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Omega Invariant of Union, Join and Corona Product of Two Graphs

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Abstract

Graphs nowadays are getting a lot of attention due to their applications in all areas of science including physics, chemistry, pharmacology, network science, neuroscience, social sciences, etc. There are several mathematical methods used in graph theory to obtain such applications. Nearly in half of them, the vertex degrees and the degree sequences play an important role. The graph products are very useful tools as they help us to calculate several properties of large graphs by means of smaller graphs. Recently, a new topological graph invariant named as omega was defined in terms of the vertex degrees, that is degree sequence. In this paper, the degree sequences of the union of some special graph classes are given. Recalling the degree sequences of the join and corona products of two special graph classes from literature, the omega invariants of the union, join and corona products of two special graphs are obtained. Also for each of these graph products, results giving the omega values and the number of faces for general graphs are given.

1 Introduction

Let $G = (V, E)$ be a finite, simple and undirected graph with $|V(G)| = n$ vertices and $|E(G)| = m$ edges. Simple means that loops or multiple edges are not allowed. The degree of a vertex $v \in V(G)$ is denoted by $d_G(v)$ or $d(v)$ if there is no probable confusion. A vertex of degree one is called a pendant vertex. As usual, we use $P_n, S_n, C_n, K_n, K_{r,s}, T_n$ and $T_{r,s}$ to denote the path, star, cycle, complete, complete bipartite, tree and tadpole graphs, respectively. The smallest and biggest vertex

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degrees in a graph G is respectively denoted by δ and Δ.

A degree sequence with multiplicities is written as

$$DS(G) = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta)}\}.$$

The degree sequences of some well-known and frequently used graph classes are as follows:

G	$DS(G)$
P_n	$\{1^{(2)}, 2^{(n-2)}\}$
C_n	$\{2^{(n)}\}$
K_n	$\{(n-1)^{(n)}\}$
S_n	$\{1^{(n-1)}, (n-1)^{(1)}\}$
$K_{r,s}$	$\{r^{(s)}, s^{(r)}\}$
T_n	$\{1^{(a_1)}, 2^{(a_2)}, \cdots, \Delta^{(a_\Delta)}\}$ $a_1+a_2+\cdots+a_\Delta=n$
$T_{r,s}$	$\{1^{(1)}, 2^{(r+s-2)}, 3^{(1)}\}$

Figure 1: Degree sequences of some graph classes

Definition 1.1 ([2]). *Let* $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta)}\}$ *be a set which also is the degree sequence of a graph G. The omega invariant* $\Omega(G)$ *of the graph G is defined only in terms of the degree sequence as*

$$\begin{aligned} \Omega(G) = & \ a_3 + 2a_4 + 3a_5 + \cdots + (\Delta - 2)a_\Delta - a_1 \\ = & \sum_{i=1}^{\Delta} (i-2)a_i. \end{aligned} \tag{1}$$

$\Omega(G)$ of several well-known graph classes such as T, P_n, C_n, S_n, K_n, $T_{r,s}$ and $K_{r,s}$, where $n = r + s$ which respectively denote a tree, path, cycle, star, complete, tadpole and complete bipartite graphs with n vertices are

$$\begin{aligned} \Omega(C_n) &= 0 \\ \Omega(P_n) &= -2 \\ \Omega(S_n) &= -2 \\ \Omega(T) &= -2 \\ \Omega(K_n) &= n(n-3) \\ \Omega(K_{r,s}) &= 2[rs - (r+s)] \\ \Omega(T_{r,s}) &= 0. \end{aligned}$$

There are several graph operations defined and used in calculating some chemical properties of molecules by means of graphs. Amongst these, the join, cartesian, corona, union, disjunction and symmetric difference are well-known. In this paper, we shall study omega invariant of union, join and corona products of two graphs. First we recall their definitions:

Definition 1.2. *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs. The union denoted by $G_1 \cup G_2$ of graphs G_1 and G_2 having disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.*

Definition 1.3. *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs with n_1 and n_2 vertices and m_1 and m_2 edges, respectively. The join denoted by $G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph union $G_1 \cup G_2$ together with all the edges joining the vertices in $V(G_1)$ and $V(G_2)$.*

Thus, for example, $K_{n_1} + K_{n_2}$ is the complete bipartite graph K_{n_1,n_2}. We then have $|V(G_1 + G_2)| = n_1 + n_2$ and $|E(G_1 + G_2)| = m_1 + m_2 + n_1 n_2$.

