Proceedings of the Jangjeon Mathematical Society
21 (2018), No. 4. pp. 759 - 763

www.jangjeon.or.kr
http://dx.doi.org/10.17777/pjms2018.21.4.759

$(1,3)$ SOLID BURST-CORRECTING OPTIMAL LINEAR CODES OVER $GF(5)$

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ABSTRACT. In 2012, Das [5] had obtained a lower bound on the necessary number of parity-check digits in an $(n = n_1 + n_2, k)$ linear codes over $GF(2)$ which are optimal in a specific sense viz. which are capable to correct single errors in the first sub-block of length n_1 and solid bursts of length 3 or less in the second sub-block of length n_2 as well as gave sufficient condition for the existence of such codes. Later, Lata and Tyagi [9] had studied these codes over $GF(3)$.
In this paper, the author examines the possibility of the existence of such codes over $GF(5)$.

2010 MATHEMATICS SUBJECT CLASSIFICATION. 94B25, 94B60.

KEYWORDS AND PHRASES. Parity-check matrix, Burst error, Syndromes, Solid burst error, Optimal codes.

1. INTRODUCTION

We know that burst error is well known error type in the literature. In many communication channels occurrence of burst errors is more than occurrence of random errors. So, from applications point of view, burst error correcting codes are more economical to use than random error correcting codes. Das [5] studied binary codes of length n which are sub divided into two sub-blocks of length n_1 and n_2, $n_1+n_2 = n$ which are capable to correct bursts of length 1 in the first sub-block and solid bursts of length 3 or less in the second sub-block. Such codes were termed as $(1, 3)$ optimal linear codes on solid bursts.

In this paper, we explore the possibility of existence of the codes of length n which are sub divided into two sub-blocks of length n_1 and n_2 where $n = n_1 + n_2$. These codes can correct bursts of length 1 in the first sub-block of length n_1 and solid bursts of length 3 or less in the second sub-block of length n_2 over $GF(5)$.

The distance between codewords as well as the weight of the codeword shall be considered in the Hamming sense. In this paper, we consider the definition of the burst which is given by Fire [7] and definition of solid burst which is the modification of the definition of burst.

Definition 1 : A burst of length b or less has been considered as an n-tuple whose only non-zero components are confined to b consecutive positions, the first and the last of which is non-zero.

Definition 2 : A solid burst of length b or less has been considered as a vector with non-zero components in some b consecutive positions and zero elsewhere.

For an (n, k) linear code the number of its starting positions is $(n-b+1)$.

The paper is organized into four sections along with introduction. Section 2 presents possibilities of occurrence of $(1,3)$ solid burst-correcting optimal linear codes over $GF(5)$. In Section 3 we discuss these codes with the help of an example. Finally we give conclusion of the paper and open problem in Section 4.

2. (1, 3) Solid Burst-Correcting Optimal Linear Codes

Das [5] gave a lower bound on the necessary number of parity check digits required for an $(n = n_1 + n_2, k)$ linear code over $GF(q)$ that corrects solid bursts of length b_1 or less in the first sub-block of length n_1 and solid bursts of length b_2 or less in the second sub-block of length n_2.

The bound proved by Das [5] is as follows:

Theorem 2.1. *The number of parity check digits for an $(n = n_1 + n_2, k)$ linear code over $GF(q)$ that corrects all solid burst errors of length b_1 or less in the first block of length n_1 and all solid burst errors of length b_2 or less in the second block of length n_2 is at least*

$$\log_q \left\{ 1 + \sum_{i=1}^{b_1} (q-1)^i (n_1 - i + 1) + \sum_{j=1}^{b_2} (q-1)^j (n_2 - j + 1) \right\}. \tag{1}$$

Equivalently, the bound in (1) *may be put as follows:*

$$q^{n-k} \geq 1 + \sum_{i=1}^{b_1} (q-1)^i (n_1 - i + 1) + \sum_{j=1}^{b_2} (q-1)^j (n_2 - j + 1). \tag{2}$$

To find the values of various parameters for which the bound (2) is tight and gives (b_1, b_2) solid burst correcting optimal codes, we must consider inequality in (2) with equality viz.

$$q^{n-k} = 1 + \sum_{i=1}^{b_1} (q-1)^i (n_1 - i + 1) + \sum_{j=1}^{b_2} (q-1)^j (n_2 - j + 1). \tag{3}$$

The equality (3) of the inequality (2) gives us the optimal case. These codes are optimal in the sense that the number of burst errors to be corrected length 1 in the first block of length n_1 and all solid burst errors of length 3 or less in the second block of length n_2 in such codes equals the total number of cosets viz. 5^{n-k}. Such codes are termed as $(1,3)$ solid burst-correcting optimal linear codes over $GF(5)$.

Let us take $q = 5$ and $b_1 = 1$, $b_2 = 3$ in (3). We get

$$5^{n-k} = 4n_1 + 84n_2 - 143. \tag{4}$$

Now, we examine the values of n_1, n_2 and k satisfying (4) for $3 \leq n-k \leq 5$. For this, we shall assign values to n_1 and find out the corresponding values of n_2 and k.

