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A NEW SEPARATION AXIOM ON BITOPOLOGICAL SPACES

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Abstract. The purpose of this paper is to introduce a new separation axiom on bitopological spaces which lies between the separation axiom T_0 and T_1. A new kind of topology has also been induced over the bitopological spaces. It is interesting to see how this topology behaves in the presence of this new separation axiom.

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1. Introduction

The concept of bitopological space was first introduced by Kelley [4] when he noticed that the asymmetric behavior of a quasi- metric space gave rise to two topologies on it. Since then, the field of bitopological spaces has been enriched by the introduction of various diverse concepts such as pairwise connectedness, pairwise compactness and its variants like pairwise Lindelof, pairwise countable compactness, pairwise paracompactness and like, and a number of pairwise version of separation axioms. We know that now, a wide variety of separation axioms have been introduced and studied for topological and bitopological spaces. The first systematic study of separation axioms topological spaces was given by Urysohn [7]. Further Van and Freundenthal [10] gave a more detailed discussion of separation axioms. Aull [1] introduced a new separation axiom between the axioms T_0 and T_1. The tools developed by Aull inspired us to introduce a new separation axiom between pairwiseT_0 and pairwise T_1 which we name as pairwise T_D* axiom. Next we introduce a new topology on a bitopological space $(X,\Im_1, \Im_2)$ as the coarsest of all topologies in which every subset of X open in both the topologies is both closed and open. The set X along with this topology gives rise to a new topological space which we denote as $(X,\Im_R)$. Finally we establish the correspondence between the separation axiom T_D* and the topology $\Im_R$.

2. Preliminaries

We denote a bitopological space as $(X,\Im_1, \Im_2)$ where X is a nonempty set equipped with two arbitrary topologies $\Im_1$ and $\Im_2$. The closure and interior of a subset of X have their general sense. To make the article self contained, we recall the following well known definitions:

Definition 2.1: A bitopological space (X, $\Im_1$, $\Im_2$) is pairwise T_0 if for every pair of distinct elements of X there exists an open set in any of the topologies containing only one of the points.

Definition 2.2: A bitopological space (X,$\Im_1$, $\Im_2$) is pairwise T_1 if for every pair of distinct elements of X say x and y there exists an open set U in $\Im_i$ containing x but not y and an open set say V in $\Im_j$ containing y but not x, for i, j $\in$ {1,2} i $\neq$ j.

It is obvious that if a bitopological space is T_1 then every singleton is closed both in $\Im_1$ and $\Im_2$.

Definition 2.3: A subset A of X in a bitopological space (X,$\Im_1$, $\Im_2$) is called bi - open if A is both $\Im_1$-open and $\Im_2$-open.

Definition 2.4: A bitopological space (X,$\Im_1$, $\Im_2$) is pairwise $T_{1/2}$ if for every pair of distinct elements of X say x and y there exists an open or closed set say U in $\Im_i$ containing x but not y and an open or closed set say V in $\Im_j$ containing y but not x, i ,j $\in$ {1,2} i $\neq$ j.

Pairwise $T_1 \Rightarrow$ Pairwise $T_{1/2} \Rightarrow$ Pairwise T_0

3. A Separation axiom between pairwise T_0 and pairwise T_1

Definition 3.1: A bitopological space (X, $\Im_1$, $\Im_2$) is pairwise T_D* if for every element x of X there exists an open set U in $\Im_i$ such that U- {x} = $P_1 \cap P_2$ where $P_i \in \Im_i$ and x $\notin$ ($P_1 \cup P_2$) $\forall$ i $\in$ {1,2}.

Definition 3.2: A bitopological space (X, $\Im_1$, $\Im_2$) is pairwise T_D* if for every element x of X there exists an open set U in $\Im_i$ such that U$\cap$($\Im_i$-cl{x} $\cup$ $\Im_j$-cl{x})={x}.