Definition 1.4. *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs such that $V_1 = \{u_1, u_2, \cdots, u_{n_1}\}$, $|E(G_1)| = m_1$ and $V_2 = \{v_1, v_2, \cdots, v_{n_2}\}$, $|E(G_2)| = m_2$. The corona product of the graphs G_1 and G_2 is denoted by $G_1 \circ G_2$ and defined as the graph having vertex set*

$$V(G_1 \circ G_2) = \{(u_i, v_j),\ i = 1, 2, \cdots, n_1; j = 1, 2, \cdots, n_2\}$$

.

Then it follows from the definition of corona product that $G_1 \circ G_2$ has $n_1(1 + n_2)$ vertices and also $m_1 + n_1 m_2 + n_1 n_2$ edges.

The remaining of this paper is planned as follows: In Section 2, omega invariant and its basic properties are given. In Section 3, union of two given graphs is considered and after recalling the degree sequences and calculating omega invariant of the union of two graphs which are path, cycle, star, complete, complete bipartite and tadpole graphs. The general results for the union of any two graphs are obtained. The omega invariant of union is calculated and the formula for the number of faces of union graph is obtained. A lower bound is given for the omega value of the union. In Section 4, omega invariant of join operation is considered. After recalling the degree sequences of the join of two graphs which are path, cycle, star, complete, complete bipartite and tadpole graphs, a relation between the omega invariant of the join of two given graphs and omega values of two given graphs is obtained. A lower bound for the omega of the join is given and also the number of faces of the join product is formulized in two different ways. Similar relations are obtained for the corona product in Section 5.

2 Ω invariant

In this section, for a realizable degree sequence D having a realization G or for a given graph G, we recall and study some fundamental properties of the number $\Omega(G)$ or $\Omega(D)$, respectively, which are defined and studied recently in [2]. The idea appeared from the number a_1 of pendant vertices of a tree T:

$$a_1 = 2 + a_3 + 2a_4 + 3a_5 + 4a_6 + \cdots + (\Delta - 2)a_\Delta, \tag{2}$$

where a_i denotes the number of vertices having degree i. We can see that Eqn. (2) might be rewritten as

$$a_3 + 2a_4 + 3a_5 + 4a_6 + \cdots + (\Delta - 2)a_\Delta - a_1 = -2. \tag{3}$$

Generalizing this last equation, $\Omega(D)$ was defined in [2] as recalled in Eqn. (1). We first recall some important properties of Ω. Many results are related to disconnected graphs. The following result shows that Ω invariant of a graph G is additive over the set of components of the graph G:

Theorem 2.1. *[2] Let G be a disconnected graph having c components $G_1, G_2, \cdots, G_c$. Then*

$$\Omega(G) = \sum_{i=1}^{c} \Omega(G_i).$$

The following relation is very important in finding $\Omega(G)$ of a given G and is used in many results related to Ω:

Theorem 2.2. *[2] For a graph G,*

$$\Omega(G) = 2(m - n).$$

That is, for every graph G, $\Omega(G)$ must be even. Therefore, when $\Omega(D)$ is an odd number for a set D consisting of non-negative integers, then we can easily say that D is not realizable, which can be thought as a new realizability test.

The cyclomatic number of a connected graph G which is the number r of independent cycles in G can also be given in terms of $\Omega(G)$:

Theorem 2.3. *[2] Let $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta)}\}$ be a set of positive integers. If D is realizable as a connected planar graph G, then the number r of faces is*

$$r = \frac{\Omega(G)}{2} + 1.$$

This result is important in many applications. The next result is a direct generalization of Theorem 2.3 to disconnected graphs:

Corollary 2.1. *[2] Let $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta)}\}$ be a realizable set as a graph G having c components. The total number r of the faces of G is*

$$r = \frac{\Omega(G)}{2} + c.$$

Every graph has edges, vertices and faces as its pieces. These are sometimes given special names like bridges, cut vertices, loops, pendant vertices, chords, pendant edges, multiple edges etc. For the definitions not recalled here, see [1], [3], [4], [7], [8].

