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SUBMANIFOLD OF (κ, μ)- CONTACT METRIC MANIFOLD AS A RICCI SOLITON

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ABSTRACT. We study invariant and anti-invariant submanifolds of (κ, μ)-contact metric manifolds as Ricci solitons and show that the nature of Ricci soliton depends on the value of k. We also show that, in the submanifold as Ricci soliton, structure tensor ϕ anti commutes with the Ricci operator Q.

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1. INTRODUCTION

Blair et al. [4] introduced a class of contact metric manifolds, in which the structure vector field ξ satisfies the (κ, μ)-nullity condition. A contact metric manifold belonging to this class is called a (κ, μ) -manifold. A full classification of (κ, μ)-contact metric manifolds was given by Boeckx [5]. The (κ, μ)-contact metric manifolds are invariant under D_a-homothetic transformation and have been studied widely by several authors (See [3, 11, 16, 18]).

In 1982, Hamilton [19] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. Then Ricci flow has become a powerful tool for the study of Riemannian manifolds, especially for those manifolds with positive curvature. A solution to the Ricci flow is called Ricci soliton if it moves only by a one parameter group of diffeomorphisms and scaling. It has become more important after Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904. A Ricci soliton emerges as the limit of the solutions of the Ricci flow.

A Ricci soliton (g, V, λ) defined on (M, g) as $(L_V g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) = 0$, where L_V denotes the Lie-derivative of Riemannian metric g along a vector field V, λ is a constant and X, Y are arbitrary vector fields on M. A Ricci soliton is said to shrinking or steady or expanding to the extent that λ is negative, zero or positive respectively.

In 2008, Sharma [26] started the study of Ricci Solitons in K-Contact manifolds. Also in 2009, Ghosh et al. [17] studied gradient Ricci Solitons in a (κ, μ)-contact metric manifold. Ricci solitons and gradient Ricci solitons on some kinds of almost contact metric manifolds of dimension three

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were studied by many authors. De et al. [13] and Turan et al. [27] investigated Ricci solitons and gradient Ricci solitons on three-dimensional normal almost contact metric manifolds and three-dimensional transSasakian manifolds respectively. Moreover, Ghosh [14, 15] and Cho [8] classified Ricci solitons on three-dimensional Kenmotsu manifolds and 3-dimensional contact metric manifolds respectively. In addition, Ricci solitons on f-Kenmotsu manifolds and $N(k)$-quasi-Einstein manifolds were also studied by Calin and Crasmareanu [6] and Crasmareanu [9] respectively. Nagaraja et al. [23, 24, 25, 21, 20, 22] studied Ricci solitons in Kenmotsu manifolds, f-Kenmotsu manifolds, $N(\kappa)$-manifolds and (κ, μ) manifolds under D-homothetic deformations.

In this paper, we study submanifold of (κ, μ)-contact metric manifold as a Ricci soliton. We prove that a submanifold of (κ, μ)-contact metric manifold as a shrinking or expanding Ricci soliton is non flat and if the soliton is steady then the $(1,1)$-tensor ϕ anti-commutes with the Ricci operator Q. In particular for $\lambda = -2$, submanifold is locally isometric to $E^{(n+1)}(0) \times S^n(4)$ for $n > 1$. Further, we extend the study to invariant and anti-invariant submanifolds of (κ, μ)-contact metric manifold as Ricci solitons. For invariant submanifold of (κ, μ)-contact metric manifold the nature of the soliton depends on the constant k. In the case of anti-invariant submanifold, the submanifold is cyclic parallel and if it is Ricci pseudosymmetric then it is either Sasakian or $L_S = k - \frac{r}{2n(2n+1)}$.

