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FIXED POINT RESULTS FOR CYCLIC $(\alpha \circ \beta)$ - CONTRACTION IN FUZZY METRIC SPACES

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Abstract. Firstly, we give a fruitful idea of cyclic $(\alpha\circ\beta)$-contraction for coupled maps on fuzzy metric space. Afterward, the existence theorems of coupled points of cyclic contraction mappings have been established on fuzzy metric space. Moreover, we utilize the notion of closed subset, complete subspace, weakly commuting mappings and continuous mappings for proving results.

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1. Introduction and preliminaries

Fixed point theory is significant theory in mathematics. There are valuable number of its applications in diverse branches of science and technology. An vital point in the development of the present day idea of instability of uncertainty was the production of a seminal paper by Zadeh [10] in which he coined the idea of a fuzzy set. Later on, Kramosil and Michalek [6] initially presented the idea of of a fuzzy metric space. It serves as a beacon for the construction of this theory in fuzzy metric space. After slightly modification in this concept, the definition of fuzzy metric space is reintroduced by Georage and Veeramani [1]. We are concerned with the definition of fuzzy metric space is given by Georage and Veeramani [1].
One of the most significant result was given by Kirk et al. [26] in which they gave the concept of cyclic representation and cyclic contractions. It should be observed that cyclic contractions need not be continuous, which is a vital advantage of this method. Pacurar and Rus [13] presented the idea of cyclic contraction and demonstrated a result for the cyclic contraction on complete metric space. Some fixed point results involving cyclic weaker contraction were demonstrated by Nashine and Kadelburg [9]. For more details, please see [1] - [27].

Definition 1.1. [25] *A operator $\alpha : [0, 1] \to [0, 1]$ is defined as comparison operator if α is non-decreasing, left continuous and $\alpha(t) > t$ for all $t \in (0, 1)$ and having properties as $\alpha(1) = 1$ and $\lim_{n\to\infty} \alpha^n(t) = 1$.*

Definition 1.2. [13] *Let X and Z be subsets of $Y \neq \phi$ which are closed. A pair of mappings $E, S : \Delta \to \Delta$ where $\Delta = X \cup Z$, is supposed to have cyclic form if $E(X) \subseteq Z, S(Z) \subseteq X$.*

Karapinar et al. [5] defined the following concept of cyclic representation,

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Definition 1.3. [5] *Let (Y, D) be a nonempty metric space. Let $q \in N$, $B_1, B_2, \cdots, B_q$ be closed subsets of $Y \neq \phi$ and $Y = \cup_{i=1}^{q} B_i$. Let $E, H : Y \to Y$ be self mappings. Then cyclic representation such as $Y = \cup_{i=1}^{q} B_i$ with respect to (E, H) if*

$$E(B_1) \subset H(B_2), E(B_2) \subset H(B_3), \cdots, E(B_{q-1}) \subset H(B_q), E(B_q) \subset H(B_1).$$

2. Main Results

Firstly, we will define the idea of cyclic $(\alpha \circ \beta)$-contraction for coupled maps on fuzzy metric space. After that coupled fixed point results based on this contraction on fuzzy metric space will be proved.

Definition 2.1. *Let X and Z be two closed subsets of $Y \neq \phi$ and $(Y, \mathcal{F}, *)$ be a fuzzy metric space, which is complete. Let $E : \Delta \times \Delta \to \Delta$ be a map, where $\Delta = X \cup Z$ which hold following conditions:*

i. a cyclic representation as $\Delta = X \cup Z$ w.r.t E,
ii. there exists an operator β as

$$\beta\left(\mathcal{F}(E(\gamma, \delta), E(\omega, \upsilon), t)\right) \geq \alpha\left(\beta(\mathcal{F}(\gamma, \omega, t))\right),$$
$$\beta\left(\mathcal{F}(E(\delta, \gamma), E(\upsilon, \omega), t)\right) \geq \alpha\left(\beta(\mathcal{F}(\delta, \upsilon, t))\right)$$

for any $\gamma, \omega \in X$, $\delta, \upsilon \in Z$ and α is a comparison operator and operator $\beta : [0,1] \to [0,1]$ is defined such that β is non-decreasing, continuous, $\beta(\omega) > 0$ for $\omega > 0$ and $\beta(\omega) = \omega$ if $\omega = \{0,1\}$ and $\beta(\omega) < \omega$ for all $\omega \in (0,1)$, then E is called cyclic $(\alpha \circ \beta)$-contraction.

