

A GENERALIZED NET MODEL OF DECISION MAKING PROCESS

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Abstract: In this paper, a generalized net model of a decision making process is described. It is an extension of the existing generalized net models of such systems.

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1 Introduction

In the series of papers [12 – 17, 22, 23] and the monograph [10], new procedures for MultiCriteria Decision Making (MCDM) are described. They are based on the apparatus of the Intuitionistic Fuzzy Sets (IFS; see [6, 8]). In [12] and [13], IF-interpretations of multi-measurement tool and multi-person MCDM procedures are given, respectively. In them, the evaluations of the multi-measurement tools and of multi-persons are in the form $\langle m, n \rangle$, where m and n are the positive and negative evaluation degrees, $m, n \in [0, 1]$ and $m, n \leq 1$. In [22, 23], these procedures are described by the tools of Generalized Nets (GNs; see [5, 7]) and they are extended in [15] for the case in which the experts can use not only the previous given criteria, but also own ones. Now, the experts can order the used criteria in their own way and can ignore some of them. The next extension of the MCDM procedure is described in [14]. Now, the experts have their own scores in the form $\langle p, q \rangle$, where p and q correspond to the successfully and non-successfully finished procedures from the respective expert, $p, q \in [0, 1]$ and $p, q \leq 1$. The number of the refused in the middle experts' procedures corresponds to the number $1 - p - q$. Procedures for self-evaluation of the experts with the aim to determine their scores on the base of subjective opinions are discussed in [10, 17]. A method for evaluation of the proximity of the criteria used by the experts is described in [16]. It is based on the intercriteria analysis [11]. In the case that there are two close criteria, one of them – the more expensive/difficult for measurement can be omitted and in the next MCDM procedure it will not be given to the experts.

2 A generalized net model

In Group Decision Making (GDM) a set of experts in a given field is involved in a decision process concerning the selection of the best alternative(s) among a set of predefined ones. For the first time in [21] the process of GDM has been described by GNs. An evaluation of the alternatives is performed independently by each decision maker: the experts express their evaluations on the basis of some decision scheme, which can be either implicitly assumed or explicitly specified in the form of a set of predefined criteria [18, 19]. In both cases, the aim is to obtain an evaluation (performance judgement or rating) of the alternatives by each expert.

Here, following and extending [10], we shall construct a GN-model of the process of multi-person multi-criteria decision making. In the intuitionistic fuzzy interpretation of this process from [13] each expert is asked to evaluate each alternative in terms of its performance with respect to each predefined criterion: the experts evaluations are expressed as a pair of numeric values, interpreted in the intuitionistic fuzzy framework: these numbers express a “positive” and a “negative” evaluation respectively. Each expert is also assigned a pair of values, which express the expert’s reliability (confidence in her/his expertise with respect to each criterion). Distinct reliability values are associated with distinct criteria. *The proposed formulation is based on the assumption of alternatives’ independency.* The second phase of a group decision process is the definition of a collective evaluation for each alternative: once the alternatives have been evaluated, the main problem is to aggregate the experts’ performance judgements to obtain an overall rating for each alternative.

The following basic notation is adopted in the paper:

$E = \{E_1, E_2, \dots, E_m\}$ is the set of the experts involved in the decision process;

$A = \{A_1, A_2, \dots, A_p\}$ is the set of the considered alternatives;

$C = \{C_1, C_2, \dots, C_s\}$ is the set of the criteria used for evaluating the alternatives.

The GN, that we shall describe below has four types of tokens – α -, β -, γ - and δ -tokens. The second, third and fourth tokens are unique, while α -tokens are m in number. They will represent the experts. The tokens are ordered by some criterion (e.g., alphabetically following experts’ names), but their order is not important. Each one of the α -tokens ($\alpha_1, \alpha_2, \dots, \alpha_m$) enters place l_1 with the initial characteristic:

$x_0^{\alpha_i}$ = “expert’s name, his/her own (current) reliability score $\langle \delta_i, \varepsilon_i \rangle$ such

that $\langle \delta_i, \varepsilon_i \rangle \in [0, 1]^2$ and $\delta_i + \varepsilon_i \leq 1$; his/her own (current) number of participations in expert investigations γ_i , a list of the new criteria that

the expert likes to offer; a list of the criteria that he/she does not like;

a list of the alternatives that he/she does not like to estimate”.

