

INDEX MATRICES AND OLAP-CUBE
PART 4: A PRESENTATION OF THE OLAP “DRILL ACROSS”
OPERATION BY INDEX MATRICES

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ABSTRACT. Online analytical processing (OLAP) tools are conceived to ease the navigation through the data, saved in multidimensional structures. The operation “Drill Across” retrieve facts on common dimensions of the multiple data-cubes. In the current paper an interpretation of the OLAP “Drill Across” operation using the apparatus of index matrices is presented. Also some practical examples of this operation by language MDX(MultiDimensional eXpressions) [13] are given.

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1. INTRODUCTION

Data warehouse [11, 12] is a subject oriented, nonvolatile collection of data used to support strategic decision-making. The term “OLAP (Online Analytical Processing)” was introduced in 1993 by Codd [10]. The purpose of OLAP systems is to facilitate solving data analysis problems. The Online Analytical Processing ([12], OLAP) is defined as a set of principles that provide a dimensional framework for decision support. The most important characteristic of OLAP [1] is its multidimensional view. This allows one to apply specific storage techniques in order to reduce response times. Most operations are concerned with analyzing data only from one data-cube with the exception of “Drill Across”. This operation retrieves facts [1] from multiple cubes if they have common dimensions. The common dimensions are used to essentially perform a join between the two cubes. The other dimensions do not appear in the result. This may not be adequate in all situations. This operation is studied in [1, 9].

In the current article, which is a continuation of articles [8, 18, 19], “Drill-Across” operation to OLAP-cubes is researched and defined in the terms of the index matrix (IM) concept, which was introduced in 1984 in [2]. The practical examples in the

article are performed using Multidimensional expressions language (MDX). MDX provides a syntax for querying and manipulating the multidimensional data [13].

For the needs of the present research we remind the definition of the three-dimensional extended index matrix (3D-EIM) and some operations over them in section 2. In sections 3 will be presented the definition of an operation “Drill across” using the apparatus of the IMs and applications of this operation in OLAP-cube will be considered.

2. SHORT REMARKS ON 3D-EXTENDED INDEX MATRIX

Let us start with a definition of a 3D-extended index matrix [3, 7]. It was extended in [17].

2.1. Definition of 3D-EIM and some operations over them.

2.1.1. *Definition of 3D-EIM.* An Intuitionistic Fuzzy Pair (IFP) [4, 6] is an object of the form $\langle a, b \rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process. Its components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. Let \mathcal{I} be a fixed set of indices,

$$\mathcal{I}^n = \{\langle i_1, i_2, \dots, i_n \rangle | (\forall j : 1 \leq j \leq n)(i_j \in \mathcal{I})\}$$

and

$$\mathcal{I}^* = \bigcup_{1 \leq n \leq \infty} \mathcal{I}^n.$$

Let \mathcal{X} be a fixed set of some objects. In particular cases, they can be either real numbers, or only the numbers 0 or 1, or logical variables, propositions or predicates, IFPs, functions etc.

A “3D-Extended Index Matrix” (3D-EIM) with index sets K, L and $H(K, L, H \subset \mathcal{I}^*)$ and elements from set \mathcal{X} is called the object:

$$[K, L, H, \{a_{k_i, l_j, h_g}\}] = \left\{ \begin{array}{c|cccccc} h_g & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & a_{k_1, l_1, h_g} & \vdots & a_{k_1, l_j, h_g} & \dots & a_{k_1, l_n, h_g} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_i & a_{k_i, l_1, h_g} & \dots & a_{k_i, l_j, h_g} & \dots & a_{k_i, l_n, h_g} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_m & a_{k_m, l_1, h_g} & \dots & a_{k_m, l_j, h_g} & \dots & a_{k_m, l_n, h_g} \end{array} \right\}, \quad | h_g \in H$$

where $K = \{k_1, k_2, \dots, k_m\}$, $L = \{l_1, l_2, \dots, l_n\}$, $H = \{h_1, h_2, \dots, h_f\}$, and for $1 \leq i \leq m$, $1 \leq j \leq n$, $1 \leq g \leq f : a_{k_i, l_j, h_g} \in \mathcal{X}$.

Following [3, 17], let $3D-EIM_R$ be the set of all 3D-EIMs with elements being real numbers; $3D-EIM_{\{0,1\}}$ be the set of all $(0, 1)$ -3D-EIMs with elements being 0 or 1; $3D-EIM_P$ be the set of all 3D-EIMs with elements – predicates; $3D-EIM_{IFP}$ be the set of all 3D-EIMs with elements – IFPs and $3D-EIM_{FE}$ – the set of all 3D-EIMs with elements – 1-argument functions $\in F$ ¹.

2.1.2. Operations over 3D-EIMs.

- **Projection**

Let us have 3D-EIM $A = [K, L, H, \{a_{k_i, l_j, h_g}\}]$ and let $M \subseteq K$, $N \subseteq L$ and $U \subseteq H$. Then,

$$pr_{M, N, U} A = [M, N, U, \{b_{k_i, l_j, h_g}\}],$$

where for each $k_i \in M$, $l_j \in N$ and $h_g \in U$ $b_{k_i, l_j, h_g} = a_{k_i, l_j, h_g}$.

- **Composition**

Let the index set \mathcal{I}^* and the set \mathcal{X} be fixed and let 3D-EIMs A_1, A_2, \dots, A_n over both sets be given. Let for s ($1 \leq s \leq n$):

$$A_s = [K^s, L^s, H^s, \{a_{s, k_{s,i}, l_{s,j}, h_{s,g}}\}]$$

$$\begin{array}{c|ccccc} h_{s,g} \in H^s & l_{s,1} & \dots & l_{s,j} & \dots & l_{s,n_s} \\ \hline k_{s,1} & a_{s,k_{s,1},l_{s,1},h_{s,g}} & \dots & a_{s,k_{s,1},l_{s,j},h_{s,g}} & \dots & a_{s,k_{s,1},l_{s,n_s},h_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{s,i} & a_{s,k_{s,i},l_{s,1},h_{s,g}} & \dots & a_{s,k_{s,i},l_{s,j},h_{s,g}} & \dots & a_{s,k_{s,i},l_{s,n_s},h_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{s,m} & a_{s,k_{s,m},l_{s,1},h_{s,g}} & \dots & a_{s,k_{s,m},l_{s,j},h_{s,g}} & \dots & a_{s,k_{s,m},l_{s,n_s},h_{s,g}} \end{array}.$$