Theorem 2.1. *Let X and Z be subsets of $Y \neq \phi$ which are closed and $(Y, \mathcal{F}, *)$ be a fuzzy metric space, which is complete and $a * b = min\,(a, b)$. Let $E : \Delta \times \Delta \to \Delta$ and $H, W : \Delta \to \Delta$ be functions, where $\Delta = X \cup Z$ which hold following conditions:*

i. $H(\Delta) \cap W(\Delta) \supset E(\Delta \times \Delta)$,
ii. E is cyclic $(\alpha \circ \beta)$ -contraction,
iii. for all $\gamma, \omega \in X$ and $\delta, \upsilon \in Z$ and $t > 0$,

$$\beta\left(\mathcal{F}(E(\gamma, \delta), E(\omega, \upsilon), t)\right) \geq \alpha\left\{\beta\left(\begin{array}{c} \mathcal{F}(H\gamma, W\omega, t) * \mathcal{F}(H\gamma, E(\gamma, \delta), t) * \\ \mathcal{F}(H\gamma, E(\omega, \upsilon), t) * \mathcal{F}(W\omega, E(\omega, \upsilon), t) \end{array}\right)\right\},$$

iv. H, W are two continuous functions and the pairs (E, H), (E, W) are weakly commuting.

Then there exist a fixed point of E, H and W which is unique in $X \cap Z$.

Proof. Let $\gamma_0 \in X$ and $\delta_0 \in Z$ be any two arbitrary elements. From (i) condition, we get sequences $\{\gamma_r\}$, $\{\omega_r\}$ in X and $\{\delta_r\}$, $\{\upsilon_r\}$ in Z as

$$\left.\begin{array}{r} E(\gamma_r, \delta_r) = H\gamma_{r+1} = \omega_r,\ E(\delta_r, \gamma_r) = H\delta_{r+1} = \upsilon_r, \\ E(\gamma_{r+1}, \delta_{r+1}) = W\gamma_{r+2} = \omega_{r+1},\ E(\delta_{r+1}, \gamma_{r+1}) = W\delta_{r+2} = \upsilon_{r+1} \end{array}\right\}. \tag{1}$$

By taking condition (iii), we have

$$\beta\left(\mathcal{F}(\omega_{r+2},\omega_{r+1},t)\right) = \beta\left(\mathcal{F}(E(\gamma_{r+2},\delta_{r+2}),E(\gamma_{r+1},\delta_{r+1}),t)\right) \geq \alpha\left\{\beta\begin{pmatrix}\mathcal{F}(H\gamma_{r+2},W\gamma_{r+1},t)* \\ \mathcal{F}(H\gamma_{r+2},E(\gamma_{r+2},\delta_{r+2}),t)* \\ \mathcal{F}(H\gamma_{r+2},E(\gamma_{r+1},\delta_{r+1}),t)* \\ \mathcal{F}(W\gamma_{r+1},E(\gamma_{r+1},\gamma_{r+1}),t)\end{pmatrix}\right\}.$$

This implies that

$$\beta\left(\mathcal{F}(\omega_{r+2},\omega_{r+1},t)\right) \geq \alpha\left\{\beta\left(\ \mathcal{F}(\omega_{r+2},\omega_{r+1},t)*\mathcal{F}(\omega_{r+1},\omega_r,t)\ \right)\right\}.$$