Theorem 3.1: Both the Definitions of pairwise T_D* axiom are equivalent.
Proof: Let x $\in$ X and let U$\in$ $\Im_i$ such that U contains x and U-{x} = $P_1 \cap P_2$ where $P_i \in \Im_i$ and x $\notin$($P_1 \cup P_2$) . Let y $\in$ $P_1 \cap P_2$ Then no P_i contains x. Therefore $P_i \cap \Im_i$-cl{x} = $\phi \Rightarrow$ ($P_1 \cap P_2$)$\cap$ $\Im_i$-cl{x } = ϕ for i $\in$ {1,2} $\Rightarrow$ ($P_1 \cap P_2$)$\cap$ $\Im_i$-cl{x })$\cap \Im_j$-cl{x})= $\phi \Rightarrow$ U$\cap$ $\Im_i$-cl{x}$\cup$ $\Im_j$-cl{x})={x}.

Conversely, let U$\cap$($\Im_i$ -cl{x} $\cup$ $\Im_j$ -cl{x})={x} $\Rightarrow$ X-(U-{x}) = (X-U) $\cup$ {x} = (X-U) $\cup$ ($\Im_i$- cl{x} $\cup$ $\Im_j$-cl{x}) = ((X-U) $\cup$ $\Im_i$-cl{x}) $\cup$ $\Im_j$-cl{x} = $F_1 \cup F_2$ where X-$F_i \in \Im_i$ and x $\in$ $F_1 \cap F_2$ $\Rightarrow$ U-{x} = $P_1 \cap P_2$ where $P_i \in \Im_i$ and x $\notin$($P_1 \cup P_2$) .
Theorem 3.2 : In a bitopological space (X, $\Im_1$, $\Im_2$), pairwise $\mathbf{T}_1 \Rightarrow$ pairwise $T_D* \Rightarrow$ pairwise $\mathbf{T}_0$.

Proof: Let (X, $\Im_1$, $\Im_2$) be a pairwise T_1 bitopological space. Then each singleton is closed in both the topologies. Therefore for every $\Im_i$ neighborhood U of x in X, U-{x} is $\Im_i$ open in X. Then U-{x} = (U-{x}) ∩ (X-{x}) satisfying the condition of pairwise T_D* axiom.
Now let (X, $\Im_1$, $\Im_2$) be a pairwise T_D* bitopological space. Let x,y be two distinct elements of X. Then there exists a $\Im_i$ neighborhood U of x in X such that U-{x} = P_1∩ P_2 such that each P_i is open in $\Im_i$ and none of them contains x. If y ∈ X-U then we are done. If y ∈ U then both $P_1$and P_2 are $\Im_1$and $\Im_2$ neighborhoods of y not containing x. Hence (X, $\Im_1$, $\Im_2$) is pairwise T_0 .

Theorem 3.3: In a pairwise T_D* bitopological space, the derived set of each element is closed in both the topologies.
Proof: We just need to show that x∉ $\Im_i$-cl({x}$_i$') where ({x}$_i$') denotes the derived set of {x} in the topology $\Im_i$. Since(X, $\Im_1$, $\Im_2$) is a pairwise T_D* bitopological space, there exists an open neighborhood of x in $\Im_i$ such that U ∩($\Im_i$-cl{x} ∩$\Im_j$-cl{x}) = {x} ⇒ U ∩$\Im_i$-cl{x} ={x} ⇒ U ∩ ({x}$_i$') = ϕ. Hence x does not belong to $\Im_i$-cl({x}$_i$') leading to the fact that derived set of each element is closed in both the topologies.

Theorem 3.4 : In a pairwise T_D* bitopological space (X, $\Im_1$, $\Im_2$), for every pair of distinct elements of X say x and y there exists an open set P_i in $\Im_i$ such that P_1∩ P_2 contains either only x but not y or only y but not x, for i, j ∈ {1,2} i ≠ j.
Proof: Since(X, $\Im_1$, $\Im_2$) is a pairwise T_D* bitopological space and x, y are two distinct elements of X, there exists a $\Im_i$ neighborhood U of x in X such that U-{x} = P_1∩ P_2 such that each P_i is open in $\Im_i$ and none of them contains x. If y ∉ U then U∩X serves the purpose. If y ∈ U then both P_1 and P_2 are $\Im_1$and $\Im_2$ neighborhoods of y not containing x.