Following [21] the definition of the operation “composition” is:

$$\flat\{A_s | 1 \leq s \leq n\} = [\bigcup_{s=1}^n K^s, \bigcup_{s=1}^n L^s, \bigcup_{s=1}^n H^s,$$

$$\{\langle c_{1,t_{1,u},v_{1,w},d_{1,e}}, c_{2,t_{2,u},v_{2,w},d_{2,e}}, \dots, c_{n,t_{n,u},v_{n,w},d_{n,e}} \rangle\}],$$

where for r ($1 \leq r \leq n$):

$$c_{r,t_{r,u},v_{r,w},d_{r,e}} = \begin{cases} a_{r,k_{r,i},l_{r,j},h_{r,g}}, & \text{if } t_{r,u} = k_{r,i} \in K^r, v_{r,w} = l_{r,j} \in L^r \\ \perp, & \text{and } d_{r,e} = h_{r,g} \in H^r \\ & \text{otherwise} \end{cases}$$

where symbol \perp denotes the lack of some components in the definition or empty matrix elements.

We give an example from [3]. Let the EIMs A_1, A_2 have the forms

$$A_1 = \begin{array}{c|cccc} & d & e & f & g \\ \hline a & 1 & 2 & \perp & 3 \\ b & 4 & \perp & 5 & \perp \\ c & 6 & 7 & \perp & 8 \end{array}, \quad A_2 = \begin{array}{c|ccc} & d & i & f \\ \hline a & 11 & \perp & 12 \\ c & \perp & 13 & 14 \\ h & 15 & \perp & \perp \end{array},$$

respectively.

Then

$$A = \flat\{A_1, A_2\} = \begin{array}{c|ccccc} & d & e & f & g & i \\ \hline a & \langle 1, 11 \rangle & \langle 2, \perp \rangle & \langle \perp, 12 \rangle & \langle 3, \perp \rangle & \langle \perp, \perp \rangle \\ b & \langle 4, \perp \rangle & \langle \perp, \perp \rangle & \langle 5, \perp \rangle & \langle \perp, \perp \rangle & \langle \perp, \perp \rangle \\ c & \langle 6, \perp \rangle & \langle 7, \perp \rangle & \langle \perp, 14 \rangle & \langle 8, \perp \rangle & \langle \perp, 13 \rangle \\ h & \langle \perp, 15 \rangle & \langle \perp, \perp \rangle \end{array}.$$

- Automatic reduction

The definition of the operation for an EIM A is

$$@(\mathcal{A}) = [P, Q, \{b_{pr,qs,de}\}],$$

where $P \subseteq K, Q \subseteq L, R \subseteq H$ are index sets with the following properties:

$$\begin{aligned}
& (\forall k \in K - P)(\forall l \in L)(\forall h \in H)(a_{k_i, l_j, h_g} = \perp) \\
& \& (\forall k \in K)(\forall l \in L - Q)(\forall h \in H)(a_{k_i, l_j, h_g} = \perp) \\
& \& (\forall k \in K)(\forall l \in L)(\forall h \in H - K)(a_{k_i, l_j, h_g} = \perp) \\
& \& (\forall k \in K)(\forall l \in L)(\forall h \in H - L)(a_{k_i, l_j, h_g} = \perp) \\
& \& (\forall k \in K - H)(\forall l \in L)(\forall h \in H)(a_{k_i, l_j, h_g} = \perp) \\
& \& (\forall k \in K)(\forall l \in L - H)(\forall h \in H)(a_{k_i, l_j, h_g} = \perp) \\
& \& (\forall p_r = a_i \in P)(\forall q_s = b_j \in Q)(\forall d_e = h_g \in H) \\
& \quad (b_{p_r, q_s, d_e} = a_{k_i, l_j, h_g}).
\end{aligned}$$

2.2. Definition of 3D-Multilayer extended index matrix and some operations over them.

2.2.1. Definition of 3D-multilayer extended index matrix (3D-MLEIM).

Let us present the definition of 3D-MLEIM A [15, 21] with P -levels (layers) of use of dimension K , Q -levels(layers) of use of dimension L and R -levels(layers) of use of dimension H as follows:

$$A = [(K, P), (L, Q), (H, R), \{a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}}\}]$$

$$= \left\{ \begin{array}{c|ccccc} H_g^{(R)} \in H & L_1^{(Q)} & \dots & L_j^{(Q)} & \dots & L_n^{(Q)} \\ \hline K_1^{(P)} & a_{K_1^{(P)}, L_1^{(Q)}, H_g^{(R)}} & \dots & a_{K_1^{(P)}, L_j^{(Q)}, H_g^{(R)}} & \dots & a_{K_1^{(P)}, L_n^{(Q)}, H_g^{(R)}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_i^{(P)} & a_{K_i^{(P)}, L_1^{(Q)}, H_g^{(R)}} & \dots & a_{K_i^{(P)}, L_j^{(Q)}, H_g^{(R)}} & \dots & a_{K_i^{(P)}, L_n^{(Q)}, H_g^{(R)}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_m^{(P)} & a_{K_m^{(P)}, L_1^{(Q)}, H_g^{(R)}} & \dots & a_{K_m^{(P)}, L_j^{(Q)}, H_g^{(R)}} & \dots & a_{K_m^{(P)}, L_n^{(Q)}, H_g^{(R)}} \end{array} \right\},$$

where

$$K = \{K_1^{(P)}, K_2^{(P)}, \dots, K_i^{(P)}, \dots, K_m^{(P)}\},$$

$$K_i^{(P)} = \left\{ K_{i,1}^{(P-1)}, K_{i,2}^{(P-1)}, \dots, K_{i,x}^{(P-1)}, \dots, K_{i,I}^{(P-1)} \right\} \text{ for } 1 \leq i \leq m$$

.....