In more details, the couple of real numbers $\langle \delta_i, \varepsilon_i \rangle$ for i -th expert can be changed by a couple of sets $\langle \{\delta_{i,j} | 1 \leq j \leq q\}, \{\varepsilon_{i,j} | 1 \leq j \leq q\} \rangle$, such that for all possible i, j : $\delta_{i,j} + \varepsilon_{i,j} \leq 1$, where $\langle \delta_{i,j}, \varepsilon_{i,j} \rangle$ is the score of the i -th expert about the j -th criterion.

In the initial time-moment of GN functioning, the first α -token and the β - and γ -tokens enter places l_1 , l_2 and l_3 , respectively. The two later tokens have initial characteristics

$$x_0^\beta = \text{"list of the alternatives, i.e. } A_1, A_2, \dots, A_p \text{"}$$

and

$$x_0^\gamma = \text{"list of the estimation criteria, i.e., } C_1, C_2, \dots, C_s \text{"}.$$

Now let us give several examples of the apparatus involved. For instance, if we have to analyze the news bulletins in two TVs, a public and a private one, we will need some evaluation criteria, like the broadcast's actuality, pluralism, operativeness, entertainment, ratio of the informative to the interpretative discourse, taste for sensations. The experts who will propose their opinions about the quality of the TV news are, of course, the journalists, media experts, sociologists and art critics.

Another interesting example of the multi-person multi-criteria decision making is screening candidates for a job position. The experts here will be company representatives, specialists in the area of the certain job, and sometimes psychologists. The relevant criteria, which frame the interview, will be the submitted CV, the education and certificates in the area, the former experience at the same or a similar position, and the good look and communicativeness.

The third example of this instrumentation are the elections for President, Parliament or municipal government. The criteria according to which the evaluation shall be performed, are both from objective and subjective character: the objective criteria are the social order and form of government, the standard of living of the people in the country, some crises and tension in the society. The subjective criteria represent the pre-election campaigns, slogans and promises, or the failures and success of the previous government. The experts who will estimate the chances of each party during the votes are politologists, sociologists, historians, politicians, and journalists.

In the last illustration experts give their opinion about whether a new commodity shall be launched at the market, and on what conditions. The experts shall be economists, advertisers and image makers, specialists from the trade branch of the product. Some of their criteria are the following: high qualitative and quantitative indices, price, saturation of the market with similar commodities, durability and warranty period.

These examples are only a small part of all possible applications of the instrumentation of multi-person multi-criteria decision making. They reflect the wide variety of themes which this new approach may cover.

Let everywhere below, $1 \leq i \leq m$.

The first GN-transition has the form (see Fig. 1; for index matrices see Spr3):

$$Z_1 = \langle \{l_1, l_2, l_3, l_7\}, \{l_{4,1}, \dots, l_{4,m}, l_{5,1}, \dots, l_{5,m}, l_6, l_7\},$$

	$l_{4,1}$...	$l_{4,m}$	$l_{5,1}$...	$l_{5,m}$	l_6	l_7
l_1	$W_{1,1}$...	$W_{1,m}$	false	...	false	false	false
l_2	false	...	false	true	...	true	false	false
l_3	false	false	false	false	...	false	false	true
l_7	false	false	false	false	...	false	$W_{7,6}$	$W_{7,7}$

where

$W_{1,i}$ = “the current token is numbered by “ i ””,

$W_{7,6}$ = “on the previous time-step, the last α -token entered the GN”,

$W_{7,7} = \neg W_{7,6}$.

The α -tokens enter sequentially places $l_{4,1}, \dots, l_{4,m}$ without a new characteristic. The β -token splits to m tokens β_1, \dots, β_m that enter places $l_{5,1}, \dots, l_{5,m}$ without a new characteristic. Token γ enters place l_7 and it will stay there by the moment in which all α -tokens have entered the GN. In each time-step, in which a new α -token (let it be α_i) enters the GN, it will obtain a characteristic

$$x_i^\gamma = x_{i-1}^\gamma \cup \text{“the set of the new criteria that the } i\text{-th expert likes to offer”}.$$