4. A topology Induced on A Bitopological Space

In this section we induce a topology on a bitopological space.
Definition 4.1: Let (X, $\Im_1$, $\Im_2$) be a bitopological space. Define a function $R_{(1,2)}$: P(X) → P(X) as $R_{(1,2)}$(A) ={y ∈ X : ∀ U ∈$\Im_1$ and V ∈ $\Im_2$: y ∈ U-V, U ∩ A ≠ V ∩ A}.

Definition 4.2: Let(X, $\Im_1$, $\Im_2$) be a bitopological space. The function $R_{(1,2)}$ can also be defined as:
$R_{(1,2)}$: P(X) →P(X) as $R_{(1,2)}$(A) ={y ∈ X : ∀ U ∈$\Im_1$: y ∈ U, $\Im_2$-cl{y}∩ A ∩ U≠ ϕ }.
It can be easily seen that both the definitions of the $R_{(1,2)}$ function are equivalent:
Suppose x ∈ {y ∈ X : ∀ U ∈ $\Im_1$and V ∈ $\Im_2$: y ∈ U-V, U ∩ A ≠ V ∩ A}.
Then ∀ $\Im_1$neighborhood U of x and ∀ $\Im_2$ open set V not containing x we have U ∩ A ≠ V ∩ A ⇒ (U ∩ A) - (V ∩ A) ≠ ϕ⇒ U ∩ (X-V) ∩ A ≠ ϕ⇒∀ $\Im_1$ neighborhood U of x and ∀ $\Im_2$ closed set F containing x, U ∩ F ∩ A ≠

ϕ . In particular, if we take F = $\Im_2$-cl{x} then $\Im_2$-cl{x}$\cap$ A $\cap$ U$\neq \phi \Rightarrow$ x $\in$ {y $\in$ X : $\forall$ U $\in \Im_1$: y $\in$ U, $\Im_2$-cl{y}$\cap$ A $\cap$ U$\neq \phi$ }.
Conversely, if x $\in$ {y $\in$ X : $\forall$ U $\in \Im_1$: y $\in$ U, $\Im_2$-cl{y}$\cap$ A $\cap$ U$\neq \phi$ } then $\Im_2$-cl{x}$\cap$ A $\cap$ U$\neq \phi$ $\forall$ $\Im_1$ neighborhood U of x. Since $\Im_2$ -cl{x} is intersection of all $\Im_2$ closed sets of X containing x it implies that $\forall$ $\Im_1$neighborhood U of x and $\forall$ $\Im_2$ closed set F containing x, U $\cap$ F $\cap$ A $\neq \phi$. Hence x $\in$ {y $\in$ X : $\forall$ U $\in \Im_1$ and V $\in \Im_2$: y $\in$ U-V, U $\cap$ A $\neq$ V $\cap$ A}.

Definition 4.3 : In a bitopological space (X, $\Im_1$, $\Im_2$), a subset A of X is called $R_{(1,2)}$ closed if $R_{(1,2)}(A)$ = A.
The fact that every $\Im_2$-closed set is $R_{(1,2)}$ closed can easily be concluded from definition 1(a).However, it is interesting to note that $\Im_2$-open set is $R_{(1,2)}$ closed as can be shown as below :
If A is $\Im_2$-open then X-A is $\Im_2$-closed and therefore for every x $\in$ X-A, $\Im_2$ - cl{x} $\subset$ (X- A)$\Rightarrow\Im_2$-cl{x}$\cap$ A $\cap$ U$\neq \phi$ $\forall$ U $\in \Im_1$: x $\in$ U $\Rightarrow$ A is $R_{(1,2)}$ closed.

Definition 4.4: Let(X, $\Im_1$, $\Im_2$) be a bitopological space. Define a function $R_{(2,1)}$: P(X) $\rightarrow$P(X) as $R_{(2,1)}$ (A) ={y $\in$ X : $\forall$ U $\in \Im_2$ and V $\in \Im_1$: y $\in$ U-V, U $\cap$ A $\neq$ V $\cap$ A}.