Thus two cases arise, which are discuses below:
Case 1: If $\mathcal{F}(\omega_{r+1},\omega_r,t) > \mathcal{F}(\omega_{r+2},\omega_{r+1},t)$, one can have

$$\beta\left(\mathcal{F}(\omega_{r+2},\omega_{r+1},t)\right) \geq \alpha\left\{\beta\left(\mathcal{F}(\omega_{r+2},\omega_{r+1},t)\right)\right\} > \beta\left(\mathcal{F}(\omega_{r+2},\omega_{r+1},t)\right).$$

This is not possible.
Case 2: If $\mathcal{F}(\omega_{r+1},\omega_r,t) < \mathcal{F}(\omega_{r+2},\omega_{r+1},t)$, we get

$$\beta\left(\mathcal{F}(\omega_{r+2},\omega_{r+1},t)\right) \geq \alpha\left\{\beta\left(\mathcal{F}(\omega_{r+1},\omega_r,t)\right)\right\}.$$

From this, one can get
$\mathcal{F}(\omega_{r+1},\omega_r,t) > \alpha^r\left\{\beta\left(\mathcal{F}(\omega_1,\omega_0,t)\right)\right\}$.
Taking $r \to \infty$, we have $\mathcal{F}(\omega_{r+1},\omega_r,t) \to 1$.
This implies $\{\omega_r\}$ is a Cauchy sequence . One can easily get $\{\upsilon_r\}$ is a Cauchy sequence. Since completeness of X and Z gives that $\exists q \in X$ and $m \in Z$ such that

$$\omega_r \to q, \upsilon_r \to m.$$

We conclude that

$$\{E(\gamma_r,\delta_r)\} \to l,\ \{E(\delta_r,\gamma_r)\} \to m. \tag{2}$$

From condition (i), we have

$$\mathcal{F}(\omega_r,E(q,m),t) \geq \beta(\mathcal{F}(\omega_r,E(q,m),t)) \geq \alpha(\beta(\mathcal{F}(\gamma_r,q,t)))$$

As $r \to \infty$ and using property of α, one can have

$$\left.\begin{aligned} E(q,m) = q, \\ similarly\ E(m,q) = m\end{aligned}\right\}. \tag{3}$$

From (i), subsequences of $\{E(\gamma_r,\delta_r)\}$ converges to same limit. This shows that $\{H\gamma_r\} \to q$ and $\{W\gamma_r\} \to q$.
In the same way , we have

$$\left.\begin{aligned} \{H\delta_r\} \to m, \\ \{W\delta_r\} \to m\end{aligned}\right\}. \tag{4}$$

This implies

$$HE(\gamma_r,\delta_r) \to Hq,\ HE(\delta_r,\gamma_r) \to Hm$$

and

$$H^2\gamma_r \to Hq,\ H^2\delta_r \to Hm.$$

From the definition of weak compatibility for the pair (E,H), we obtain

$$\mathcal{F}(E(H\gamma_r,H\delta_r),H(E\gamma_r,E\delta_r),t) \geq \mathcal{F}(E(\gamma_r,\delta_r),H\gamma_r,t).$$

By taking $r \to \infty$, we obtain

$$\left.\begin{array}{r} E(H\gamma_r, H\delta_r) \to Hq, \\ E(H\delta_r, H\gamma_r) \to Hm \end{array}\right\}. \tag{5}$$

Again using condition (iii), and as $r \to \infty$, one can obtain

$$Hq = q, Hm = m. \tag{6}$$

Now assuming (iv) condition, we get

$$WE(\gamma_r, \delta_r) \to Wq, \ WE(\delta_r, \gamma_r) \to Wm$$

and

$$W^2\gamma_r \to Wq, \ W^2\delta_r \to Wm.$$

From the definition of weak compatibility for the pair (E, W), we have

$$\mathcal{F}(E(W\gamma_r, W\delta_r), W(E\gamma_r, E\delta_r), t) \geq \mathcal{F}(E(\gamma_r, \delta_r), W\gamma_r, t).$$