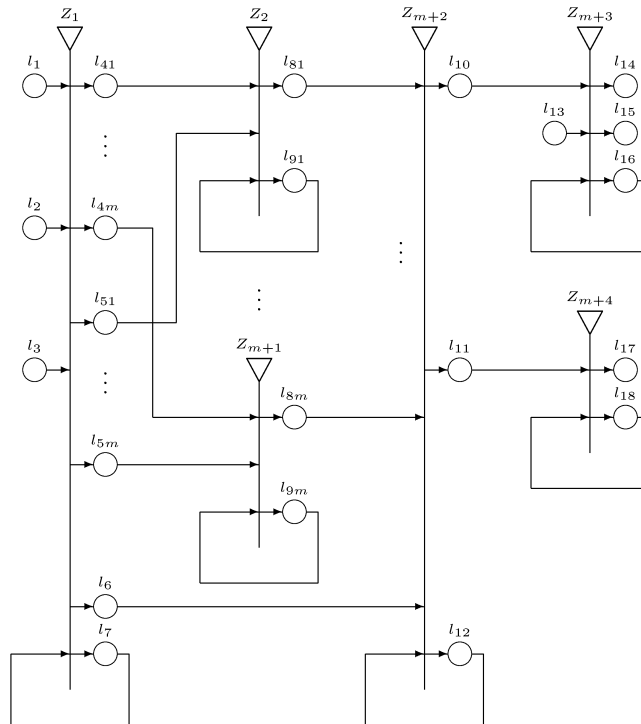


Figure 1.

Let the final number of the criteria that the experts will use be q , where $q \geq s$. The case $s = q$ corresponds of the situation when all experts accept the initial

criteria and do not offer new one. In practice, this is the standard case for data mining procedures, while here, we describe the more general case, in which the experts can offer new criteria, too.

When all α -tokens enter the GN, if predicate $W_{7,7} = true$, then token γ stays in place l_7 without a new characteristic. Only when predicate $W_{7,7} = false$, i.e., when predicate $W_{7,6} = true$, token γ enter place l_6 without any characteristic, too.

Transitions Z_2, \dots, Z_{m+1} have similar forms. For $1 \leq i \leq m$:

$$Z_{i+1} = \langle \{l_{4,i}, l_{5,i}, l_{9,i}\}, \{l_{8,i}, l_{9,i}\}, \begin{array}{c|cc} & l_{8,i} & l_{9,i} \\ \hline l_{4,i} & false & W_{4,9} \\ l_{5,i} & false & W_{5,9} \\ l_{9,i} & true & false \end{array} \rangle,$$

where

$W_{4,9} = W_{5,9} =$ "all α -tokens have entered the GN".

When predicates $W_{4,9} = W_{5,9} = true$, the α_i - and β_i -tokens from places $l_{4,i}$ and $l_{5,i}$ enter place $l_{9,i}$ and unite as an α_i -token that obtains as a first characteristic the IM

$$S_i = \begin{array}{c|cccc} & A_1 & A_2 & \dots & A_p \\ \hline C_1 & & & & \\ & \langle \alpha_{j,k}^i \beta_{j,k}^i \rangle & & & \\ C_2 & & & & \\ & (1 \leq i \leq m, & & & \\ \vdots & & & & \\ & 1 \leq j \leq q, 1 \leq k \leq p) & & & \\ C_q & & & & \end{array}$$

where: $\alpha_{j,k}^i, \beta_{j,k}^i \in [0, 1]$ and $\alpha_{j,k}^i + \beta_{j,k}^i \leq 1$. It corresponds to i -th expert's evaluations of the given alternatives (in the initial β -token characteristic) about the criteria in the current γ -token characteristic.

The predicates $W_{4,9}$ and $W_{5,9}$ guarantee that all new experts' criteria are collected in the current and in practically, final γ -token characteristic.

The form of the IM S_i can be the present one (in it all alternatives exist) or with small number of columns, if the alternatives that the i -th expert does not like to evaluate. The other possibility is, if some expert does not like to evaluate some alternative, on the respective places in the IM to put $\langle 0, 0 \rangle$.