$$K_u^{(1)} = \left\{ K_{u,1}^{(0)}, K_{u,2}^{(0)}, \dots, K_{u,U}^{(0)} \right\}$$

i.e. p -th layer of dimension K of the multilayer matrix, where $(1 \leq p \leq P)$, is represented by

$$K_{u_*}^{(p)} = \left\{ K_{u_{*,1}}^{(p-1)}, K_{u_{*,2}}^{(p-1)}, \dots, K_{u_{*,U_*}}^{(p-1)} \right\} \text{ for } 1 \leq p \leq P$$

$$L = \{L_1^{(Q)}, L_2^{(Q)}, \dots, L_j^{(Q)}, \dots, L_n^{(Q)}\},$$

$$L_j^{(Q)} = \{L_{j,1}^{(Q-1)}, L_{j,2}^{(Q-1)}, \dots, L_{j,y}^{(Q-1)}, \dots, L_{j,J}^{(Q-1)}\} \text{ for } 1 \leq j \leq n$$

$$L_v^{(1)} = \{L_{v,1}^{(0)}, L_{v,2}^{(0)}, \dots, l_{v,V}^{(0)}\}$$

i.e. q -th layer of dimension Q of the multilayer matrix is represented by

$$\begin{aligned} L_{v_*}^{(q)} &= \{L_{v_{*,1}}^{(q-1)}, L_{v_{*,2}}^{(q-1)}, \dots, L_{v_{*,V_*}}^{(q-1)}\} \text{ for } 1 \leq q \leq Q \\ H &= \{H_1^{(R)}, H_2^{(R)}, \dots, H_g^{(R)}, \dots, H_f^{(R)}\}, \\ H_g^{(R)} &= \{H_{g,1}^{(R-1)}, H_{g,2}^{(R-1)}, \dots, H_{g,z}^{(R-1)}, \dots, H_{g,G}^{(R-1)}\} \text{ for } 1 \leq g \leq f \\ &\dots \\ H_w^{(1)} &= \{H_{w,1}^{(0)}, H_{w,2}^{(0)}, \dots, H_{w,W}^{(0)}\} \end{aligned}$$

i.e. r -th layer of dimension H of the multilayer matrix is represented by

$$H_{w_*}^{(r)} = \{H_{w_{*,1}}^{(r-1)}, H_{w_{*,2}}^{(r-1)}, \dots, H_{w_{*,W_*}}^{(r-1)}\} \text{ for } 1 \leq r \leq R$$

and $(K, L, H \subset \mathcal{I}^*)$, and for $1 \leq i \leq I$, $1 \leq j \leq J$, $1 \leq g \leq G$, $1 \leq p \leq P$, $1 \leq q \leq Q$, $1 \leq r \leq R$, $1 \leq d \leq I$, $1 \leq b \leq J$, $1 \leq c \leq G : a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}} \in \mathcal{X}$, $K_{i,0}^{(p)} \notin K$, $L_{j,0}^{(q)} \notin L$ and $H_{g,0}^{(r)} \notin H$.

2.2.2. Operations with 3D-MLEIMs.

- **Projection**

Let us have 3D-MLEIM $A = [(K, P), (L, Q), (H, R), \{a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}}\}]$. In [8, 21] are extended the definitions of the operation "Projection" over matrix A as follows:

$$\begin{aligned} pr_{(K_i^{(P)}, p\text{-layer}), (L_j^{(Q)}, q\text{-layer}), (H_g^{(R)}, r\text{-layer})} A \\ = [(K_i^{(P)}, p\text{-layer}), (L_j^{(Q)}, q\text{-layer}), (H_g^{(R)}, r\text{-layer}), \{b_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}}\}], \end{aligned}$$

where for each $K_{i,d}^{(p)} \in \{K_i^{(P)}, p\text{-layer}\}$, $L_{j,b}^{(q)} \in \{L_j^{(Q)}, q\text{-layer}\}$ and $H_{g,c}^{(r)} \in \{H_g^{(R)}, r\text{-layer}\}$

$$b_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}} = a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}},$$

where $K_i^{(P)} \subset K$, $1 \leq p \leq P$, $L_j^{(Q)} \subset L$, $1 \leq q \leq Q$ and $H_g^{(R)} \subset H$, $1 \leq r \leq R$.

Let there be given index sets, whose members are also index sets:

$$K_* \subseteq K \text{ and } K_* = \{K_{v_1}^{(P)}, \dots, K_{v_x}^{(P)}, \dots, K_{v_t}^{(P)}\},$$

$$P_* = \{p_1, \dots, p_x, \dots, p_t\}, \text{ where } 1 \leq p_x \leq P \text{ for } 1 \leq x \leq t,$$

$$L_* \subseteq L \text{ and } L_* = \{L_{u_1}^{(Q)}, \dots, L_{u_y}^{(Q)}, \dots, L_{u_s}^{(Q)}\},$$

$$Q_* = \{q_1, \dots, q_y, \dots, q_s\}, \text{ where } 1 \leq q_y \leq Q \text{ for } 1 \leq y \leq s,$$

$$H_* \subseteq H \text{ and } H_* = \{H_{w_1}^{(R)}, \dots, H_{w_z}^{(R)}, \dots, H_{w_e}^{(R)}\},$$

$$R_* = \{r_1, \dots, r_z, \dots, r_e\}, \text{ where } 1 \leq r_z \leq R \text{ for } 1 \leq z \leq e.$$

We denote the dimension of some index set G by $\dim(G) = u$. Let

$$\dim(K_*) = \dim(P_*) = t, \dim(L_*) = \dim(Q_*) = s, \dim(H_*) = \dim(R_*) = e.$$

Then

$$pr_{(K_*, P_*), (L_*, Q_*), (H_*, R_*)} A = [(K_*, P_*), (L_*, Q_*), (H_*, R_*), \{b_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}}\}],$$

where for each $K_{i,d}^{(p)} \in \{K_{v_x}^{(P)}, p_x - \text{layer}\}$ for $1 \leq x \leq t$, $L_{j,b}^{(q)} \in \{L_{u_y}^{(Q)}, q_y - \text{layer}\}$ for $1 \leq y \leq s$ and $H_{g,c}^{(r)} \in \{H_{w_z}^{(R)}, r_z - \text{layer}\}$ for $1 \leq z \leq e$,

$$b_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}} = a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}}.$$