3 Ω invariant of union

It is well-known and easy to see that the union of two graphs is abelian. We now want to calculate the degree sequence and omega invariant of the union of two special graphs such as path, cycle, star, complete, complete bipartite and tadpole graphs. Hence we can obtain the following table giving these required information.

G_1	G_2	$D(G_1 \cup G_2)$	$\Omega(G_1 \cup G_2)$
P_{n_1}	P_{n_2}	$\{1^{(4)}, 2^{(n_1+n_2-4)}\}$	-4
P_{n_1}	C_{n_2}	$\{1^{(2)}, 2^{(n_1+n_2-2)}\}$	-2
P_{n_1}	K_{n_2}	$\{1^{(2)}, 2^{(n_1-2)}, (n_2-1)^{(n_2)}\}$	m^2-3m-2
P_{n_1}	S_{n_2}	$\{1^{(n_2+1)}, 2^{(n_1-2)}, (n_2-1)^{(1)}\}$	-4
P_{n_1}	$K_{r,s}$	$\{1^{(2)}, 2^{(n_1-2)}, r^{(s)}, s^{(r)}\}$	2[rs-(r+s)]-2
P_{n_1}	$T_{r,s}$	$\{1^{(3)}, 2^{(r+s+n_1-4)}, 3^{(1)}\}$	-2
C_{n_1}	C_{n_2}	$\{2^{(n_1+n_2)}\}$	0
C_{n_1}	K_{n_2}	$\{2^{(n_1)}, (n_2-1)^{(n_2)}\}$	m(m-3)
C_{n_1}	S_{n_2}	$\{1^{(n_2-1)}, 2^{(n_1)}, (n_2-1)^{(1)}\}$	-2
C_{n_1}	$K_{r,s}$	$\{2^{(n_1)}, r^{(s)}, s^{(r)}\}$	2[rs-(r+s)]
C_{n_1}	$T_{r,s}$	$\{1^{(1)}, 2^{(r+s+n_1-2)}, 3^{(1)}\}$	0
K_{n_1}	K_{n_2}	$\{(n_1-1)^{(n_1)}, (n_2-1)^{(n_2)}\}$	n(n-3)+m(m-3)
K_{n_1}	S_{n_2}	$\{1^{(n_2-1)}, (n_2-1)^{(1)}, (n_1-1)^{(n_1)}\}$	n(n-3)-2
K_{n_1}	$K_{r,s}$	$\{(n_1-1)^{(n_1)}, r^{(s)}, s^{(r)}\}$	n(n-3)+2[rs-(r+s)]
K_{n_1}	$T_{r,s}$	$\{1^{(1)}, 2^{(r+s-2)}, 3^{(1)}, (n_1-1)^{(n_1)}\}$	n(n-3)
S_{n_1}	S_{n_2}	$\{1^{(n_1+n_2-2)}, (n_1-1)^{(1)}, (n_2-1)^{(1)}\}$	-4
S_{n_1}	$K_{r,s}$	$\{1^{(n_1-1)}, (n_1-1)^{(1)}, r^{(s)}, s^{(r)}\}$	-2+2[rs-(r+s)]
S_{n_1}	$T_{r,s}$	$\{1^{(n_1)}, 2^{(r+s-2)}, 3^{(1)}, (n_1-1)^{(1)}\}$	-2
$K_{r,s}$	$K_{t,m}$	$\{r^{(s)}, s^{(r)}, t^{(m)}, m^{(t)}\}$	2[rs-(r+s)+tm-(t+m)]
$K_{r,s}$	$T_{a,b}$	$\{1^{(1)}, 2^{(r+s-2)}, 3^{(1)} r^{(s)}, s^{(r)}\}$	2[rs-(r+s)]
$T_{r,s}$	$T_{a,b}$	$\{1^{(2)}, 2^{(r+s+a+b-4)}, 3^{(2)}\}$	0

Figure 2: Degree sequences and Ω invariants of the union of two graphs

In this table, m and n denote the order and size of the union graph. After listing the degree sequences of the union of the above special graph classes and observing their omega values, we can now give some general results on the omega invariant of the union of two graphs. First we state that omega of the union of two graphs is equal to the sum of the omega values of the graphs:

Theorem 3.1. *Let G_1 and G_2 be two connected graphs. Then*

$$\Omega(G_1 \cup G_2) = \Omega(G_1) + \Omega(G_2). \tag{4}$$

Proof. $G_1 \cup G_2$ has components G_1 and G_2. By the additivity of Ω, we get the result. $\square$

Therefore we obtain the omega invariant of the union of any two graphs G_1 and G_2 as follows:

Theorem 3.2. *Let G_1 and G_2 be two connected graphs. Let $D(G_1) = \{1^{(a_1)}, 2^{(a_2)}, \cdots, \Delta^{(a_\Delta)}\}$ and $D(G_2) = \{1^{(b_1)}, 2^{(b_2)}, \cdots, \nabla^{(b_\nabla)}\}$ be the degree sequences of G_1 and G_2, respectively. Then*

$$\Omega(G_1 \cup G_2) = \sum_{i=1}^{\Delta} i \cdot a_i + \sum_{j=1}^{\nabla} j \cdot b_j - 2(n_1 + n_2). \tag{5}$$

Proof. It follows from the definition of omega invariant and from the degree sequence of the union of two graphs. $\square$

The following is an immediate consequence of Theorem 3.2:

Corollary 3.1. *Let G_1 and G_2 be two graphs as in Theorem 3.2. Then*

$$r(G_1 \cup G_2) = \frac{1}{2}\sum_{i=1}^{\Delta} i \cdot a_i + \frac{1}{2}\sum_{j=1}^{\nabla} j \cdot b_j + n_1 + n_2 + 1.$$

As each component of a graph is a maximal connected graph itself, we know that omega of each component is ≥ -2 implying the following important result:

Corollary 3.2. *If G is the union of two connected graphs G_1 and G_2, then*

$$\Omega(G_1 \cup G_2) \geq -4$$

.

Proof. Recall that for any connected graph, the omega invariant is an even integer ≥ -2. Therefore as G has two connected components and as the omega of each component is ≥ -2, the result follows by Theorem 3.1. $\square$

This result can easily be generalized to the following:

Corollary 3.3. *If G is the union of k connected graphs $G_1, G_2, \cdots, G_k$, then*

$$\Omega(G_1 \cup G_2 \cup \cdots \cup G_k) \geq -2k.$$

G_1	G_2	$D(G_1+G_2)$
P_{n_1}	P_{n_2}	$\{(n_2+1)^{(2)}, (n_2+2)^{(n_1-2)}, (n_1+1)^{(2)}, (n_1+2)^{(n_2-2)}\}$
P_{n_1}	C_{n_2}	$\{(n_2+1)^{(2)}, (n_2+2)^{(n_1-2)}, (n_1+2)^{(s)}\}$
P_{n_1}	K_{n_2}	$\{(n_2+1)^{(2)}, (n_2+2)^{(n_1-2)}, (n_1+n_2-1)^{(n_2)}\}$
P_{n_1}	S_{n_2}	$\{(n_2+1)^{(2)}, (n_2+2)^{(n_1-2)}, (n_1+1)^{(n_2-1)}, (n_1+n_2-1)\}$
P_{n_1}	$K_{r,s}$	$\{(r+s+1)^{(2)}, (r+s+2)^{(n_1-2)}, (n_1+r)^{(s)}, (n_1+s)^{(r)}\}$
P_{n_1}	$T_{r,s}$	$\{(r+s+1)^{(2)}, (r+s+2)^{(n_1-2)}, (n_1+1), (n_1+2)^{(r+s-2)}, (n_1+3)\}$
C_{n_1}	C_{n_2}	$\{(n_2+2)^{(n_1)}, (n_1+2)^{(n_2)}\}$
C_{n_1}	K_{n_2}	$\{(n_2+2)^{(n_1)}, (n_1+n_2-1)^{(n_2)}\}$
C_{n_1}	S_{n_2}	$\{(n_2+2)^{(n_1)}, (n_1+n_2-1), (n_1+1)^{(n_2-1)}\}$
C_{n_1}	$K_{r,s}$	$\{(r+s+2)^{(n_1)}, (n_1+r)^{(s)}, (n_1+s)^{(r)}\}$
C_{n_1}	$T_{r,s}$	$\{(r+s+2)^{(n_1)}, (n_1+1), (n_1+2)^{(r+s-2)}, (n_1+3)\}$
K_{n_1}	K_{n_2}	$\{(n_1+n_2-1)^{(n_1+n_2)}\}$
K_{n_1}	S_{n_2}	$\{(n_1+n_2-1)^{(n_1)}, (n_1+1)^{(n_2-1)}, (n_1+n_2-1)\}$
K_{n_1}	$K_{r,s}$	$\{(n_1+r+s-1)^{(n_1)}, (n_1+r)^{(s)}, (n_1+s)^{(r)}\}$
K_{n_1}	$T_{r,s}$	$\{(n_1+r+s-1)^{(n_1)}, (n_1+1), (n_1+2)^{(r+s-2)}, (n_1+3)\}$
S_{n_1}	S_{n_2}	$\{(n_2+1)^{(n_1-1)}, (n_1+n_2-1)^{(2)}, (n_1+1)^{(n_2-1)}\}$
S_{n_1}	$K_{r,s}$	$\{(r+s+1)^{(n_1-1)}, (n_1+r+s-1), (n_1+r)^{(s)}, (n_1+s)^{(r)}\}$
S_{n_1}	$T_{r,s}$	$\{(r+s+1)^{(n_1-1)}, (n_1+r+s-1),$ $(n_1+1), (n_1+2)^{(r+s-2)}, (n_1+3)\}$
$K_{r,s}$	$K_{t,m}$	$(r+t+m)^{(s)}, (s+t+m)^{(r)}, (r+s+m)^{(t)}, (r+s+t)^{(m)}\}$
$K_{r,s}$	$T_{a,b}$	$(r+a+b)^{(s)}, (s+a+b)^{(r)}, (r+s+2)^{(a+b-2)}, (r+s+1), (r$ $+s+3)\}$
$T_{r,s}$	$T_{a,b}$	$\{(a+b+2)^{(r+s-2)}, (a+b+1), (a+b+3), (r+s+$ $2)^{(a+b-2)}, (r+s+1), (r+s+3)\}$