2. Preliminaries

Let M be $(2m+1)$-dimensional almost contact metric manifold with the structure tensors (ϕ, ξ, η, g), where ϕ is a tensor field of type $(1,1)$, ξ is a unit vector field, η a 1-form and g is a Riemannian metric on M [2]. Then

$$\begin{aligned} (1) \qquad & \phi^2 = -I + \eta \otimes \xi, \ \eta(\xi) = 1, \ \phi\xi = 0, \ \eta \cdot \phi = 0, \\ & g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \ g(X, \xi) = \eta(X), \end{aligned}$$

for any vector fieelds X, Y on M. Let Φ denote the 2-form in M and is given by
$\Phi(X,Y) = g(X, \phi Y)$. For real constants k, μ, the (κ, μ)-nullity distribution of a contact metric manifold is a distribution [4]

$$\begin{aligned} (2) \qquad N(\kappa,\mu) : p \longrightarrow N_p(\kappa,\mu) = \{ & Z \in T_pM : R(X,Y)Z \\ & = \kappa(g(Y,Z)X - g(X,Z)Y) \\ & + \mu(g(Y,Z)hX - g(X,Z)hY)\}, \end{aligned}$$

for any vector fieelds X, Y on M, where R denotes the Riemannian curvature tensor and T_pM denotes the tangent vector space of M at ant point $p \in M$. If the characteristic vector field ξ of a contact metric manifold belongs to the (κ, μ)-nullity distribution, then

$$(3) \qquad R(X,Y)\xi = k(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY).$$

A contact metric manifold with $\xi \in N(\kappa, \mu)$ is called a (κ, μ)-contact metric manifold. In a (κ, μ)-contact metric manifold the following relations hold:

$$(4) \qquad (\nabla_X \phi)Y = g(X + hX, Y)\xi - \eta(Y)(X + hX),$$

(5) $$h\xi = 0,$$

(6) $$h^2 = (k-1)\phi^2,$$

(7) $$\nabla_X \xi = -\phi X - \phi h X,$$

(8) $$(\nabla_X h)Y = \{(k-1)g(X,\phi Y) + g(X, h\phi Y)\}\xi + \eta(Y)[h(\phi X + \phi h X)] - \mu\eta(X)\phi h Y,$$

for all vector fieelds X, Y on M, where h is a symmetric tensor. Now let N be $(2n+1)$- dimensional immersed submanifold of M. Then the Gauss and Weingarten formulas are respectively given by

(9) $$\nabla_X Y = \tilde{\nabla}_X Y + \sigma(X,Y)$$

and

(10) $$\nabla_X V = -A_V X + \tilde{\nabla}^{\perp}_X V,$$

for all vector fieelds X, Y on N, and a normal vector field V on N, where σ denotes the second fundamental form, $\tilde{\nabla}^{\perp}$ the normal connection and A the shape operator. The second fundamental form and shape operator are related by

(11) $$g(A_V X, Y) = g(\sigma(X,Y), V),$$

where g denotes the induced metric on N as well as the Riemannian metric g on M.
The covariant derivative of σ is defined by

(12) $$(\nabla_X \sigma)(Y,Z) = \tilde{\nabla}^{\perp}_X \sigma(Y,Z) - \sigma(\tilde{\nabla}_X Y, Z) - \sigma(Y, \tilde{\nabla}_X Z).$$

for any vector fields X, Y, Z on N.
If $R_N(X,Y)Z$ and $R_M(X,Y)Z$ denote the Riemannian curvature tensor on the ambient manifold M and the submanifold N respectively, then we have

(13) $$R_M(X,Y)Z = R_N(X,Y)Z + (\nabla_X\sigma)(Y,Z) - (\nabla_Y\sigma)(X,Z) + A_{\sigma(X,Z)}Y - A_{\sigma(Y,Z)}X,$$

for all vector fields X, Y, Z on N [7].

The invariant and anti-invariant submanifolds depend on the behaviour of almost contact metric structure ϕ. A submanifold N of an almost contact metric manifold is said to be invariant if the structure vector field ξ is tangent to N at every point of N and $\phi(T_xN) \subset T_xN$ and the submanifold is called anti invariant in M if $\phi(T_xN) \subset (T_xN)^{\perp}$ for each point x of N, where T_xN and $T_xN^{\perp}$ denote respectively the tangent space and normal spaces of N at x [1].