• Composition

Let index set \mathcal{I}^* and the set \mathcal{X} be fixed and let 3D-MLEIMs A_1, A_2, \dots, A_n over both sets be given. Let for s ($1 \leq s \leq n$):

$$\left\{ \begin{array}{c|cccc} H_{s,g}^{(R_s)} \in H_s & L_{s,1}^{(Q_s)} & \dots & L_{s,j}^{(Q_s)} & \dots & L_{s,n}^{(Q_s)} \\ \hline K_{s,1}^{(P_s)} & a_{s,K_{s,1}^{(P_s)}, L_{s,1}^{(Q_s)}, H_{s,g}^{(R_s)}} & \dots & a_{s,K_{s,1}^{(P_s)}, L_{s,j}^{(Q_s)}, H_{s,g}^{(R_s)}} & \dots & a_{s,K_{s,1}^{(P_s)}, L_{s,n}^{(Q_s)}, H_{s,g}^{(R_s)}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{s,i}^{(P_s)} & a_{s,K_{s,i}^{(P_s)}, L_{s,1}^{(Q_s)}, H_{s,g}^{(R_s)}} & \dots & a_{s,K_{s,i}^{(P_s)}, L_{s,j}^{(Q_s)}, H_{s,g}^{(R_s)}} & \dots & a_{s,K_{s,i}^{(P_s)}, L_{s,n}^{(Q_s)}, H_{s,g}^{(R_s)}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{s,m}^{(P_s)} & a_{s,K_{s,m}^{(P_s)}, L_{s,1}^{(Q_s)}, H_{s,g}^{(R_s)}} & \dots & a_{s,K_{s,m}^{(P_s)}, L_{s,j}^{(Q_s)}, H_{s,g}^{(R_s)}} & \dots & a_{s,K_{s,m}^{(P_s)}, L_{s,n}^{(Q_s)}, H_{s,g}^{(R_s)}} \end{array} \right\},$$

where

$$\begin{aligned} K_s &= \{K_{s,1}^{(P_s)}, K_{s,2}^{(P_s)}, \dots, K_{s,i}^{(P_s)}, \dots, K_{s,m}^{(P_s)}\}, \\ K_{s,i}^{(P_s)} &= \{K_{s,i,1}^{(P_s-1)}, K_{s,i,2}^{(P_s-1)}, \dots, K_{s,i,x}^{(P_s-1)}, \dots, K_{s,i,I}^{(P_s-1)}\} \text{ for } 1 \leq i \leq m \\ &\dots \\ K_{s,u}^{(1)} &= \{K_{s,u,1}^{(0)}, K_{s,u,2}^{(0)}, \dots, K_{s,u,U}^{(0)}\} \end{aligned}$$

i.e. p_s -th layer of dimension K_s of the multilayer matrix, where ($1 \leq p_s \leq P_s$), is represented by

$$\begin{aligned} K_{s,u_*}^{(p_s)} &= \{K_{s,u_*,1}^{(p_s-1)}, K_{s,u_*,2}^{(p_s-1)}, \dots, K_{s,u_*,U_*}^{(p_s-1)}\} \text{ for } 1 \leq p_s \leq P_s \\ L_s &= \{L_{s,1}^{(Q_s)}, L_{s,2}^{(Q_s)}, \dots, L_{s,j}^{(Q_s)}, \dots, L_{s,n}^{(Q_s)}\}, \\ L_{s,j}^{(Q_s)} &= \{L_{s,j,1}^{(Q_s-1)}, L_{s,j,2}^{(Q_s-1)}, \dots, L_{s,j,y}^{(Q_s-1)}, \dots, L_{s,j,J}^{(Q_s-1)}\} \text{ for } 1 \leq j \leq n \\ &\dots \\ L_{s,v}^{(1)} &= \{L_{s,v,1}^{(0)}, L_{s,v,2}^{(0)}, \dots, l_{s,v,V}^{(0)}\} \end{aligned}$$

i.e. q_s -th layer of dimension Q_s of the multilayer matrix is represented by

$$\begin{aligned} L_{s,v_*}^{(q_s)} &= \{L_{s,v_*,1}^{(q_s-1)}, L_{s,v_*,2}^{(q_s-1)}, \dots, L_{s,v_*,V_*}^{(q_s-1)}\} \text{ for } 1 \leq q_s \leq Q_s \\ H_s &= \{H_{s,1}^{(R_s)}, H_{s,2}^{(R_s)}, \dots, H_{s,g}^{(R_s)}, \dots, H_{s,f}^{(R_s)}\}, \\ H_{s,g}^{(R_s)} &= \{H_{s,g,1}^{(R_s-1)}, H_{s,g,2}^{(R_s-1)}, \dots, H_{s,g,z}^{(R_s-1)}, \dots, H_{s,g,G}^{(R_s-1)}\} \text{ for } 1 \leq g \leq f \\ &\dots \\ H_{s,w}^{(1)} &= \{H_{s,w,1}^{(0)}, H_{s,w,2}^{(0)}, \dots, H_{s,w,W}^{(0)}\} \end{aligned}$$

i.e. r_s -th layer of dimension H of the multilayer matrix is represented by

$$H_{s,w_*}^{(r_s)} = \{H_{s,w_*,1}^{(r_s-1)}, H_{s,w_*,2}^{(r_s-1)}, \dots, H_{s,w_*,W_*}^{(r_s-1)}\} \text{ for } 1 \leq r_s \leq R_s$$

and $(K_s, L_s, H_s \subset \mathcal{I}^*)$, and for $1 \leq i \leq I$, $1 \leq j \leq J$, $1 \leq g \leq G$, $1 \leq p_s \leq P_s$, $1 \leq q_s \leq Q_s$, $1 \leq r_s \leq R_s$, $1 \leq s \leq n$, $1 \leq d \leq I$, $1 \leq b \leq J$, $1 \leq c \leq G$:
 $a_{s, K_{s,i,d}^{(p_s)}, L_{s,j,b}^{(q_s)}, H_{s,g,c}^{(r_s)}} \in \mathcal{X}$,

Let us extend the definition of the operation “composition” as follows:

$$\wp\{A_s | 1 \leq s \leq n\} = [(\bigcup_{s=1}^n K_s, \bigcup_{s=1}^n P_s), (\bigcup_{s=1}^n L_s, \bigcup_{s=1}^n Q_s), (\bigcup_{s=1}^n H_s, \bigcup_{s=1}^n R_s),$$

$$\{\langle \phi_{1, K_{1,i*,d*}^{(p*1)}, L_{1,j*,b*}^{(q*1)}, H_{1,g*,c*}^{(r*1)}}, \phi_{2, K_{2,i*,d*}^{(p*2)}, L_{2,j*,b*}^{(q*2)}, H_{2,g*,c*}^{(r*2)}}, \dots, \phi_{n, K_{n,i*,d*}^{(p*n)}, L_{n,j*,b*}^{(q*n)}, H_{n,g*,c*}^{(r*n)}} \rangle\}],$$

where for s ($1 \leq s \leq n$):

$$\phi_{s, K_{s,i*,d*}^{(p*s)}, L_{s,j*,b*}^{(q*s)}, L_{s,j*,b*}^{(q*s)}} = \begin{cases} a_{s, K_{s,i,d}^{(p_s)}, L_{s,j,b}^{(q_s)}, H_{s,g,c}^{(r_s)}}, & \text{if } K_{s,i*,d*}^{(p_s)} = K_{s,i,d}^{(p_s)} \in K_s^{p_s}, \\ & L_{s,j*,b*}^{(q_s)} = L_{s,j,b}^{(q_s)} \in L_s^{q_s}, \\ & \text{and } H_{s,g*,c*}^{(r_s)} = H_{s,g,c}^{(r_s)} \in H_s^{r_s}. \\ \perp, & \text{otherwise} \end{cases}.$$

3. AN IMPLEMENTATION OF THE OLAP OPERATION “DRILL ACROSS” BY INDEX MATRICES

In the current section an implementation of the OLAP operation “Drill across” is presented by IMs and some examples of its practical application are given.