By taking $r \to \infty$, we obtain

$$\left.\begin{array}{r} E(W\gamma_r, W\delta_r) \to Wq, \\ E(W\delta_r, W\gamma_r) \to Wm \end{array}\right\}. \tag{7}$$

Again using condition (iii), we get

$$\beta\left(\mathcal{F}(E(W\gamma_r, W\delta_r), E(\gamma_r, \delta_r), t)\right) \geq \alpha \left\{ \beta \left(\begin{array}{c} \mathcal{F}(WW\gamma_r, W\gamma_r, t)* \\ \mathcal{F}(WW\gamma_r, E(W\gamma_r, W\delta_r), t)* \\ \mathcal{F}(WW\gamma_r, E(\gamma_r, \delta_r), t)* \\ \mathcal{F}(W\gamma_r, E(\gamma_r, \delta_r), t) \end{array} \right) \right\}.$$

As $r \to \infty$, one can obtain

$$Wq = q, \ Wm = m. \tag{8}$$

From (3), (6) and (8), we obtain

$$E(q, m) = Hq = Wq = q, E(m, q) = Hm = Wm = m.$$

This show that functions E, H and W have common coupled fixed point. we will assert that $q = m$. Let suppose that $q \neq m$, then we can assume $\mathcal{F}(q, m, t) \neq 1$
With the help of (iii) condition, we have

$$\beta\left(\mathcal{F}(q, m, t)\right) > \beta\left(\mathcal{F}(q, m, t)\right).$$

This is a contradiction, therefore $q = m$ and

$$E(q, q) = Hq = Wq = q. \tag{9}$$

Since $X \cap Z \neq \phi$ and from (9), it follows that $q \in X \cap Z$. This show that there is a unique common fixed point of mappings E, H and W in $X \cap Z$ with help of condition (iii) of this theorem. □

Corollary 2.2. *Let X and Z be two subsets of $Y \neq \phi$ which are closed and $(Y, \mathcal{F}, *)$ be a complete fuzzy metric space. Let $E : \Delta \times \Delta \to \Delta$ and $H : \Delta \to \Delta$ be two functions, where $\Delta = X \cup Z$ which hold following conditions as:*

i. $E(\Delta \times \Delta) \subset H(\Delta)$,

ii. E is cyclic $(\alpha \circ \beta)$-contraction,
iii. for all $\gamma, \omega \in X$ and $\delta, \upsilon \in Z$ and $t > 0$,

$$\beta\left(\mathcal{F}(E(\gamma,\delta),E(\omega,\upsilon),t)\right) \geq \alpha\left\{\beta\left(\begin{array}{c}\mathcal{F}(H\gamma,H\omega,t)*\mathcal{F}(H\gamma,E(\gamma,\delta),t)*\\ \mathcal{F}(H\gamma,E(\omega,\upsilon),t)*\mathcal{F}(H\omega,E(\omega,\upsilon),t)\end{array}\right)\right\},$$

iv. H is a continuous function and the pair (E,H) is weakly commuting.

Then E, H and W have a unique fixed point in $X \cap Z$.

Proof. By assuming $H = W$ in Theorem 2.1, we get above result. □

The next result is as generalization of Theorem 2.1. we are considering finite numbers of closed subset of non empty set Y. The proof of following result follows the same method as in Theorem 2.1.