Figure 3: Degree sequences of the join of two graphs

4 Ω of join

Secondly, we determine the omega values of the join of two graphs. Let G_1 and G_2 be two connected graphs with degree sequences $DS(G_1) = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta))}\}$ and $DS(G_2) = \{1^{(b_1)}, 2^{(b_2)}, 3^{(b_3)}, \cdots, \nabla^{(b_\nabla)}\}$, orders n_1 and n_2 and size m_1 and m_2, respectively. Then the join of G_1 and G_2 has degree sequence $DS(G_1 + G_2) = \{(n_2+1)^{(a_1)}, (n_2+2)^{(a_2)}, (n_2+3)^{(a_3)}, \cdots, (n_2+\Delta)^{(a_\Delta)}, (n_1+1)^{(b_1)}, (n_1+2)^{(b_2)}, (n_1+3)^{(b_3)}, \cdots, (n_1+\nabla)^{(b_\nabla)}\}$. The degree sequences of the join of two graphs which are path, cycle, star, complete, complete bipartite and tadpole graphs were listed in [6]: We can now calculate the omega invariant of the join of any two graphs in general:

Theorem 4.1. *Let G_1 and G_2 be two connected graphs as above. Then the omega of $G_1 + G_2$ is*

$$\Omega(G_1 + G_2) = 2n_1n_2 + \Omega(G_1) + \Omega(G_2). \tag{6}$$

Proof. Let $DS(G_1) = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta))}\}$ and $DS(G_2) = \{1^{(b_1)}, 2^{(b_2)}, 3^{(b_3)}, \cdots, \nabla^{(b_\nabla)}\}$ and $\mid V(G_1) \mid = n_1$, $\mid V(G_2) \mid = n_2$. Then

$$\begin{aligned}
\Omega(G_1 + G_2) &= a_1(n_2-1) + a_2n_2 + a_3(n_2+1) + \cdots + a_\Delta(n_2+\Delta-2) \\
&\quad + b_1(n_1-1) + b_2n_1 + b_3(n_1+1) + \cdots + b_\nabla(n_1+\nabla-2) \\
&= \sum_{i=1}^{\Delta} a_i(n_2+i-2) + \sum_{j=1}^{\nabla} b_j(n_1+j-2) \\
&= (n_2-2)\sum_{i=1}^{\Delta} a_i + \sum_{i=1}^{\Delta} i \cdot a_i + (n_1-2)\sum_{j=1}^{\nabla} b_j + \sum_{j=1}^{\nabla} j \cdot b_j \\
&= (n_2-2)n_1 + (n_1-2)n_2 + 2m_1 + 2m_2 \\
&= 2n_1n_2 + \Omega(G_1) + \Omega(G_2).
\end{aligned}$$