For this purposes we use the OLAP cube “Bookshops”, constructed in [18]. It contains information for the book sales in different bookshops (managed by different regional managers) in different locations. The structure of the cube “Bookshops” is visualized on Fig. 1. The fact table “Sales” and the dimensional tables Books{Id, Title, Publisher, Genre}, Bookshops{Id, Bookshop Name, Regional Manager, Owner} and Location{Id, Town, Country} are constructed. The measures are “NumberSales” and “Sales Count”. The hierarchical structures of the dimensions are presented in [18].

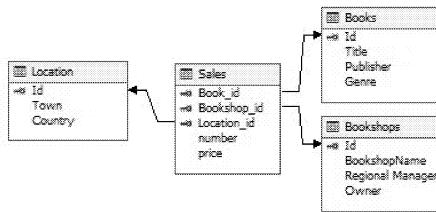


FIG. 1. Star schema “Bookshops”

In terms of the IMs the above example of OLAP-cube becomes the following: a matrix $A = [(K, P), (L, Q), (H, R), \{a_{K_{i,d}^{(p)}, L_{j,b}^{(q)}, H_{g,c}^{(r)}}\}]$ (3D-MLEIM with P -levels (layers) of use of a dimension K , Q -levels (layers) of use of a dimension L and R -levels (layers) of use of a dimension H with structure defined in [18]).

3.1. Definition.

The “Drill across” combines the measures of two union comparable OLAP cubes with the same dimensions and levels, i.e. cubes with equal number of dimensions of the same domain and different measures. The operation allows performing a comparison between the measures. “Drill across” between two cubes where they have the same or equal dimension attributes is natural join between corresponding dimension attributes [14]. If the dimension attributes are conformed but they have intersecting values, then the data may be lost during the join.

3.2. Presentation of the operation “Drill across” by IMs.

In the case of 3D-EIMs:

Let us be given two 3D-IMs

$$A_s = [K^s, L^s, H^s, \{a_{s,k_{s,i},l_{s,j},h_{s,g}}\}]$$

$$\begin{array}{c|ccccc} h_{s,g} \in H^s & l_{s,1} & \dots & l_{s,j} & \dots & l_{s,n_s} \\ \hline k_{s,1} & a_{s,k_{s,1},l_{s,1},h_{s,g}} & \dots & a_{s,k_{s,1},l_{s,j},h_{s,g}} & \dots & a_{s,k_{s,1},l_{s,n_s},h_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{s,i} & a_{s,k_{s,i},l_{s,1},h_{s,g}} & \dots & a_{s,k_{s,i},l_{s,j},h_{s,g}} & \dots & a_{s,k_{s,i},l_{s,n_s},h_{s,g}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{s,m} & a_{s,k_{s,m},l_{s,1},h_{s,g}} & \dots & a_{s,k_{s,m},l_{s,j},h_{s,g}} & \dots & a_{s,k_{s,m},l_{s,n_s},h_{s,g}} \end{array}$$

for $s = 1, 2$.

Let $K_{s,*}, L_{s,*}, H_{s,*}$ be such that:

$$K_{s,*} \subseteq K^s \text{ and } K_{s,*} = \{K_{s,v_1}, \dots, K_{s,v_x}, \dots, K_{s,v_t}\},$$

$$L_{s,*} \subseteq L^s \text{ and } L_{s,*} = \{L_{s,u_1}, \dots, L_{s,u_y}, \dots, L_{s,u_e}\},$$

$$H_{s,*} \subseteq H^s \text{ and } H_{s,*} = \{H_{s,w_1}, \dots, H_{s,w_z}, \dots, H_{s,w_d}\} \text{ for } s = 1, 2.$$

The operation “Drill across” can be represented in terms of the IMs, using the operation “Composition”, as follows:

$$\triangleright \{pr_{(K_{1,*}, L^1, H^1)} A_1, pr_{(K_{2,*}, L^2, H^2)} A_2\} \oplus_{\vee, \wedge} \triangleright \{pr_{(K^1, L_{1,*}, H^1)} A_1, pr_{(K^2, L_{2,*}, H^2)} A_2\} \oplus_{\vee, \wedge}$$

$$\triangleright \{pr_{(K^1, L^1, H_{1,*})} A_1, pr_{(K^2, L^2, H_{2,*})} A_2\} \oplus_{\vee, \wedge} \triangleright \{pr_{(K_{1,*}, L_{1,*}, H_{1,*})} A_1, pr_{(K_{2,*}, L_{2,*}, H_{2,*})} A_2\}$$

In the case of 3D-MLEIMs:

Let us be given two 3D-MLEIMs A_1, A_2 . Let for s ($1 \leq s \leq 2$):

$$A_s = [(K_s, P_s), (L_s, Q_s), (H_s, R_s), \{a_{s,K_{s,i,d}^{(ps)},L_{s,j,b}^{(qs)},H_{s,g,c}^{(rs)}}\}]$$