The $(m + 2)$ -nd GN-transition has the form:

$$Z_{m+2} = \langle \{l_6, l_{8,1}, \dots, l_{8,m}, l_{12}\}, \{l_{10}, l_{11}, l_{12}\},$$

$$\begin{array}{c|ccc} & l_{10} & l_{11} & l_{12} \\ \hline l_6 & false & false & true \\ l_{8,1} & true & false & false \\ \vdots & \vdots & \vdots & \vdots \\ l_{8,m} & true & false & false \\ l_{12} & false & true & false \end{array} \rangle.$$

The α -tokens without any characteristic, simultaneously enter place l_{10} that has capacity m (in practice, all other places can have capacity 1).

Token γ from place l_6 enters place l_{12} and obtains as a characteristic the following IM

$$S = \begin{array}{c|cccc} & A_1 & A_2 & \dots & A_p \\ \hline C_1 & & & & \\ C_2 & & \langle \alpha_{j,k} \beta_{j,k} \rangle & & \\ & & (1 \leq j \leq q, & & \\ \vdots & & & & \\ C_q & & 1 \leq k \leq p) & & \end{array}$$

where $\alpha_{j,k}$ and $\beta_{j,k}$ can be calculated by different formulas, with respect to some specific aims. For example, such formulas are the following:

$$\left\{ \begin{array}{l} \alpha_{j,k} = \frac{\sum_{i=1}^m \delta_i \cdot \alpha_{j,k}^i}{\sum_{i=1}^m \delta_i} \\ \beta_{j,k} = \frac{\sum_{i=1}^m \varepsilon_i \cdot \beta_{j,k}^i}{\sum_{i=1}^m \varepsilon_i} \end{array} \right.$$

(here only the average degrees of expert reliability participate),

$$\left\{ \begin{array}{l} \alpha_{j,k} = \frac{\sum_{i=1}^m \delta_{i,j} \cdot \alpha_{j,k}^i}{\sum_{i=1}^m \delta_{i,j}} \\ \beta_{j,k} = \frac{\sum_{i=1}^m \varepsilon_{i,j} \cdot \beta_{j,k}^i}{\sum_{i=1}^m \varepsilon_{i,j}} \end{array} \right.$$

(here only the experts' degrees of reliability estimated by the corresponding criteria participate).

$$\left\{ \begin{array}{l} \alpha_{j,k} = \frac{\sum_{i=1}^m \bar{\alpha}_{j,k}^i}{m} \\ \beta_{j,k} = \frac{\sum_{i=1}^m \bar{\beta}_{j,k}^i}{m} \end{array} \right. ,$$

where $\bar{\alpha}_{j,k}^i$ and $\bar{\beta}_{j,k}^i$ can also be calculated by various formulas, according to particular goals and expert knowledge. For example, such formulas can be:

$$\left\{ \begin{array}{l} \bar{\alpha}_{j,k}^i = \gamma_i \cdot \frac{\alpha_{j,k}^i \cdot \delta_{i,j} + \beta_{j,k}^i \cdot \varepsilon_{i,j}}{\gamma_i + 1} \\ \bar{\beta}_{j,k}^i = \gamma_i \cdot \frac{\alpha_{j,k}^i \cdot \varepsilon_{i,j} + \beta_{j,k}^i \cdot \delta_{i,j}}{\gamma_i + 1} \end{array} \right.$$

or

$$\begin{cases} \bar{\alpha}_{j,k}^i = \alpha_{j,k}^i \cdot \frac{\delta_{i,j} + 1 - \varepsilon_{i,j}}{2} \\ \bar{\beta}_{j,k}^i = \beta_{j,k}^i \cdot \frac{\varepsilon_{i,j} + 1 - \delta_{i,j}}{2} \end{cases}$$

The first formula takes into account not only the rating of each expert by the different criteria, but also the number of times he has given an opinion (his first time is neglected, since he has not had a rating then). Obviously, the so constructed elements of the IM satisfy the inequality: $\alpha_{j,k} + \beta_{j,k} \leq 1$. This IM contains the average experts estimations taking into account experts ratings.

More general case is, if every one of the criteria $C_j (1 \leq j \leq q)$ has its own priority, denoted by $\varphi_j \in (0, 1]$. This information will be put in the initial characteristic of token γ , while the criteria offered by the experts will have, e.g., priority $\langle 0, 0 \rangle$ or other small value, fixed before the start of the procedure.