Definition 4.5: In a bitopological space (X, $\Im_1$, $\Im_2$) the function $R_{(2,1)}$ can also be Defined as: $R_{(2,1)}$: P(X) $\rightarrow$ P(X) as $R_{(2,1)}$ (A) ={y $\in$ X : $\forall$ U $\in$ $\Im_2$: y $\in$ U, $\Im_1$-cl{y}$\cap$ A $\cap$ U$\neq \phi$ }.

Definition 4.6: In a bitopological space (X, $\Im_1$, $\Im_2$), A $\subset$ X is called $R_{(2,1)}$ closed if $R_{(2,1)}$ (A) = A.
Again it can be easily verified that both the Definitions of the $R_{(2,1)}$ function are equivalent. Also Definition 2(a) asserts that every $\Im_1$ closed subset A of X is $R_{(2,1)}$ closed as well as open.

Definition 4.7: In a bitopological space (X, $\Im_1$, $\Im_2$), A$\subset$ X is called R closed if $R_{(1,2)}$(A)=A= $R_{(2,1)}$(A).
*Hence A is R closed if $\forall$ x $\notin$ A, there exist U_1,$U_2$$\in$ $\Im_1$and V_1,$V_2$$\in$ $\Im_2$ such that x $\in$ U_1-V_1 and x $\in$ V_2-U_2 W$_i$th $U_1$$\cap$ A = $V_1$$\cap$A and $U_2$$\cap$A = $V_2$$\cap$A. equivalently, one can say that A is R closed if $\forall$ x $\notin$ A there exist U $\in$ $\Im_1$and V $\in$ $\Im_2$ such that x $\in$ U and x $\in$ V and $\Im_2$-cl{x}$\cap$ A $\cap$ U= ϕ = $\Im_1$-cl{x}$\cap$ A $\cap$ V.

Theorem 4.1: The R operator holds following assertions:

(1) A $\subset$ $R_{(i,j)}$(A) $\subset$ $\Im_i$-cl(A), i, j $\in$ (1,2) i $\neq$j. Also A $\subset$ R(A) $\subset$ $\Im_1$-cl(A) $\cap$ $\Im_2$-cl(A)
(2) B$\subset$A$\Rightarrow$R(B)$\subset$ R(A)
(3) R(R(A)) = R(A)
(4) R(A$\cup$B) = R(A) $\cup$R(B)

Proof:

(1) Let x $\notin$ R(A) then there exist U_1,$U_2$$\in$ $\Im_1$and V_1,V_2 $\in$ $\Im_2$ such that x $\in$ U_1-V_1 and x $\in$$V_2$-$U_2$ with $U_1$$\cap$A = $V_1$$\cap$A and $U_2$$\cap$A = $V_2$$\cap$A.

This forces to conclude that x $\notin$ A for if x $\in$ A then x $\in$ $U_1 \cap A$ whereas x $\notin$ $V_1 \cap A$ leading to the conclusion that $U_1 \cap A \neq V_1 \cap A$. Hence A $\subset$ R(A) .

Now let x $\notin$ $\Im_i$-cl(A) , i $\in$ {1,2} then there exists a neighborhood of x say $U_i \in \Im_i$ such that$U_i \cap A = \phi$. Taking $\phi = U_j \in \Im_j$, j $\in$ {1,2}, i $\neq$j we get x $\notin$ R(A).

(2) Let x $\notin$ R(A) then there exist $U_1, U_2 \in \Im_1$and $V_1, V_2 \in \Im_2$ such that x $\in$ U_1-V_1 and y $\in$ V_2-U_2 with $U_1 \cap A = V_1 \cap A$ and $U_2 \cap A = V_2 \cap A$. Then $(U_1 \cap A) \cap B = (V_1 \cap A) \cap B$ and $(U_2 \cap A) \cap B = (V_2 \cap A) \cap B$ $\Rightarrow$ $U_1 \cap B = V_1 \cap B$ and $U_2 \cap B = V_2 \cap B$ $\Rightarrow$ x $\notin$ R(B).