Let there be given index sets, whose members are also index sets:

$$K_{s,*} \subseteq K_s \text{ and } K_{s,*} = \{K_{s,v_1}^{(P_s)}, \dots, K_{s,v_x}^{(P_s)}, \dots, K_{s,v_t}^{(P_s)}\},$$

$$P_{s,*} = \{p_{s,1}, \dots, p_{s,x}, \dots, p_{s,t}\}, \text{ where } 1 \leq p_{s,x} \leq P_s \text{ for } 1 \leq x \leq t,$$

$$L_{s,*} \subseteq L_s \text{ and } L_{s,*} = \{L_{s,u_1}^{(Q_s)}, \dots, L_{s,u_y}^{(Q_s)}, \dots, L_{s,u_\beta}^{(Q_s)}\},$$

$$Q_{s,*} = \{q_{s,1}, \dots, q_{s,y}, \dots, q_\beta\}, \text{ where } 1 \leq q_{s,y} \leq Q_s \text{ for } 1 \leq y \leq \beta,$$

$$H_{s,*} \subseteq H_s \text{ and } H_{s,*} = \{H_{s,w_1}^{(R_s)}, \dots, H_{s,w_z}^{(R_s)}, \dots, H_{s,w_e}^{(R_s)}\},$$

$$R_{s,*} = \{r_{s,1}, \dots, r_{s,z}, \dots, r_{s,e}\}, \text{ where } 1 \leq r_{s,z} \leq R_s \text{ for } 1 \leq z \leq e.$$

Then the operation “Drill across” can be represented in terms of the IMs, using the operation “Composition”, as follows:

$$\begin{aligned}
 & \flat\{pr_{((K_1,*,P_1,*),(L_1,Q_1),(H_1,R_1))}A_1, pr_{((K_2,*,P_2,*),(L_2,Q_2),(H_2,R_2))}A_2\} \\
 & \oplus_{\vee,\wedge}\flat\{pr_{((K_1,P_1),(L_1,*,Q_1,*),(H_1,R_1))}A_1, pr_{((K_2,P_2),(L_2,*,Q_2,*),(H_2,R_2))}A_2\} \\
 & \oplus_{\vee,\wedge}\flat\{pr_{((K_1,P_1),(L_1,Q_1),(H_1,*,R_{1,*}))}A_1, pr_{((K_2,P_2),(L_2,Q_2),(H_2,*,R_{2,*}))}A_2\} \\
 & \oplus_{\vee,\wedge}\flat\{pr_{((K_1,*,P_{1,*}),(L_{1,*},Q_{1,*}),(H_{1,*},R_{1,*}))}A_1, pr_{((K_2,*,P_{2,*}),(L_{2,*},Q_{2,*}),(H_{2,*},R_{2,*}))}A_2\}.
 \end{aligned}$$

The operation can be applied on the corresponding sublayers of the two MLEIMs A_1 and A_2 in the following form:

$$\begin{aligned}
 & \flat\{pr_{((K_1,*,p_{1,x}),(L_1,Q_1),(H_1,R_1))}A_1, pr_{((K_2,*,p_{2,x}),(L_2,Q_2),(H_2,R_2))}A_2\} \\
 & \oplus_{\vee,\wedge}\flat\{pr_{((K_1,P_1),(L_1,*,q_{1,y}),(H_1,R_1))}A_1, pr_{((K_2,P_2),(L_2,*,q_{2,y}),(H_2,R_2))}A_2\} \\
 & \oplus_{\vee,\wedge}\flat\{pr_{((K_1,P_1),(L_1,Q_1),(H_1,*,r_{1,z}))}A_1, pr_{((K_2,P_2),(L_2,Q_2),(H_2,*,r_{2,z}))}A_2\} \\
 & \oplus_{\vee,\wedge}\flat\{pr_{((K_1,*,p_{1,x}),(L_{1,*},q_{1,y}),(H_{1,*},r_{1,z}))}A_1, pr_{((K_2,*,p_{2,x}),(L_{2,*},q_{2,y}),(H_{2,*},r_{2,z}))}A_2\}.
 \end{aligned}$$

It is possible after this operation to obtain many empty matrix elements, in which case it is necessary to apply the operation “Reduction” [3, 20, 21].

3.3. Examples for operation “Drill across”.

Operation “Drill across” combines two cubes with the same dimensions and different measures. Originally the “Drill across” and the other set operations are not directly included in MDX, since the “From” clause supports one cube. The only way of adding a measure is at the design time. We can drill across if the fact tables are modeled in the same cube. In the examples we present the “Drill across” operation using the free “Power BI Excel add-in” called “Power Query” (or “Power Pivot” in Microsoft Office Excel 2007, OLAP add-in). It simulates the “Drill across” operation from one cube and construct two new cubes using the same dimensions but with different measures in each of them. The connection between Microsoft Analyses Services and Microsoft Excel is made. OLAP cube Bookshops is selected. The working process is presented on Fig. 2

The screenshot shows the Microsoft Excel interface with the Power Pivot add-in open. On the left, the 'PivotTable Field List' pane displays various data sources and their hierarchies. In the center, a PivotTable is visible with data for Sales, Books, Bookshops, Location, and more. On the right, the 'OLAP PivotTable Extensions' dialog box is open, showing the MDX query generated for the current report:

```

SELECT NON EMPTY
CrossJoin(Hierarchize(DrilldownMember({{{DrilldownLevel({[Bookshops].[Hierarchy].[All]})}}}, {[Bookshops].[Hierarchy].[Regional Manager].&[Richard Gray]})), {[Measures].[Number],[Measures].[Sales Count]}) DIMENSION PROPERTIES PARENT_UNIQUE_NAME,[Bookshops].[Hierarchy].[Regional Manager].[Owner],[Bookshops].[Hierarchy].[Bookshop Name].[Regional Manager] ON COLUMNS , NON EMPTY CrossJoin(Hierarchize(DrilldownMember({{{DrilldownLevel({[Books].[Hierarchy].[All]})}}}), {[Books].[Hierarchy].[Publisher].&[HarperCollins],[Books].[Hierarchy].[Publisher].&[Orion]}), Hierarchize({DrilldownLevel({[Location].[Hierarchy].[All]})}) ) DIMENSION PROPERTIES PARENT_UNIQUE_NAME,[Books].[Hierarchy].[Publisher].[Genre],[Books].[Hierarchy].[Title].[Publisher] ON ROWS FROM [Bookshops] CELL PROPERTIES VALUE, FORMAT_STRING, BACK_COLOR, FORE_COLOR, FONT_FLAGS

```

FIG. 2. The working process in Power Pivot add-in for Excel

The parameters for the query are set using the PowerTable Filed List. In the OLAP PivotTable Extensions property for the generated report the user can see the automatically written MDX query.

- *MDX query 1:* The following query presents the number of the books in stock and the number of the sold books by owner of bookshop, country and genre of the book (Fig. 2). More precisely the operation “Drill across” in OLAP PivotTable extension for MS Excel include the dimensions “Location” → level “Country”, dimension Books → level “Genre”, dimension Bookshops → level “Owner” and Measures “Number” (Books In Stock) and “Sales count”.