Theorem 2.3. *Let $X_1, X_2, \cdots, X_n$ be closed subsets of $Y \neq \phi$ and $(Y, \mathcal{F}, *)$ be a fuzzy metric space, which is complete. Let $E : \Delta \times \Delta \to \Delta$ and $H, W : \Delta \to \Delta$ be two functions, where $\Delta = \cup_{i=1}^{n} X_i$ which hold following conditions such as: for all $t > 0$,*

i. $E(\Delta \times \Delta) \subset H(\Delta) \cap W(\Delta)$,
ii. E is cyclic $(\alpha \circ \beta)$ -contraction,
iii. for all $\gamma, \omega \in X_i$ and $\delta, \upsilon \in X_{i+1}$

$$\beta\left(\mathcal{F}(E(\gamma,\delta),E(\omega,\upsilon),t)\right) \geq \alpha\left\{\beta\left(\begin{array}{c}\mathcal{F}(H\gamma,W\omega,t)*\mathcal{F}(H\gamma,E(\gamma,\delta),t)*\\ \mathcal{F}(H\gamma,E(\omega,\upsilon),t)*\mathcal{F}(W\omega,E(\omega,\upsilon),t)\end{array}\right)\right\}$$

iv. H, W are two continuous function and the pairs (E,H) and (E,W) are weakly commuting.

Then E, H and W have a unique fixed point in $\cap_{i=1}^{n} X_i$.

Theorem 2.4. *Let X and Z be subsets of $Y \neq \phi$ which are closed and $(Y, \mathcal{F}, *)$ be a fuzzy metric space, which is complete. Let $E : \Delta \times \Delta \to \Delta$ be a functions, where $\Delta = X \cup Z$ which satisfies following conditions such as:*

i. $E : \Delta \times \Delta \to \Delta$ is cyclic $(\alpha \circ \beta)$ contraction,
ii. a cyclic representation as $\Delta = X \cup Z$ w.r.t E .

Then there is a unique fixed point of E in $X \cap Z$.

Proof. Let $\gamma_0 \in X$ and $\delta_0 \in Z$ be elements and let sequences $\{\gamma_r\}$ and $\{\delta_r\}$ be defined as

$$\gamma_{r+1} = E(\gamma_r, \delta_r), \ \ \delta_{r+1} = E(\delta_r, \gamma_r) \tag{10}$$

for some $r \geq 0, \gamma_r \in X$ and $\delta_r \in Z$.
From (i), we have

$$\begin{aligned}\beta(\mathcal{F}(\gamma_r,\gamma_{r+1},t)) &= \beta(\mathcal{F}(E(\gamma_{r-1},\delta_{r-1}),E(\gamma_r,\gamma_r),t))\\ &\geq \alpha(\beta(\mathcal{F}(\gamma_{r-1},\gamma_r,t))).\end{aligned}$$

Using induction, we get

$$\mathcal{F}(\gamma_r,\gamma_{r+1},t) \geq \beta(\mathcal{F}(\gamma_r,\gamma_{r+1},t)) \geq \alpha^r(\beta(\mathcal{F}(\gamma_0,\gamma_1,t)))$$

For any $q > 0$, we have

$$\mathcal{F}(\gamma_r, \gamma_{r+q}, t) \geq \alpha^r \left(\beta\left(\mathcal{F}(\gamma_0, \gamma_1, \frac{t}{q})\right)\right) * \cdots * \alpha^{r+p-1}\left(\beta\left(\mathcal{F}(\gamma_0, \gamma_1, \frac{t}{q})\right)\right).$$

By taking $r \to \infty$ and using property of α, we have $\{\gamma_r\}$ is a Cauchy sequence .

From (i), we have

$$\beta(\mathcal{F}(\delta_r, \delta_{r+1}, t)) = \beta(\mathcal{F}(E(\delta_{r-1}, \gamma_{r-1}), E(\delta_r, \gamma_r), t)) \geq \alpha(\beta(\mathcal{F}(\delta_{r-1}, \delta_r, t))).$$

This implies that

$$\mathcal{F}(\delta_r, \delta_{r+1}, t) \geq \beta(\mathcal{F}(\delta_r, \delta_{r+1}, t)) \geq \alpha^r(\beta(\mathcal{F}(\delta_0, \delta_1, t))).$$