Definition 2.1. *The Ricci tensor S of a Riemannian manifold M is said to be cyclic-parallel if*

(14) $$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y)) = 0.$$

Definition 2.2. *The D-conformal curvature tensor B on a Riemannian manifold (M,g), $(n>4)$ is defined as* [10]

(15)
$$\begin{aligned}B(X,Y)Z &= R(X,Y)Z+\frac{1}{n-3}[S(X,Z)Y-S(Y,Z)X+g(X,Z)QY\\&-g(Y,Z)QX+S(Y,Z)\eta(X)\xi-S(X,Z)\eta(Y)\xi+\eta(Y)\eta(Z)QX\\&-\eta(X)\eta(Z)QY]-\frac{K-2}{n-3}[g(X,Z)Y-g(Y,Z)X]\\&+\frac{K}{n-3}[g(X,Z)\eta(Y)\xi-g(Y,Z)\eta(X)\xi+\eta(X)\eta(Z)Y\\&-\eta(Y)\eta(Z)X],\end{aligned}$$

where $K=\frac{r+2(n-1)}{n-2}$, Q is the Ricci operator, S is the Ricci tensor and r is the scalar curvature of M.

A Riemannian manifold (M,g) is called Ricci pseudosymmetric if the tensor $R.S$ and the Tachibana tensor $Q(g,S)$ are linearly dependent, where

(16) $$(R(X,Y).S)(Z,U)=-S(R(X,Y)Z,U)-S(Z,R(X,Y)U),$$

(17) $$Q(g,S)(Z,U;X,Y)=-S((X\wedge_g Y)Z,U)-S(Z,(X\wedge_g Y)U),$$

and

(18) $$(X\wedge_g Y)=g(Y,Z)X-g(X,Z)Y,$$

for all vector fields X,Y,Z,U of M, R denotes the curvature tensor of M. A Riemannian manifold (M,g) is called Ricci pseudo-symmetric if and only if

(19) $$(R(X,Y).S)(Z,U)=L_SQ(g,S)(Z,U;X,Y),$$

holds on $U_S=\{x\in M|S-\frac{r}{n}g\neq 0\ at\ x\}$, for some function L_S on U_S. The concircular curvature tensor C is defined by

(20) $$C(X,Y)Z=R(X,Y)Z-\frac{r}{n(n-1)}[g(Y,Z)X-g(X,Z)Y].$$

A Riemannian manifold (M,g) is said to be concircular Ricci pseudosymmetric if its concircular curvature tensor C satisfies

(21) $$(C(X,Y).S)(Z,U)=L_SQ(g,S)(Z,U;X,Y),$$

on $U_S=\{x\in M|S-\frac{r}{n}g\neq 0\ at\ x\}$, for some function L_S on U_S.

3. Ricci soliton on submanifolds of (κ,μ)-contact metric manifold

Throughout this section N is a submanifold of (κ,μ)-contact metric manifold M. A smooth vector field V on a Riemannian manifold (M,g) is said to define a Ricci soliton [19] if it satisfies

(22) $$\frac{1}{2}\mathcal{L}_V g+Ric=\lambda\, g,$$

where $\mathcal{L}_V g$ is the Lie-derivative of the metric tensor g with respect to V, Ric is the Ricci tensor of (M,g) and λ is a constant.

A Ricci soliton (M, g, V, λ) is called shrinking, steady or expanding according to $\lambda > 0$, $\lambda = 0$ or $\lambda < 0$ respectively.

Suppose (g, ξ, λ) is a Ricci soliton on the submanifold N of (k, μ)-contact metric manifold M with contact structure (ϕ, ξ, η, g). Then from (22), we write

$$g(\tilde{\nabla}_Y \xi, Z) + g(Y, \tilde{\nabla}_Z \xi) + 2\ S_N(Y, Z) - 2\ \lambda\ g(Y, Z) = 0.$$

Using (9) and (7) in the foregoing equation, we obtain

$$S_N(Y, Z) = g(\phi hY + \lambda Y, Z). \tag{23}$$

Theorem 3.1. *If (N, g, ξ, λ) is a shrinking or expanding Ricci soliton then N is not flat.*