Code:

```

SELECT NON EMPTY CrossJoin(Hierarchize(DrilldownMember
({{{DrilldownLevel({[Bookshops].[Hierarchy].[All]})}}}, {[Bookshops].[Hierarchy].[Regional Manager].&[Richard Gray]})), {[Measures].[Number],[Measures].[Sales Count]}) DIMENSION PROPERTIES PARENT_UNIQUE_NAME,[Bookshops].[Hierarchy].[Regional Manager].[Owner],[Bookshops].[Hierarchy].[Bookshop Name].[Regional Manager] ON COLUMNS , NON EMPTY CrossJoin(Hierarchize(DrilldownMember
({{{DrilldownLevel({[Books].[Hierarchy].[All]})}}}}), {[Books].[Hierarchy].[Publisher].&[HarperCollins],[Books].[Hierarchy].[Publisher].&[Orion]}), Hierarchize({DrilldownLevel({[Location].[Hierarchy].[All]})}) ) DIMENSION PROPERTIES PARENT_UNIQUE_NAME,[Books].[Hierarchy].[Publisher].[Genre],[Books].[Hierarchy].[Title].[Publisher] ON ROWS FROM [Bookshops] CELL PROPERTIES VALUE, FORMAT_STRING, BACK_COLOR, FORE_COLOR, FONT_FLAGS

```

```
[Books].[Hierarchy].[Publisher].[Genre],
[Books].[Hierarchy].[Title].[Publisher] ON ROWS
FROM [Bookshops2]
CELL PROPERTIES VALUE, FORMAT_STRING,
LANGUAGE, BACK_COLOR, FORE_COLOR, FONT_FLAGS
```

Result: The results of MDX query are presented on the Fig. 3 and 4.

Row Labels	Column Labels			Stamen Dimitrov			Valeri Rodev			Total Number	Total Sales Count
	Number	Olivia Gomez	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count		
Children Books	6	4		30	13	6	4	42	21		
Bulgaria				30	13			30	13		
England	6	4						6	4		
Turkey						6	4	6	4		
Computer Books	37	15		125	42	25	14	187	71		
Bulgaria				125	42			125	42		
England	37	15						37	15		
Turkey						25	14	25	14		
Cooking Books	3	2		15	6	7	2	25	10		
Bulgaria				15	6			15	6		
England	3	2						3	2		
Turkey						7	2	7	2		
Grand Total	46	21		170	61	38	20	254	102		

FIG. 3. Operation “Drill across” in the case of roll-up (higher layer of granularity)

Row Labels	Column Labels			Total
	Olivia Gomez	Stamen Dimitrov	Valeri Rodev	
Number,Sales Count				
Children Books	6,4	30,13	6,4	42,21
Bulgaria	,	30,13	,	30,13
England	6,4	,	,	6,4
Turkey	,	,	6,4	6,4
Computer Books	37,15	125,42	25,14	187,71
Bulgaria	,	125,42	,	125,42
England	37,15	,	,	37,15
Turkey	,	,	25,14	25,14
Cooking Books	3,2	15,6	7,2	25,10
Bulgaria	,	15,6	,	15,6
England	3,2	,	,	3,2
Turkey	,	,	7,2	7,2
Grand Total	46,21	170,61	38,20	254,102

FIG. 4. Operation “Drill across” in the case of roll-up

- *MDX query 2:* The next MDX-query present the same query but the dimension are partially drilled-down. The operation “Drill across” in OLAP PivotTable extension for MS Excel selects dimensions “Location” → level “Town”, dimension Books → level “Genre”, dimension Bookshops → level “Owner” and Measures “Number” (Books In Stock) and “Sales Count”. Obviously the data for the countries are presented by towns also (Fig. 5).

Code:

```
SELECT NON EMPTY CrossJoin(Hierarchize
(DrilldownMember({{{ DrilldownLevel({[Bookshops].[Hierarchy].[All]})}}}),
{[Bookshops].[Hierarchy].[Regional Manager].&[Richard Gray]}),
{[Measures].[Number],[Measures].[Sales Count]})  

DIMENSION PROPERTIES  

PARENT_UNIQUE_NAME,  

[Bookshops].[Hierarchy].[Regional Manager].[Owner],
```

*[Bookshops].[Hierarchy].[Bookshop Name].[Regional Manager]
 ON COLUMNS,
 NON EMPTY CrossJoin(Hierarchize(DrilldownMember
 ({DrilldownLevel({[Books].[Hierarchy].[All]})})),
 {[Books].[Hierarchy].[Publisher].&[HarperCollins],
 [Books].[Hierarchy].[Publisher].&[Orion]}),
 Hierarchize(DrilldownMember
 ({DrilldownLevel({[Location].[Hierarchy].[All]})})),
 {[Location].[Hierarchy].[Country].&[Bulgaria],
 [Location].[Hierarchy].[Country].&[England], [Location].[Hierarchy].[Country].&[Turkey]}))
 DIMENSION PROPERTIES PARENT_UNIQUE_NAME,
 [Location].[Hierarchy].[Town].[Country],
 [Books].[Hierarchy].[Publisher].[Genre],
 [Books].[Hierarchy].[Title].[Publisher] ON ROWS
 FROM [Bookshops2]
 CELL PROPERTIES VALUE, FORMAT_STRING,
 LANGUAGE, BACK_COLOR, FORE_COLOR, FONT_FLAGS*

Result: The results of MDX query are presented on the Fig. 5.

Column Labels		Richard Gray Number		Richard Gray Sales Count		Olivia Gomez Number		Olivia Gomez Sales Count		Stamen Dimitrov		Valeri Rodev		Total Number		Total Sales Count		
Row Labels	Number	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count	Number	Sales Count
Children Books	6	4	6	4	5	4	50	13	6	4	42	21	30	13	50	13	4	4
Bulgaria							30	13					13	4				
Burgas							13	4					9	5				
Plovdiv							9	5					8	4				
Sofia							8	4					6	4				
England	6	4	6	4	6	4	125	42	25	14	187	71	25	14	125	42	14	14
London	6	4	6	4	6	4	125	42					25	14	25	14		
Turkey							25	14					27	14	27	14		
Mersin							75	14					75	14	37	15		
Computer Books	37	15	37	15	37	15	37	15	15	15	15	15	25	14	25	14	37	15
Bulgaria							125	42	25	14	187	71	25	14	125	42	14	14
Burgas							125	42					25	14	25	14		
Plovdiv							25	14					27	14	27	14		
Sofia							75	14					75	14	37	15		
England	37	15	37	15	37	15	37	15	15	15	15	15	25	14	25	14	37	15
London	37	15	37	15	37	15	37	15	15	15	15	15	25	14	25	14	37	15
Turkey													25	14	25	14		
Mersin													25	14	25	14		
Cooking Books	3	2	3	2	3	2	15	6	7	2	25	10	15	6	15	6	2	2
Bulgaria							15	6					7	2	7	2		
Burgas							7	2					6	2	6	2		
Plovdiv							6	2					2	2	2	2		
Sofia							2	2					2	2	2	2		
England	3	2	3	2	3	2	3	2	2	2	2	2	7	2	7	2	2	2
London	3	2	3	2	3	2	3	2	2	2	2	2	7	2	7	2	2	2
Turkey													7	2	7	2		
Mersin													7	2	7	2		
Grand Total	46	21	46	21	46	21	46	21	170	61	38	20	254	102				