(3) By (i) it is clear that R(A) $\subset$ R(R(A)). Now let x $\notin$ R(A) then there exist $U_1, U_2 \in \Im_1$and $V_1, V_2 \in \Im_2$ such that x $\in$ U_1-V_1 and x $\in$ V_2-U_2 with $U_1 \cap A = V_1 \cap A$ and $U_2 \cap$ A = $V_2 \cap$ A. We claim that $U_1 \cap R(A) = V_1 \cap R(A)$ and $U_2 \cap R(A) = V_2 \cap R(A)$. For if $U_1 \cap R(A) \neq V_1 \cap R(A)$ then there exists an element y $\in$ X such that y $\in$ $U_1 \cap R(A)$ and y $\notin$ $V_1 \cap R(A)$. Thus y $\in$ $U_1 \in \Im_1$, y $\in$ R(A) and y $\notin$ $V_1 \in \Im_2$ .But since y $\in$ R(A) we must have $U_1 \cap A \neq V_1 \cap A$ which is a contradiction. Therefore we have R(R(A)) = R(A).

(4) To prove this part we make use of the second definition of R(A). By (ii) R(A) $\cup$R(B) $\subset$ R(A$\cup$B). Now let x $\notin$ R(A)$\cup$R(B). Then there exist U, P $\in$ $\Im_1$and V, Q $\in$ $\Im_2$ containing x such that $\Im_2$-cl {x}$\cap$ A $\cap$ U= ϕ = $\Im_1$-cl{x}$\cap$ A $\cap$ V and $\Im_2$-cl{x}$\cap$ B $\cap$ P= ϕ = $\Im_1$-cl{x}$\cap$ B $\cap$ P. Consider E=U $\cap$ P and F= V $\cap$ Q then, $\Im_2$-cl{x}$\cap$ (A$\cup$B) $\cap$ E= ϕ = $\Im_1$-cl{x}$\cap$ (A$\cup$B) $\cap$ F implying that R(A$\cup$B) = R(A) $\cup$ R(B).

Hence the R operator satisfies all the conditions of Kuratowski's closure operator and therefore defines a topology on the bitopological space (X, $\Im_1$, $\Im_2$). We denote this topology by $\Im_R$ and the topological space equipped with this topology is denoted as (X,$\Im_R$). The members of $\Im_R$ are defined as :

If A $\in$ P(X) then A $\in$ $\Im_R$ if $\forall$ x $\in$ A there exist a $\Im_1$ neighborhood U of x and a $\Im_2$ neighborhood V of x satisfying the condition: x $\in$ ($\Im_1$-cl{x}$\cap$U) $\cup$ ($\Im_2$-cl{x}$\cap$V)$\in$ A.

Further it can also be verified that both $R_{(1,2)}$ and $R_{(2,1)}$ operators also satisfy the conditions of Kuratowski's closure operator and therefore define topologies on a bitopological space (X, $\Im_1$, $\Im_2$) such that the topology generated by $R_{(1,2)}$ is finer than $\Im_2$ and the topology generated by $R_{(2,1)}$ is finer than $\Im_1$.

The above discussion indicates to the fact that a subset A of a bitopological space (X, $\Im_1$, $\Im_2$) is both open and closed in $\Im_R$ if it is open (closed) in both the topologies $\Im_1$ and $\Im_2$. Also (X,$\Im_R$) is coarsest of all the topologies in which a subset of X open (closed) in both the topologies is clopen.

Consider the following collection of subsets of X:

$\mathbf{B}_{(1,2)}$ = {U-V : U $\in$ $\Im_2$ and V $\in$ $\Im_1$}

$\mathbf{B}_{(2,1)}$ = {U-V : U $\in$ $\Im_1$ and V $\in$ $\Im_2$}

$\mathbf{B}$ = {(U-V)$\cup$(V-U): U $\in$ $\Im_1$ and V $\in$ $\Im_2$}

Then, $\mathbf{B}_{(1,2)}$, $\mathbf{B}_{(2,1)}$ and $\mathbf{B}$ serve as base for topologies generated by $R_{(1,2)}$, $R_{(2,1)}$ and R operator respectively.

Example 4.1: Let X be the set of all positive integers. Let $A_n = \{m \in X : m \geq n\}$. Let $\Im_1 = \{A_n : n \geq 1\} \cup \{\phi\}$ and $\Im_2$ = Discrete topology on X. Then (X, $\Im_R$) is also discrete topology.