Proof. Suppose the submanifold N admitting Ricci soliton is flat. Then we have

$$R_N(X, Y)Z = 0,$$

which implies

$$S_N(Y, Z) = 0.$$

From (22), we get

$$g(\phi hY + \lambda Y, Z) = 0. \tag{24}$$

Taking $Y = \xi$ in (24), we obtain

$$\lambda \eta(Z) = 0.$$

Hence the theorem is proved. □

Theorem 3.2. *If (N, g, ξ, λ) is a steady Ricci soliton then the Ricci operator Q and the structure tensor ϕ satisfy $Q\phi + \phi Q = 0$.*

Proof. From (23), we write

$$QY = \phi hY + \lambda Y. \tag{25}$$

Applying ϕ both sides of (25), we get

$$\phi QY = -hY + \lambda \phi Y. \tag{26}$$

Next replace Y by ϕY in (25), we obtain

$$Q\phi Y = hY + \lambda \phi Y. \tag{27}$$

Adding equations (26) and (27) proves the theorem. □

Theorem 3.3. *If (N, g, ξ, λ) is a Ricci soliton and if N is D-conformally flat then for $\lambda = -2$, N is locally isometric to $E^{(n+1)}(0) \times S^n(4)$ for $n > 1$.*

Proof. If the submanifold N for $n > 1$ is D-conformally flat then from (15) we have

$$
\begin{aligned}
(28)\quad R_N(X,Y)Z = &-\frac{1}{2(n-1)}[S_N(X,Z)Y - S_N(Y,Z)X + g(X,Z)QY \\
&- g(Y,Z)QX + S_N(Y,Z)\eta(X)\xi - S_N(X,Z)\eta(Y)\xi \\
&+ \eta(Y)\eta(Z)QX - \eta(X)\eta(Z)QY] + \frac{K-2}{2(n-1)}[g(X,Z)Y \\
&- g(Y,Z)X] - \frac{K}{2(n-1)}[g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi \\
&+ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X].
\end{aligned}
$$

Using (23) and (25) in (28), a simple calculation gives

$$
\begin{aligned}
(29)\quad R_N(X,Y)Z = &\frac{\lambda-K}{2(n-1)}g(X,Z)\eta(Y)\xi + \frac{K-\lambda}{2(n-1)}g(Y,Z)\eta(X)\xi \\
&+ \frac{\lambda-K}{\eta}(X)\eta(Z)Y + \frac{K-2-2\lambda}{2(n-1)}g(X,Z)Y \\
&+ \frac{2\lambda-K+2}{2(n-1)}g(Y,Z)X - \frac{1}{2(n-1)}[g(\phi hX,Z)Y \\
&- g(\phi hY,Z)X + g(X,Z)\phi hY - g(Y,Z)\phi hX \\
&+ g(\phi hY,Z)\eta(X)\xi - g(\phi hX,Z)\eta(Y)\xi + \eta(Y)\eta(Z)\phi hX \\
&- \eta(X)\eta(Z)\phi hY].
\end{aligned}
$$

Putting $Z = \xi$ in (29), we obtain

$$R_N(X,Y)\xi = \frac{\lambda+2}{2(n-1)}[\eta(Y)X - \eta(X)Y].$$

Hence the theorem is proved. □

Theorem 3.4. *Let N be an invariant submanifold of (κ,μ)-contact metric manifold M admitting Ricci soliton. Then soliton is shrinking, expanding and steady according as $k \in (0,1]$, $k < 0$ and $k = 0$ respectively.*

Proof. For an invariant submanifold N of (κ,μ)-contact metric manifold, we have

$$
\begin{aligned}
(30)\quad S_N(Y,Z) = &[2(n-1) - n\mu]g(Y,Z) + [2(n-1)+\mu]g(hY,Z) \\
&+ [2(1-n) + n(2k+\mu)]\eta(Y)\eta(Z).
\end{aligned}
$$