FIG. 5. Operation “Drill across” with drilled-down Location dimension (lower layer of granularity for Location dimension)

On the Fig. 6 and 7 the operation “Drill across” is visualized using PivotTable and PivotChart in OLAP PivotTable extension for MS Excel. The PivotTable contains dimension “Location” → level “Country”, dimension Books → level “Genre”, dimension Bookshops → level “Owner” on the rows and Measures “Number” (Books In Stock) and “Sales Count” on the columns (PivotTable; PivotChart – the opposite case).

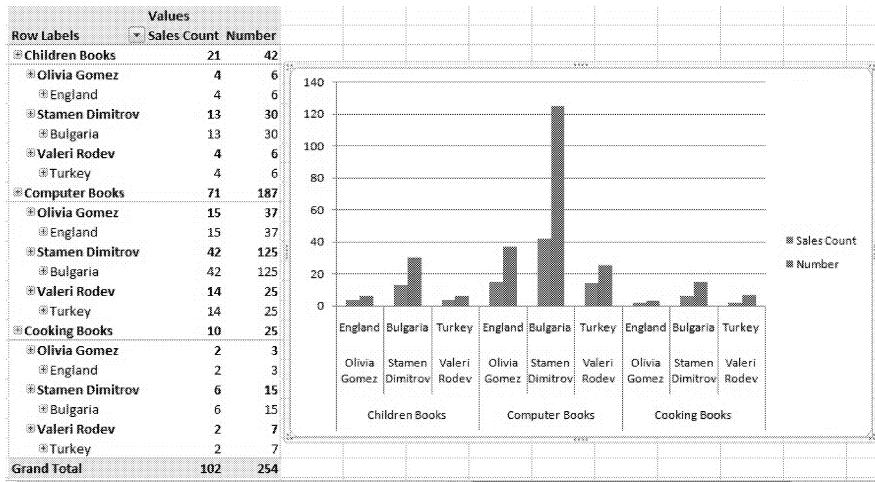


FIG. 6. Operation “Drill across” PivotTable and PivotChart in OLAP PivotTable extension for MS Excel

Row Labels	Sales Count,Number
Children Books	21,42
Olivia Gomez	4,6
England	4,6
Stamen Dimitrov	13,30
Bulgaria	13,30
Valeri Rodev	4,6
Turkey	4,6
Computer Books	71,187
Olivia Gomez	15,37
England	15,37
Stamen Dimitrov	42,125
Bulgaria	42,125
Valeri Rodev	14,25
Turkey	14,25
Cooking Books	10,25
Olivia Gomez	2,3
England	2,3
Stamen Dimitrov	6,15
Bulgaria	6,15
Valeri Rodev	2,7
Turkey	2,7
Grand Total	102,254

FIG. 7. Operation “Drill across” PivotTable and PivotChart in OLAP PivotTable extension for MS Excel

The last operation “Drill across” can be made in the field of Analyses Services Multidimensional and Data Mining Project (Multidimensional Project) in Microsoft Visual Studio. The dimensions and the measures are presented on the column and the values are placed on the rows (Fig. 8).

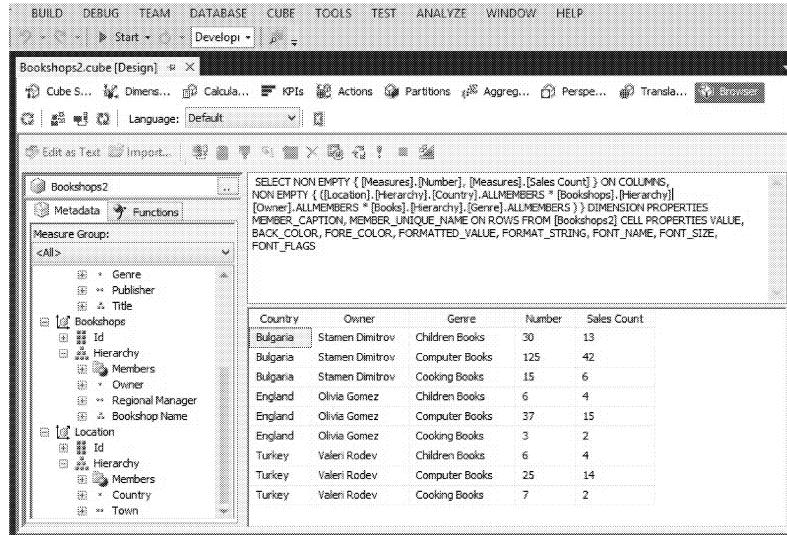


FIG. 8. Operation “Drill across” in the field of Analyses Services Multidimensional and Data Mining Project (Multidimensional Project), Microsoft Visual Studio

4. CONCLUSION

In the presented paper we used the index matrices tool to represent an OLAP operation “Drill across”. The tools of Analysis Services and Power Pivot add-in for MS Excel are used to present the examples in the paper.

The outlined approach for extracting knowledge from the information stored in OLAP-cubes has the following advantages:

- The defined operations can be applied to data with explicit parameters, as well as to fuzzy or intuitionistic fuzzy parameters;
- The defined operations can be expanded to retrieve information to other types of two-dimensional or multi-dimensional data cubes [5].

Nowadays the attention is focused over the modeling the OLAP concept using 3-Dimensional and n-Dimensional IMs [5, 16], and the use of these operations in the future development for large data analysis. This paper is the fourth part of series of articles investigated the OLAP operations by index matrices. In the future the authors will finish the studies and some fields of application will be discussed.

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