Definition 4.8: A subset A of X is called R-dense if R-cl(A) = X. In this case X is called R-hull of A.

Theorem 4.2: A subset A of X is R-dense in X if $\forall$ U $\in \Im_1$ and V $\in \Im_2$ we have U$\cap$A $\neq$V$\cap$A.

Definition 4.9: A bi open filter $\mathfrak{F}$ on a bitopological space (X, $\Im_1$,$\Im_2$) is defined as a collection of non-empty subsets of X :
(i) $\mathfrak{F} \subseteq \Im_1 \cup \Im_2$ and $\mathfrak{F} \cap \Im_i \neq \phi$ $\forall$ i $\in$ {1,2}
(ii) E, F $\in \mathfrak{F}$: E, F $\in \Im_i$ E$\cap$ F $\in \Im_i$ $\forall$i $\in$ {1,2}
(iii) G $\in \mathfrak{F}$ and G $\subseteq$H with G, H $\in \Im_i$ H $\in \mathfrak{F}$ $\forall$ i $\in$ {1,2}

Theorem 4.3: Let (X, $\Im_1$,$\Im_2$) be a pairwise T_0 bitopological space and let Y $\subseteq$ X such that R- cl(Y) = X then two different elements in X have different biopen filter.
Proof: Let p, q$\in$ X : p $\neq$ q. Let $\mathfrak{U}_p$= {U $\cap$ Y : U is $\Im_i$ neighborhood of p $\forall$ i $\in$ {1,2} } and $\mathfrak{U}_q$ = {U$\cap$ Y : U is $\Im_i$ neighborhood of q $\forall$ i $\in$ {1,2} }. Then both $\mathfrak{U}_p$ and $\mathfrak{U}_q$ are biopen filter on (X, $\Im_1$, $\Im_2$). Since X is pairwise T_0, there exists U $\in \Im_i$, I $\in$ {1,2} containing one of p and q but not the other. Without loss of generality we may assume that U contains p then U$\cap$ Y $\in\mathfrak{U}_p$. If U$\cap$ Y $\in \mathfrak{U}_q$ then there exists V $\in \Im_i$, i $\in$ {1,2} such that U$\cap$Y = V$\cap$Y. A contradiction to the fact that Y is R-dense in X.

Theorem 4.4 : If (X, $\Im_1$, $\Im_2$) is a pairwise T_0 bitopological space then the induced topological space (X,$\Im_R$) is T_1.
Proof: Let p, q $\in$ X : p$\neq$ q. To show that (X, $\Im_R$) is T_1 it suffices to show that $\Im_R$-cl{p} = {p} $\forall$ p $\in$ X.
By definition, if q $\in \Im_R$-cl{p} then $\Im_2$-cl{q} $\cap$ {p} $\cap$ $U_p \neq \phi \neq \Im_1$-cl{q} $\cap$ {p} $\cap$ V_p $\forall$ $\Im_1$ neighborhood U_p and $\forall$ $\Im_2$ neighborhood of V_p of p $\Rightarrow$ every $\Im_1$ neighborhood and every $\Im_2$ neighborhood of p contains q and every $\Im_1$ neighborhood and every $\Im_2$ neighborhood of q contains p which is contrary to the fact that (X, $\Im_1$, $\Im_2$) is a pairwise T_0.

5. Relation Between $T_{D}*$ and $\Im_R$

Theorem 5.1 : If a bitopological space (X, $\Im_1$, $\Im_2$) is pairwise T_D* then (X, $\Im_R$) is discrete.
Proof: Let x $\in$ X. Then $\Im_R$-cl(X-{x}) = {y $\in$ X | $\Im_i$-cl(y) $\cap$ X-{x} $\cap$ $U_j \neq$ ϕ and $\Im_j$-cl(y) $\cap$ X-{x} $\cap$ $U_i \neq \phi$ } for every $\Im_i$ neighborhood U_i of y and $\Im_j$ neighbourhood U_j of y.
If x $\in \Im_R$ cl(X-{x}) then $\Im_i$-cl(x) $\cap$ X-{x} $\cap$ $U_j \neq \phi$ and $\Im_j$-cl(x) $\cap$ X-{x} $\cap$ $U_i \neq \phi$ for every $\Im_i$ neighborhood U_i of x and $\Im_j$ neighborhood U_j of x. Let z $\in \Im_i$-cl(x) $\cap$(X-{x})$\cap$ U_j $\Rightarrow$ z $\in \Im_i$- cl(x), z $\neq$ x and z $\in$ U_j. Since X is pairwise D*, x has a neighborhood U_j such that U_j −{x}= $P_1 \cap P_2$ such

