_{d,2n}}(v_j, v_l),$$

where $1 \leq i < k \leq n$ and $n+1 \leq j < l \leq 2n$.
By Lemma 3.1,

$$\begin{aligned} A_1 &= \sum_{v_i v_{n+j} \in E(F_{d,2n})} 1 + \sum_{v_i v_{n+j} \notin E(F_{d,2n})} 3 \\ &= m + n(n-d)3 \qquad (3.2) \\ &= nd + n(n-d)3 \\ &= 3n^2 - 2nd. \end{aligned}$$

and

$$
\begin{aligned}
A_2 &= \sum_{v_i v_k \notin E(F_{d,2n})} 2 + \sum_{v_j v_l \notin E(F_{d,2n})} 2 \\
&= 2\left(n(n-1) - \frac{(n-1)n}{2}\right) + 2\left(n(n-1) - \frac{n(n-1)}{2}\right) \\
&= 2n^2 - 2n.
\end{aligned} \tag{3.3}
$$

□

Using results (3.2) and (3.3) in (3.1) , we get the required result.
The below table gives the other distance based topological indices for $F_{d,2n}$.

Table 1 :Distance based Topological Indices.

The Weiner Polarity index of $F_{d,2n} = W_p(F_{d,2n}) = n(n-d)$
The Reciprocal Complementary Weiner index of $F_{d,2n} = RCW(F_{d,2n}) = \dfrac{8nd + 27n^2 - 15n}{60}$
Old Harary index of $F_{d,2n} = H_{old}(F_{d,2n}) = \dfrac{13n^2 - 32nd - 9n}{36}$
Harary Index of $F_{d,2n} = H(F_{d,2n}) = \dfrac{5n^2 + 4nd - 3n}{6}$

3.2 Degree Based Topological Indices

If $G = (V, E)$ represents a graph with vertex set $V = V(G)$ and edge set $E = E(G)$ then the number of edges incident on vertex v is called its degree and is denoted $d_v(G)$ or d_v. The concept of degree in graph theory is closely related (but not identical) to the concept of valence in Chemistry.

The first degree based structure descriptors was put forward by Milan Randić in his paper [18] "On characterization of molecular branching". This topological index is the most studied, applied and popular among all the topological indices.

The topological indices considered in this subsection are: Zagreb index, The forgotten index, Narumi-Katayama index, The Atom-bond connectivity index, Augmented Zagreb index, Geometric-Arithmetic index, Sum-connectivity index and Harmonic index. A detailed survey about all these topological indices can be found in [9] and the references mentioned there in. The table below gives the mathematical expressions of the topological indices listed above for the Fibonaaci graph $F_{d,2n}$.

Table 2 : Degree based Topological Indices.

First general Zagreb index of $F_{d,2n}$= $M_1^{\alpha}(F_{d,2n}) = 2nd^{\alpha}$
Second Zagreb index of $F_{d,2n} = M_2(F_{d,2n}) = nd^3$
F-index of $F_{d,2n}$= $F(F_{d,2n}) = 2nd^3$
First Zagreb co-index of $F_{d,2n} = \overline{M_1}(F_{d,2n}) = 2nd(2n-d-1)$
Second Zagreb co-index of $F_{d,2n}$= $\overline{M_2}(F_{d,2n}) = nd^2(2n-d-1)$
The reduced second Zagreb index of $F_{d,2n} = RM_2(G) = nd(d-1)^2$
The first multiplicative Zagreb index of $F_{d,2n} = \Pi_1(F_{d,2n}) = d^{4n}$
The second multiplicative Zagreb index of $F_{d,2n} = \Pi_2(F_{d,2n}) = d^{2nd}$
Narumi-katayama index of $F_{d,2n}$ =$NK(F_{d,2n}) = d^{2n}$
Randić index of $F_{d,2n} = R_{\alpha}(F_{d,2n}) = nd^{2\alpha+1}$
The general Randić co-index of $F_{d,2n} = \overline{R_{\alpha}}(F_{d,2n}) = (2n^2-n-nd)d^{2\alpha}$
The ABC-index or Atom-Bond-Connectivity index of $F_{d,2n} = ABC(F_{d,2n}) = n\sqrt{2(d-1)}$
The Harmonic index of $F_{d,2n} = H(F_{d,2n}) = n$
The Augmented Zagreb index of $F_{d,2n} = AZI(F_{d,2n}) = \frac{nd^7}{(2d-2)^3}$
The Geometric-Arithmetic index of $F_{d,2n} = GA(F_{d,2n}) = nd$
The Sum-Connectivity index of $F_{d,2n} = SCI_{\alpha}(F_{d,2n}) = 2nd^{\alpha+1}$
General Sum-Connectivity Co-index of $F_{d,2n} = \overline{SCI_{\alpha}}(F_{d,2n}) = n(2n-d-1)(2d)^{\alpha}$

3.3 Degree and Distance based Topological indices

A large number of topological indices are based on both vertex degree and the graph distance which were modified from Weiner index. The degree distance index was first introduced by Dobrynin and Kochetova [6] . Later

Gutman [8] gave the multiplicative variant of the degree distance which is now known as Gutman index. In many applications of graph invariants it is preferred that the contribution of vertex pairs diminishes with distance. Hua and Zhang [13] proposed a new graph invariant called the reciprocal degree-distance index. Below table gives the value of these indices for the Fibonacci graph $F_{d,2n}$.

Table 3: Degree-Distance based Topological Indices.

Gutman Index of $F_{d,2n} = Gut(F_{d,2n}) = d^2W(F_{d,2n}) = nd^2(5n-2d-2)$.
The Degree-distance index of $F_{d,2n} = DD(G) = 2dW(F_{d,2n}) = 2d(5n^2-2n-2nd)$.
The additively weighted Harary index of $F_{d,2n} = H_A(F_{d,2n}) = \frac{nd}{3}(5n+4d-3)$.
Multiplicatively weighted Harary index of $F_{d,2n} = H_M(F_{d,2n}) = \frac{nd^2}{6}(5n+4d-3)$.

3.4 Eccentricity based Topological Indices

The eccentricity $\epsilon(v)$ of a vertex $v \in V(G)$ is defined as

$$\epsilon(v) = max\{d(v,w)|w \in V(G)\}.$$

The minimum eccentricity in a graph G is the radius, while the maximum eccentricity is known as the diameter. Topological indices which are based on the eccentricity of the vertices in a graph G are known as eccentricity based topological indices.

Eccentricity-connectivity index was introduced by Sharma et al. [19]. Zagreb eccentricity indices are investigated by Ghorbani and Hosseinzadeh [11].

Below table shows some eccentricity based topological indices for the Fibonacci graph $F_{d,2n}$.

Table 4: Eccentricity based Topological Indices.

The first total Eccentricity index of $F_{d,2n} = \xi(F_{d,2n}) = 6n$.
First Zagreb Eccentricity index of $F_{d,2n} = E_1(F_{d,2n}) = 18n$.
Second Zagreb Eccentricity index of $F_{d,2n} = E_2(F_{d,2n}) = 9nd$.
Eccentricity Connectivity index of $F_{d,2n} = \xi^c(F_{d,2n}) = 6nd$.
Connective Eccentricity index of $F_{d,2n} = C^{\xi}(F_{d,2n}) = \frac{2nd}{3}$.
The Eccentric distance sum index of $F_{d,2n} = H_M(F_{d,2n}) = 30n^2 - 12nd - 12n$.
Adjacent Eccentric distance sum index of $F_{d,2n} = \xi^{sv}(F_{d,2n}) = \frac{30n^3 - 12n^2d - 12n^2}{d}$.

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