The following table (Table 1) gives the suitable values of various parameters n_1, n_2 and k for $3 \leq n-k \leq 5$ and possibilities of 5-ary $(1,3)$ solid bursts correcting optimal codes.

Table 1.

$n-k$	n_1	n_2	k	Possible Codes
3	4	3	4	(4+3,4)
4	3	9	8	(3+9,8)
	24	8	28	(24+8,28)
	45	7	48	(45+7,48)
	66	6	68	(66+6,68)
	87	5	88	(87+5,88)
	108	4	108	(108+4,108)
	129	3	128	(129+3,128)
5	19	38	52	(19+38,52)
	40	37	72	(40+37,72)
	61	36	92	(61+36,92)
	82	35	112	(82+35,112)
	103	34	132	(103+34,132)
	124	33	152	(124+33,152)
	:	:	:	:
	:	:	:	:
	:	:	:	:
	:	:	:	:
	:	:	:	:
	:	:	:	:
	649	8	652	(649+8,652)
	670	7	672	(670+7,672)
	691	6	692	(691+6,692)
	712	5	712	(712+5,712)
	733	4	732	(733+4,732)
	754	3	752	(754+3,752)

3. Discussion

Example : For various values of the parameters $n_1 = 4$, $n_2 = 3$ and $k = 4$, the matrix (5) may be considered as the parity check matrix for an $(4+3, 4)$ code for $q = 5$ where first sub-block n_1 of length 4 corrects all bursts of length 1 and the second sub-block n_2 of length 3 corrects all solid bursts of length 3 or less.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 4 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \end{bmatrix}. \tag{5}$$

It can be easily verified by error pattern-syndrome table (Table 2) that the code so obtained corrects all single solid bursts in the first sub-block n_1 of length 4 and also corrects all solid bursts of length 3 or less in the second sub-block n_2 of length 3.

Table 2. Error-Pattern and Syndrome Table

Error-Pattern	Syndrome	Error-Pattern	Syndrome	Error-Pattern	Syndrome
1000 000	110	0000 233	101	0000 210	422
0100 000	120	0000 234	111	0000 220	024
0010 000	130	0000 241	233	0000 230	121
0001 000	100	0000 242	243	0000 240	223
2000 000	220	0000 243	203	0000 310	332
0200 000	240	0000 244	213	0000 320	434
0020 000	210	0000 311	342	0000 330	031
0002 000	200	0000 312	302	0000 340	133
3000 000	330	0000 313	312	0000 410	242
0300 000	310	0000 314	322	0000 420	344
0030 000	340	0000 321	444	0000 430	441
0003 000	300	0000 322	404	0000 440	043
4000 000	440	0000 323	414	0000 011	112
0400 000	430	0000 324	424	0000 012	122
0040 000	420	0000 331	041	0000 013	132
0004 000	400	0000 332	001	0000 014	142
0000 111	022	0000 333	011	0000 021	214
0000 112	032	0000 334	021	0000 022	224
0000 113	042	0000 341	143	0000 023	234
0000 114	002	0000 342	103	0000 024	244
0000 121	124	0000 343	113	0000 031	311
0000 122	134	0000 344	123	0000 032	321
0000 123	144	0000 411	202	0000 033	331
0000 124	104	0000 412	212	0000 034	341
0000 131	221	0000 413	222	0000 041	413
0000 132	231	0000 414	232	0000 042	423
0000 133	241	0000 421	304	0000 043	433
0000 134	201	0000 422	314	0000 044	443
0000 141	323	0000 423	324	0000 100	410
0000 142	333	0000 424	334	0000 010	102
0000 143	343	0000 431	401	0000 001	010
0000 144	303	0000 432	411	0000 200	320
0000 211	432	0000 433	421	0000 020	204
0000 212	442	0000 434	431	0000 002	020
0000 213	402	0000 441	003	0000 300	230
0000 214	412	0000 442	013	0000 030	301
0000 221	034	0000 443	023	0000 003	030
0000 222	044	0000 444	033	0000 400	140
0000 223	004	0000 110	012	0000 040	403
0000 224	014	0000 120	114	0000 004	040
0000 231	131	0000 130	211		
0000 232	141	0000 140	313		

4. Conclusion and Open problem

As we know that optimal codes improve the efficiency of the communication channels as well as the rate of transmission. So, these codes are very useful from application point of view.

In this paper, we have investigated the solutions of the equation (4) for $3 \leq n-k \leq 5$. We have been able to obtain an $(4+3,4)$ code corresponding to the solutions. This justifies existence of such $(1,3)$ solid burst-correcting optimal linear codes over $GF(5)$.

However, in view of the existence of other solutions of the equation (4), the existence of corresponding codes is an open problem.

Acknowledgement. The author is very much thankful to Dr. Vinod Tyagi, Department of Mathematics, Shyam Lal College (Evening), University of Delhi for his suggestions and revising the contents of this paper.

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