Taking $r \to \infty$, $q > 0$, we have

$$\begin{aligned}\mathcal{F}(\delta_r, \delta_{r+q}, t) &\geq \mathcal{F}(\delta_r, \delta_{r+1}, \frac{t}{q}) * \mathcal{F}(\delta_{r+1}, \delta_{r+2}, \frac{t}{q}) * \cdots * \mathcal{F}(\delta_{r+q-1}, \delta_{r+q}, \frac{t}{q})\\ &\geq \alpha^r \left(\beta\left(\mathcal{F}(\delta_0, \delta_1, \frac{t}{q})\right)\right) * \alpha^{r+1}\left(\beta\left(\mathcal{F}(\delta_0, \delta_1, \frac{t}{q})\right)\right) *\\ &\cdots * \alpha^{r+p-1}\left(\beta\left(\mathcal{F}(\delta_0, \delta_1, \frac{t}{q})\right)\right).\end{aligned}$$

So, we have $\{\delta_r\}$ is a Cauchy sequence .
So, completeness of Δ implies that

(11) $$\gamma_r \to \gamma \text{ and } \delta_r \to \delta,$$

where $\gamma, \delta \in \Delta$.
Since Δ is complete, we have subsequence of $\{\gamma_r\}$ and $\{\delta_r\}$ which converges to γ, δ, respectively. Thus implies that $\gamma, \delta \in V$, where $V = X \cap Z$. It follows that V is closed and complete.
Now restrict the function E to V as it can denote as $E : V \times V \to V$. By using cyclic representation of E, for $\gamma_r \in X$ we can have δ_r such that $E(\gamma_r, \delta_r) \in Z$.
Considering the definition of fuzzy metric space and condition (i) we obtain

$$\mathcal{F}(\frac{E}{V}(\gamma, \delta), \gamma, t) \geq \alpha\left(\beta\left(\mathcal{F}(\gamma, \gamma_{r+1}, \frac{t}{2})\right)\right) * \alpha\left(\beta\left(\mathcal{F}(\gamma, \gamma_{r+1}, \frac{t}{2})\right)\right).$$

This gives $E(\gamma, \delta) = \gamma \in V$.
In same way by using cyclic representation of E, for $\delta_r \in Z$, we can have X such that $E(\delta_r, \gamma_r) \in X$.
From the concept of fuzzy metric space and condition (i) we obtain

$$\mathcal{F}(E(\delta, \gamma), \delta, t) \geq \alpha\left(\beta\left(\mathcal{F}(\delta, \delta_{r+1}, \frac{t}{2})\right)\right) * \alpha\left(\beta\left(\mathcal{F}(\delta, \delta_{r+1}, \frac{t}{2})\right)\right).$$

This gives $\frac{E}{V}(\delta, \gamma) = \delta \in V$.

$$\beta(\mathcal{F}(\gamma, \delta, t)) \geq \beta(\mathcal{F}(\gamma, \delta, t))$$

This implies that $\gamma = \delta$. Hence we proved that E has a common fixed point which is unique in $V = X \cap Z$ with help of condition (i) of this theorem. □

The next result can be prove with help of above theorem.

Theorem 2.5. *Let $X_1, X_2, \cdots, X_n$ be subsets of $Y \neq \phi$ which are closed and $(Y, \mathcal{F}, *)$ be a fuzzy metric space which is complete. Let $E : \Delta \times \Delta \to \Delta$ be a function, where $\Delta = \cup_{i=1}^{n} X_i$ which follows following conditions such as:*

i. $E : \Delta \times \Delta \to \Delta$ is cyclic $(\alpha \circ \beta)$ -contraction,

ii. a cyclic representation as $\Delta = \cup_{i=1}^{n} X_i$ w.r.t E .

Then there is a unique coupled fixed point of E in $\cap_{i=1}^{n} X_i$.