that each P_i is open in $\Im_i$ and none of them contains x. If z $\in$ U_j then P_i is a neighborhood of z not containing x $\Rightarrow$ z $\notin$ $\Im_i$cl(x). Hence X-{x} is closed implying that {x} is open in $\Im_R$. Since each singleton is open in $\Im_R$ therefore $\Im_R$ is discrete.

Theorem 5.2 : In a bitopological space (X, $\Im_1$, $\Im_2$) if (X, $\Im_R$) is discrete and each derived set is closed then (X, $\Im_1$, $\Im_2$) is pairwise D*.
Proof: Let (X, $\Im_R$) be discrete then $\Im_R$-cl(X-{x}) = X-{x}$\Rightarrow$ x has a neighborhood U_i in $\Im_i$ for i $\in$ {1,2} such that $\Im_i$-cl(x) $\cap$ (X-{x}) $\cap$ U_i = ϕ=$\Im_i$-cl(x) $\cap$ (X-{x}) $\cap$ U_j $\Rightarrow$ $\Im_j$-cl(x) $\cap$ U_i = {x}. Also $\Im_j${x}$'$ is closed $\Rightarrow$ x has a neighborhood V_i in $\Im_i$ such that $\Im_i$-cl(x) $\cap$ V_i = {x}. Taking U_i $\cap$ V_i =W_i , we get W_i $\cap$ ($\Im_i$-cl(x) $\cup$ $\Im_j$-cl(x)) ={x}. Consider X-(W_i-{x}) = (X-W_i) $\cup${x} = (X-W_i) $\cup$($\Im_j$-cl(x) $\cup$ $\Im_j$-cl(x)) = ((X-W_i) $\cup$ $\Im_j$-cl(x)) $\cup$ $\Im_j$-cl(x) = F_i $\cup F_j$ where each F_i is $\Im_i$ closed and contains x.

Corollary 5.1: In a pairwise regular bitopological space (X, $\Im_1$, $\Im_2$) if (X, $\Im_R$) is discrete then (X, $\Im_1$, $\Im_2$) is pairwise D*.

References

[1] A.S. Dow and S.W.Watson, Skula spaces, *Commentationes Mathematicae Universitatis Carolinae* 31 (1990), 27-31

[2] B. Banaschewski, J.L. Frith and C.R.A. Gilmour, On the congruence lattice of a frame, *Pacific Journal of Mathematics* 130 (1987), 209-213.

[3] C.E. Aull and W. Thron, Separation axioms between T_0 and T_1, *Indagationes Math*ematicae 24 (1963), 26–37.

[4] J. C. Kelly, Bitopological Spaces, *Proceedings of London Mathematical Society* (3) 13 (1963), 71-89.

[5] L.D. Nel and R. Wilson, Epireflections in the category of T_0-topological spaces, *Fundamenta Matematicae* 75 (1972), 69-74.

[6] L. Skula, On a reflective category of all topological spaces, *Transactions of American Mathematical Society* 142 (1969), 37-61.

[7] P. Urysohn, Uber die Machtigkeit der zusammenhangenden Mengen, *Mathematische Annalen* 94, 262- 295 (1925).

[8] R. Wilson, Epireflections in the category of T_0-topological spaces, *Fundamenta Mathematicae* 75 (1972), 69–74.

[9] S. Baron, Note on epi in T_0, *Canadian Mathematical Bulletin* 11 (1968), 503–504.

[10] W. T. Van and H. Freudenthal, Trennung durch stetige Funktionen in topologischen Raumen, *Indagationes Math*ematicae vol. 13, 359-368(1951).

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