In view of equation (23), we may write

$$
\begin{aligned}
(31)\quad g(\phi hY + \lambda Y, Z) = &[2(n-1) - n\mu]g(Y,Z) + [2(n-1)+\mu]g(hY,Z) \\
&+ [2(1-n) + n(2k+\mu)]\eta(Y)\eta(Z).
\end{aligned}
$$

Taking $Y = \xi$ in (31), we get

$$\lambda = 2nk.$$

Hence the theorem is proved. □

Theorem 3.5. *Let N be an anti-invariant submanifold of (κ,μ)-contact metric manifold M admitting Ricci soliton. If N is concircular Ricci pseudosymmetric then N is either Sasakian or $L_S = k - \frac{r}{2n(2n+1)}$.*

Proof. From (21), (16), (17) and (18), we have

(32)
$$S_N(C(X,Y)Z,U) + S_N(Z,C(X,Y)U) = L_S[g(Y,Z)S_N(X,U) - g(X,Z)S_N(Y,U) + g(Y,U)S_N(X,Z) - g(X,U)S_N(Y,Z)].$$

Putting $Z = \xi$ in (32) and using (23), (3) and (13), we get

(33)
$$g(\phi h[k\{\eta(Y)X - \eta(X)Y\} + \mu\{\eta(Y)hX - \eta(X)hY\} - \frac{r}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\}], U) + \lambda g(\phi h[k\{\eta(Y)X - \eta(X)Y\} + \mu\{\eta(Y)hX - \eta(X)hY\} - \frac{r}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\}], U) + \lambda\eta(k\{g(Y,U)X - g(X,U)Y\} + \mu\{g(Y,U)hX - g(X,U)hY\} - \frac{r}{2n(2n+1)}\{g(Y,U)X - g(X,U)Y\}) = L_S[\eta(Y)g(\phi hX + \lambda X, U) - \eta(X)g(\phi hY + \lambda Y, U) + \lambda\eta(X)g(Y,U) - \lambda\eta(Y)g(X,U)].$$

Again putting $Y = \xi$ in (33), we obtain

(34)
$$[k - \frac{r}{2n(2n+1)} - L_S]g(\phi hX, U) - \mu(k-1)g(\phi X, U) + \lambda\mu g(hX, U) = 0.$$

Replace X by hX in (34), we get

$$[k - \frac{r}{2n(2n+1)} - L_S](k-1)g(\phi X, U) - \mu(k-1)g(\phi hX, U) - \lambda\mu(k-1)g(\phi^2 X, U) = 0. \tag{35}$$

Again replacing X by ϕX in (35) and using anti-invariant property, we arrive

$$[k - \frac{r}{2n(2n+1)} - L_S](k-1)g(\phi X, \phi U) = 0.$$

□

Theorem 3.6. *An anti-invariant submanifold of (κ, μ)-contact metric manifold M admitting Ricci soliton is cyclic parallel.*

Proof. From (14), we first calculate $(\tilde{\nabla}_X S_N)(Y,Z)$.

$$(\tilde{\nabla}_X S_N)(Y,Z) = \tilde{\nabla}_X S_N(Y,Z) - S_N(\tilde{\nabla}_X Y, Z) - S_N(Y, \tilde{\nabla}_X Z). \tag{36}$$

Using (23), we get

$$(\tilde{\nabla}_X S_N)(Y,Z) = g((\tilde{\nabla}_X \phi)hY + \phi(\tilde{\nabla}_X h)Y, Z). \tag{37}$$

Again using (9), (4),(5) and (8) and the property of anti-invariant, we obtain

$$(\tilde{\nabla}_X S_N)(Y,Z) = (k-1)[-g(X,Y)\eta(Z) + g(X,Z)\eta(Y)]. \tag{38}$$

In the similar way one can find

$$(\tilde{\nabla}_Y S_N)(Z,X) = (k-1)[-g(Y,Z)\eta(X) + g(Y,X)\eta(Z)]. \tag{39}$$

$$(\tilde{\nabla}_Z S_N)(X,Y) = (k-1)[-g(Z,X)\eta(Y) + g(Z,Y)\eta(X)]. \tag{40}$$

Adding (38), (39) and (40), the theorem is proved. □

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