□

Example 4.1. *Note that $DS(C_{n_1} + S_{n_2}) = \{(n_2+2)^{(n_1)}, n_1+n_2-1, (n_1+1)^{(n_2-1)}\}$. For example, $DS(C_4 + S_6) = \{8^{(4)}, 9^{(1)}, 5^{(5)}\}$ and $\Omega(C_4 + S_6) = 4 \cdot 6 + 1 \cdot 7 + 5 \cdot 3 = 46$ and $2n_1n_2 + \Omega(G_1) + \Omega(G_2) = 2 \cdot 4 \cdot 6 + \Omega(C_4) + \Omega(S_6) = 48 + 0 + (-2) = 46$ approving the truth of Theorem 4.1.*

As a direct result of Theorem 4.1, we can restate the number of faces of the join of two graphs in terms of the orders and the numbers of faces of two graphs as follows:

Corollary 4.1.

$$r(G_1 + G_2) = n_1n_2 + \frac{1}{2}\Omega(G_1) + \frac{1}{2}\Omega(G_2) + 1.$$

Proof. It follows by the above results. □

The following is an immediate result:

Corollary 4.2.

$$r(G_1+G_2)=n_1\cdot n_2+r(G_1)+r(G_2)-1.$$

Corollary 4.3. *If G is the join of two connected graphs G_1 and G_2, then*

$$\Omega(G_1+G_2)\geq -2.$$

Proof. It follows by the same reasoning with the one in the proof of Corollary 3.2. □

5 Ω of corona product

The list of degree sequences of two special graph classes was given in [5]. Using this table, one can calculate the omega invariant of the corona product of two special graphs. For example, the degree sequence of the corona product $S_r \circ K_s$ of a star graph S_r and a complete graph K_s was given in row 16 of Table 3 of [5] as $\{s^{(rs)},(s+1)^{(r-1)}\},r+s-1\}$. Hence

$$\begin{aligned}\Omega(S_r\circ K_s) &= rs(s-2)+(r-1)(s-1)+r+s-3\\ &= rs(s-1)-3.\end{aligned}$$

In the next theorem, we obtain the omega value of the corona product of two general graphs.

Theorem 5.1. *Let G_1 and G_2 be two connected graphs, $|V(G_1)|=n_1$ and $|V(G_2)|=n_2$. Then*

$$\Omega(G_1\circ G_2)=2[m_1+n_1(m_2-1)].$$

Proof. By recalling the order and size of the corona product from Definition 1.4, we get

$$\begin{aligned}\Omega(G_1\circ G_2) &= 2(m-n)\\ &= 2[m_1+n_1(n_2+m_2)-n_1n_2-n_1]\\ &= 2[m_1+n_1m_2-n_1]\\ &= 2[m_1+n_1(m_2-1)]\end{aligned}$$

by definition of Ω.

□

As a direct result of Theorem 5.1, we have the number of faces of the corona product of two graphs as follows:

Corollary 5.1.

$$r(G_1\circ G_2)=n_1[(n_2-1)+r(G_2)]+r(G_1).$$

Proof. By Theorem 2.3, we have

$$\begin{array}{rcl} r(G_1 \circ G_2) & = & \frac{\Omega(G_1 \circ G_2)}{2} + 1 \\ & = & m_1 + n_1(m_2 - 1) + 1 \\ & = & m_1 + n_1(m_2 - 1) + 1 + n_1 n_2 - n_1 n_2 + n_1 - n_1 \\ & = & n_1[(n_2 - 1) + r(G_2)] + r(G_1) \end{array}.$$

□

Corollary 5.2. *If G is the corona product of two connected graphs G_1 and G_2, then*

$$\Omega(G_1 \circ G_2) \geq -2.$$

Proof. It follows by the same reasoning with the one in the proof of Corollary 3.2. □

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