We can determine for every alternative A_k the global estimation $\langle \alpha_k, \beta_k \rangle$, where

$$\begin{cases} \alpha_k = \frac{\sum_{j=1}^q \varphi_j \cdot \alpha_{j,k}}{\sum_{j=1}^q \varphi_j} \\ \beta_k = \frac{\sum_{j=1}^q \varphi_j \cdot \beta_{j,k}}{\sum_{j=1}^q \varphi_j} \end{cases}$$

Token γ enters place l_{11} with a characteristic

“list of all alternative values”.

The $(m + 3)$ -rd transition Z_{m+3} can be activated only when token δ enters place l_{13} with initial (an unique) characteristic an IM, with elements the objective values of the attributes about the different criteria:

	A_1	A_2	...	A_p
C_1	$\langle a_{j,k} b_{j,k} \rangle$			
C_2	$(1 \leq j \leq q,$			
\vdots				
C_q	$1 \leq k \leq p)$			

where: $a_{j,k}, b_{j,k} \in [0, 1]$ and $a_{j,k} + b_{j,k} \leq 1$.

This transition will be active until all α -tokens from place l_{10} go to place l_{14} through place l_{16} . Its form is

$$Z_{m+3} = \langle \{l_{10}, l_{13}, l_{16}\}, \{l_{14}, l_{15}, l_{16}\}, \begin{array}{c|ccc} & l_{14} & l_{15} & l_{16} \\ \hline l_{10} & true & false & false \\ l_{13} & false & false & true \\ l_{16} & false & W_{16,15} & W_{16,16} \end{array} \rangle,$$

where

$W_{16,15}$ = “the last α -token entered place l_{14} ”,

$W_{16,16} = \neg W_{16,15}$.

Sequentially, the α -tokens enter place l_{14} with final characteristic

“new expert’s rating $\langle \delta_i, \varepsilon_i \rangle$, and a new number of participations in expert investigations γ'_i ”.

The values of this characteristic are estimated by formulas

$$\gamma'_i = \gamma_i + 1,$$

and

$$\left\{ \begin{array}{l} \delta'_i = \frac{\gamma_i \cdot \delta_i + \frac{c_M - c_i}{2}}{\gamma'_i}, \\ \varepsilon'_i = \frac{\gamma_i \cdot \varepsilon_i - \frac{c_M - c_i}{2}}{\gamma'_i}, \end{array} \right. ,$$

where:

$$c_i = \frac{\sum_{j=1}^q \sum_{k=1}^p \sqrt{(\alpha_{j,k} - a_{j,k})^2 + (\beta_{j,k} - b_{j,k})^2}}{p \cdot q},$$

and

$$c_M = \frac{\sum_{i=1}^n c_i}{n}.$$

As we discussed in the beginning of the section, other formulas for the expert’s rating are also possible.

Finally, transition Z_{m+4} has the form

$$Z_{m+4} = \langle \{l_{11}, l_{18}\}, \{l_{17}, l_{18}\}, \begin{array}{c|cc} & l_{17} & l_{18} \\ l_{11} & false & true \\ l_{18} & true & false \end{array} \rangle.$$

Token γ from place l_{11} enters place l_{18} , where it obtains as a characteristic the result of the intercriteria analysis (see [11]).

On the next time-moment, token γ enters place l_{17} with the final characteristic

“final results of the decision making procedure;

list of the independent criteria that will be used in a next procedure”.

Here we shall mention a significant difference between the four examples. It constitutes in the time needed for the experts to understand how well have they made their evaluations and prognoses. In the case of election prognoses, the experts obtain their own score at the moment of the final announcement of the results of the vote. On the other hand, when a job candidate evaluated, the experts can estimate his/her work in the company in a longer period of time, including periods of adaptation and training, and first finished projects. Moreover, in such a case the expert’s appraisal may be subjective and liable to refutation.

3 Conclusion

The above described GN-model can be used for simulation, investigation and control of the processes of decision making. The future models will help us to solve tasks with large degree of uncertainty, with more complex expert activities, and with more complex algorithms for decision making.

As a conclusion, it is worth mentioning that the so-constructed GN-model from one side can be extended and from another - it can be rewritten in the sense of the GN-extensions from [1 – 4], adding characteristics as to the tokens, as well as to the places and to the arcs. For example, in future, ideas from [20] can be described by GNs and these new models can be included as subnets in the above described GN.

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