Theorem 2.6. *Let X and Z be subsets of nonempty set Y which are closed and $(Y, \mathcal{F}, *)$ be a fuzzy metric space, which is complete. Let $E : \Delta \times \Delta \to \Delta$ and $H : \Delta \to \Delta$ be two functions, where $\Delta = X \cup Z$ which hold following conditions such as:*

i. $\forall t > 0$

$$\beta\left(\mathcal{F}(E(\gamma,\delta), H\omega, t)\right) \geq \alpha \left\{ \beta \left(\begin{array}{c} \mathcal{F}(\gamma,\omega,t)* \\ \mathcal{F}(\gamma, E(\gamma,\delta), t)* \\ \mathcal{F}(\gamma, H\gamma, t) \end{array} \right) \right\},$$

ii. E and H are cyclic $(\alpha \circ \beta)$ - contraction ,

iii. a cyclic representation as $\Delta = X \cup Z$ w.r.t E and H.

Then there is a unique fixed point of E and H in $X \cap Z$.

Proof. Let $\gamma_0 \in X$ and $\delta_0 \in Z$ be any two elements and let sequences $\{\gamma_r\}$ and $\{\delta_r\}$ be defined as

$$\left.\begin{array}{r} \gamma_{r+1} = E(\gamma_r, \delta_r),\ H\gamma_{r+1} = \gamma_{r+2}, \\ \delta_{r+1} = E(\delta_r, \gamma_r),\ H\delta_{r+1} = \delta_{r+2} \end{array}\right\}. \tag{12}$$

for some $r \geq 0, \gamma_r \in X$ and $\delta_r \in Z$.
From (i), we have

$$\begin{aligned} \beta(\mathcal{F}(\gamma_r, \gamma_{r+1}, t)) &= \beta(\mathcal{F}(E(\gamma_r, \delta_r), H\gamma_{r-1}), t)) \\ &\geq \alpha \left(\beta \left(\begin{array}{c} \mathcal{F}(\gamma_r, \gamma_{r-1}, t)* \\ \mathcal{F}(\gamma_r, E(\gamma_r, \delta_r), t)* \\ \mathcal{F}(\gamma_r, H\gamma_r, t) \end{array} \right) \right), \end{aligned}$$

As $r \to \infty$, we get

$$\lim_{r\to\infty} \mathcal{F}(\gamma_r, \gamma_{r+1}, t) = 1. \tag{13}$$

So, $\{\gamma_r\}$ is a Cauchy sequence. Also, $\{\delta_r\}$ is a Cauchy sequence.

$$\left.\begin{array}{r} \gamma_r \to c \in X, \\ \delta_r \to m \in Z \end{array}\right\}. \tag{14}$$

Since E is cyclic $(\alpha \circ \beta)$ - contractions.

$$\mathcal{F}(\gamma_r, E(c,m), t) \geq \beta\left(\mathcal{F}(E(\gamma_{r-1}, \delta_{r-1}), E(c,m), t)\right).$$

Considering $r \to \infty$, we get

$$\left.\begin{aligned} c = E(c,m) \\ m = E(m,c) \end{aligned}\right\}. \tag{15}$$

Also, H is cyclic $(\alpha \circ \beta)$ - contractions, which gives

$$\mathcal{F}(\gamma_r, Hc, t) \geq \alpha\left(\beta\left(\mathcal{F}(\gamma_{r-1}, c, t)\right)\right).$$

Considering $r \to \infty$, we get $c = Hc$. In the same way, $m = Hm$. This gives $c = E(c,m) = Hc$ and $m = E(m,c) = Hm$, which implies that E and H have common coupled fixed point.
Now,

$$\beta(\mathcal{F}(c,m,t)) \geq \alpha\left\{\beta\left(\mathcal{F}(c,m,t) * \mathcal{F}(c,E(c,m),t) * \mathcal{F}(c,Hc,t)\right)\right\}.$$

This gives $c = E(c,c) = Hc$, which shows that E and H have common fixed. Since $X \cap Z = \phi$ and from above, it follows that $c \in X \cap Z$.
This show that mappings E and H have unique common fixed point in $X \cap Z$ with help of condition (i) of this theorem. □

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